Generalized Yamaguchi correlation factor for coherent quadratic phase speckle metrology systems with an aperture

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In speckle-based metrology systems, a finite range of possible motion or deformation can be measured. When coherent imaging systems with a single limiting aperture are used in speckle metrology, the observed decorrelation effects that ultimately define this range are described by the well-known Yamaguchi correlation factor. We extend this result to all coherent quadratic phase paraxial optical systems with a single aperture and provide experimental results to support our theoretical conclusions.

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Speckle is the name given to the grainy interference pattern observed when an optically rough target is illuminated with coherent light. Speckle interferometry and speckle photography (SP) can be used to determine the deformation of a loaded body.[1–4] Using SP, surface translation[1–4] and tilting[2,4–6] can be estimated.

In this Letter, we assume that every speckle field is fully developed and tends toward a complex Gaussian process.[7–9] Typically, speckle fields incident on a detector are first processed by some optical system. The behavior of this optical system will limit the measurement range and sensitivity. A diffraction-limited 4-f imaging system with an aperture in the Fourier plane can be considered as the concatenation of two optical Fourier transform[9–11] (OFT) modules producing an inverted image. A small in-plane translation of ξ and/or a tilting of κ leads to a field at the aperture plane that is identical to the field before motion apart from a spatial shift of κ and a linear phase term. It is important to note that only the field passing through the aperture can contribute to the image field distribution. The effect of the spatial shift κ is to remove some of the field from within the aperture area that is common to both distributions. This effect leads directly to a decorrelation between the two fields (prior to, and after motion) incident on the camera in the output plane. This result is commonly referred to as the Yamaguchi correlation factor and is stated mathematically in Eq. (11) of Ref. 1.

Designing metrology systems using the linear canonical transform (LCT) has many advantages both simplifying the analysis and allowing increased flexibility and greater control over the range and sensitivity of measurements.[4,5,9] Thus it would be useful to extend the applicability of the Yamaguchi correlation factor to these more general coherent paraxial optical quadratic phase systems, which can be described using a LCT.[9] Using Ref. 12, we define the 1D LCT as

$$u_a(x_a) = \left(\frac{1}{\sqrt{j\lambda B}}\right)\int_{-\infty}^{\infty} u(x_o) \times \exp\left[\frac{j\pi}{\lambda B}(Dx_a^2 + Ax_o^2 - 2x_a x_o)\right]\mathrm{d}x_o, \quad (1a)$$

$$\begin{pmatrix} x_a \\ k_a \end{pmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}\begin{pmatrix} x_o \\ k_o \end{pmatrix}, \quad (1b)$$

where x_a indicates the domain into which the function $u(x_o)$ is transformed, and λ is the wavelength of the illuminating light. The second-order statistics of the speckle field $u(x_o)$ immediately after the rough surface is assumed to be stationary and white.[1] The first term in the parentheses, Eq. (1a), is neglected now. The effect of this mapping between the input signal (spatial coordinate, x_o; spatial frequency coordinate, k_o) and output signal, (x_a, k_a), can be described using the $ABCD$ matrix notation of Collins.[13] In this matrix, $AD-BC=1$, indicating a lossless affine transformation. It is important to note that the signal $u_a(x_a)$ is, in general, in a mixed spatial–spatial frequency domain. A more general linear transform exists, which includes a fixed position or phase shifts and thus describes offsets with respect to the optical axis and the effects of gratings and prisms.[14,15] We decompose the complete metrology system matrix, into two separate $ABCD$ matrices that represent two separate LCTs,[13] M1 $\{A1, B1, C1, D1\}$, describing propagation from the input plane to the aperture plane, and $M2$ $\{A2, B2, C2, D2\}$, describing propagation from the aperture to the output plane (see Fig. 1). We assume that any optical elements, e.g., lens, are infinite in extent and attribute any losses in the system to the single limiting aperture. We note that, although Fig. 1 depicts a particular LCT system that is used later, the results apply to any LCT system.[9] If the optically rough surface is translated by an amount ξ and rotated through an angle α, then the

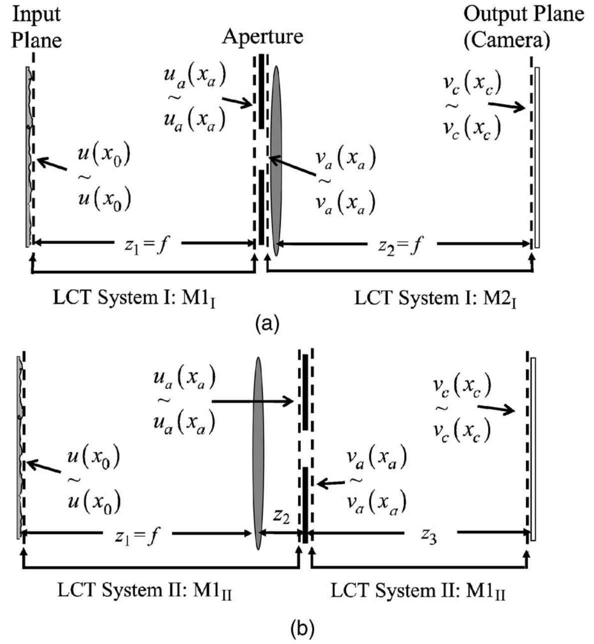

Fig. 1. Optical setup for (a) System I, (b) System II.

altered field may be represented by[5,6,9] $\tilde{u}(\tilde{x}_o) = u(x_o+\xi)\exp[j(2\pi/\lambda)\kappa x_o]$, where $\kappa \sim 2\alpha$, if $\alpha \ll 1$ rad. The field $u_a(x_a)$, incident on the aperture before motion, can be found by applying an LCT (defined by matrix M1) to $u(x_o)$. The field incident on the aperture after motion, $\tilde{u}_a(x_a)$ is similarly calculated.

The fields, $v_a(-)$ and $\tilde{v}_a(-)$, immediately after the aperture, prior to and following motion, respectively, are not the same, as the aperture ensures that some spatial and spatial-frequency information is eliminated. Let the aperture be described by the function $p(-)$. The undeformed and deformed fields immediately after the aperture $v_a(-)$ and $\tilde{v}_a(-)$ are then given by $v_a(-)=u_a(-)p(-)$ and $\tilde{v}_a(-)=\tilde{u}_a(-)p(-)$, respectively. The field incident on the camera before motion can be written as

$$v_c(x_c) = \int_{-\infty}^{\infty} v_a(x_a)\exp\left[\frac{j\pi}{\lambda B2}(D2x_c^2 + A2x_a^2 - 2x_a x_c)\right]\mathrm{d}x_a. \tag{2}$$

Similarly, we can write an expression for the field incident on the camera after the motion, $\tilde{v}_c(-)$. Following the approach taken in Ref. 1, we write the normalized covariance, $c_{I\tilde{I}}(s)$, of the intensities of the speckle field, I and $\tilde{I}$, before and after motion, respectively, as

$$c_{I\tilde{I}}(s) = \frac{|\langle v_c^*(r)\tilde{v}_c(r+s)\rangle|^2}{\sigma_I \sigma_{\tilde{I}}}, \tag{3}$$

where σ_ψ is the variance defined as $\sigma_\psi^2 = \langle\psi\rangle^2 - \langle\psi^2\rangle$, the angled brackets $\langle\,\rangle$ denote the ensemble average, and * denotes the complex conjugate. Substituting the expressions for the fields $v_a(-)$, $\tilde{v}_a(-)$, $v_c(-)$, and $\tilde{v}_c(-)$ into Eq. (5), performing some algebra, and finding the maximum value,[9] we can obtain an expression for the generalized Yamaguchi correlation factor $c_{I\tilde{I}}$, as given by

$$c_{I\tilde{I}} = \left|\frac{\int p^*(x_a)p(x_a + A1\xi + B1\kappa)\mathrm{d}x_a}{\int |p(x_a)|^2 \mathrm{d}x_a}\right|^2, \tag{4}$$

where the variable x_a indicates that the integration is occurring over the aperture plane.[9] From Eq. (4), the decorrelation in the output field is dependent on the shift of the distribution incident on the aperture, which is due to both the in-plane translation ξ, and tilting κ of the surface. It should be noted that $c_{I\tilde{I}}$ is dependent on M1 only, i.e., LCT System 1.

For a general LCT-based metrology system, Eq. (4) represents the expected decorrelation due to the aperture in the optical setup. There are, however, other factors that can contribute to decorrelation such as the camera.[3] We now outline the experimental procedure used to verify Eq. (4) using a system, which minimizes the effects at the camera.

Consider a general OFT system $\{A,B,C,D\}=\{0,f,-1/f,0\}$, where f is the focal length of the lens. Since the value of A is zero, in-plane translations of the rough surface produce no translation of the field in the output plane. However, there must be some limit to the distance we can translate the input plane after which there is no detectable correlation between the fields captured by the camera before and after motion. Since the field intensity incident on the camera should not move, the camera itself will not contribute to any resultant decorrelation. Decorrelation thus arises primarily because of the aperture.

We define two OFT systems: System I, Eq. (5a), and System II, Eq. (5b), which are identical apart from the location of the aperture [see Figs. 1(a) and 1(b)]:

$$\begin{bmatrix} 0 & f \\ -1/f & 0 \end{bmatrix} = \left\{\begin{bmatrix} 1 & z_2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix}\right\}_{M2_I} \times \begin{bmatrix} 1 & z_1 \\ 0 & 1 \end{bmatrix}_{M1_I}, \tag{5a}$$

$$= \begin{bmatrix} 1 & z_3 \\ 0 & 1 \end{bmatrix}_{M2_{II}} \left\{\begin{bmatrix} 1 & z_2 \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix}\begin{bmatrix} 1 & z_1 \\ 0 & 1 \end{bmatrix}\right\}_{M1_{II}}. \tag{5b}$$

According to matrix $M1_{\mathrm{I}}$, given in Eq. (5a), an in-plane translation of ξ (assuming no surface tilting) results in a translation of the field incident on the aperture of $\xi_a = A1_{\mathrm{I}}\xi$ where from Eq. (5a), $A1_{\mathrm{I}}=1$.

Examining System II, we see that when we multiply the component matrices of $M1_{\mathrm{II}}$, Eq. (5b) gives that $\xi_a = A1_{\mathrm{II}}\xi$ where $A1_{\mathrm{II}} = 1 - z_2/f$, and the expected rate of decorrelation is a function of z_2 and f. Clearly, different decorrelations are to be expected. To validate this prediction, we implemented Systems I and II and compared the experimental results to Eq. (4).

The CCD camera was a Sony XC ES50CE with a pixel size of $8.6\ \mu\text{m}\times 8.3\ \mu\text{m}$, a sensing area of $6.5\ \text{mm}\times 4.8\ \text{mm}$, and with no additional imaging optics. After image acquisition, the size of the grabbed image used for processing was 450×410 pixels. The correlation peak was calculated using the technique outlined in Chap. 2 of Ref. 16. The translation stage was driven with an Oriel Encoder Mike actuator, controlled using the 18113 Oriel Mike Control System. The motion stage while rated to have a positional resolution of $0.1\ \mu\text{m}$ could only be positioned with an accuracy of $\sim\pm 1\ \mu\text{m}$.[5] The target, an $8\ \text{cm}\times 4\ \text{cm}$ section of rough aluminum located perpendicular to the optical axis, was illuminated by a plane wave at an angle of 15° with light of wavelength 488 nm. System I was implemented with a lens, $f=20$ cm, and with $z_1=20$ cm, $z_2=20$ cm giving $A1_{\text{I}}=1$. The diameter of the circular aperture was $l=5$ mm. The input plane was then moved over $0<\xi<5$ mm in steps of $250\ \mu\text{m}$. In System II, $z_1=20$ cm, $z_2=5$ cm, and $z_3=15$ cm, thus $A1_{\text{II}}=0.75$, and once again $l=5$ mm. The input plane in this case was moved over $0<\xi<6.7$ mm in steps of $250\ \mu\text{m}$. Applying Eq. (6), the predicted decorrelation in this case is given by the autocorrelation of the pupil function. For the circular aperture, the autocorrelation function can be expressed analytically using one variable[10,17] where $r_a=\sqrt{x_a^2+y_a^2}$,

$$c_{\tilde{I}\tilde{I}}(r_a)=\left[\frac{2}{\pi}\cos^{-1}\left(\frac{r_a}{l}\right)-\left(\frac{r_a}{l}\right)\sqrt{1-\left(\frac{r_a}{l}\right)^2}\right]^2. \quad (6)$$

In Fig. 2, we present some results measured for System I. It is clear that the experimental data agree with the theoretical curve produced using Eq. (6). In Fig. 3, we present the results for System II. In this case, there is no one-to-one relationship ($A1_{\text{II}}=0.75$) between the input plane translation, ξ, and the resulting shift in the aperture plane distribution, ξ_a, and so we provide two scales on the horizontal axis. In System II, the input plane has to be translated more than in System I to achieve the same amount of decorrelation. The results in Figs. 2 and 3 demonstrate our ability to change the rate of decorrelation in a controlled manner shown by varying the position

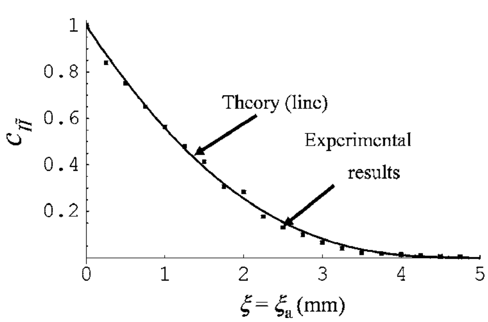

Fig. 2. Theoretical and experimental maximum correlation values as a function of input plane translation for System I. The motion of the input surface, ξ and the corresponding shift of the distribution in aperture plane, ξ_a, are identical.

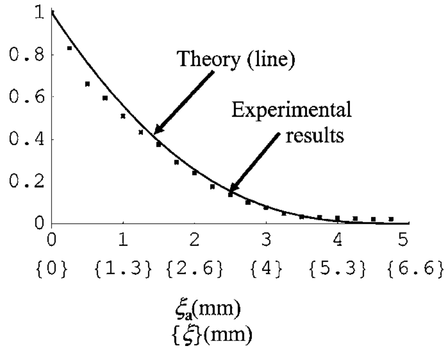

Fig. 3. Maximum correlation values for System II. The shift of the input plane is denoted as $\{\xi\}$, and the corresponding shift in the aperture plane is denoted as ξ_a.

of the aperture. Thus these experimental results confirm the validity of Eq. (4), the generalized Yamaguchi correlation factor.

Referring back to Fig. 2, it can be seen that in some cases, higher correlation values than those predicted by the theory are measured, however allowing for a worst-case error of ±0.5 mm in our estimated aperture diameter and the misalignment of optical elements, this falls well within the experimental error limits.

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