

A STUDY OF THE BENEFITS OF USING THE HISTORY OF MATHEMATICS IN TRANSITION YEAR

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In this study we designed a module of twenty lesson plans on the history of mathematics for implementation in transition year in a second level school. The module was implemented by the first author in a transition year class in an all girls school. Students provided feedback before, during and after the module and we analysed their feedback. The two main results of our qualitative analysis are that there was an instant change of all students' perception of mathematics for the better after the first lesson plan and also that there was a sustained change of all students' perception of mathematics for the better after all twenty lesson plans. This shows that the history of mathematics has a powerful immediate (and sustained) positive impact on students' perception of mathematics. We also discuss some positive changes in students' attitudes in relation to mathematics and humanity, creativity, beauty, practical power and history. We feel this paper helps to fill a gap in the current research on using the history of mathematics in second level schools in Ireland.

INTRODUCTION

The discussion about whether or not the history of mathematics should be incorporated into the teaching and learning of mathematics has been going on for many years. Some examples of suggestions on why the history of mathematics should form part of the teaching and learning of mathematics are that it may increase students' understanding and could enable them to see mathematics as a human subject (Fried, 2007; Gulikers & Blom, 2001, p. 229) and it may positively change students' and teachers' attitudes (Barbin, 2000; Lui, 2003). According to Jankvist (2009a) there are two main approaches, which are "history as a goal" and "history as a tool". He argues that there are three ways of integrating the history of mathematics into mathematics education; the illumination approach, the module approach and the history-based approach. Our research takes the form of the module approach, whereby we have designed twenty lesson plans on topics from the history of mathematics that students do not generally experience in the Irish curriculum.

According to Puig and Rojano (2004) there is "no need to further discuss the necessity or usefulness of studying the history of mathematics for mathematics education" and instead current research papers are making recommendations for "empirical investigations on the effectiveness of using history" (Jankvist, 2009b). We feel that our research helps fill the gap in this area because we analysed student feedback to our module on the history of mathematics. At second level Lit, Siu and Wong (2001) implemented an experiment over a period of three weeks in Hong Kong. The effects of teaching Pythagoras' Theorem using the history of mathematics on students in the experimental group was compared to a control group who were taught using current practise. Using statistical analysis they assessed if students' enjoyment and attitude changed after completion of the module. The results showed that the enjoyment level of the students in the experimental group rose compared to a drop in the control group. More recently a study was carried out by Jankvist (2009b) in the Danish

Upper Secondary school with 23 students. He designed two modules on “history as a goal”. The first module was on the history of error correcting codes and the second was on the history of RSA Cryptography. The module contained both the teaching material and the classroom activities for the teacher to use. The modules had a positive effect on the students. Our current research adds to the two empirical studies just mentioned above because firstly, our work took place in an Irish setting and secondly the person who implemented the module in our case was both a fully qualified second level mathematics teacher and a co-designer of the module.

The practical issues, in using the history of mathematics, for the teacher have been outlined in numerous papers (Tzanakis & Arcavi, 2000, p. 203) with the main difficulties associated with its use being the lack of teaching resources, preparation time and pedagogical knowledge on behalf of the teacher. Our project also helps to resolve these difficulties by creating many new lesson plans, along with background information on various historical topics and student activities for the teacher. There are some resources on using the history of mathematics for teaching at second level (Ó Cairbre, Watson & McKeon, 2006; Ó Cairbre, 2009).

IRISH CONTEXT

Mathematics is one of three compulsory subjects at the second level in the Irish education System. Does the curriculum include the history of mathematics? Yes, it is stated in the Junior and Leaving Certificate Syllabi that the history of mathematics is an objective of the course.

“They should be aware of the history of mathematics and hence its past, present and future role as part of our culture.” (NCCA 2000 Junior Certificate Mathematics Syllabus, Objective J, p. 4)

Currently in Ireland the NCCA, who design and publish the mathematics curriculum for all levels of the Irish Education System, are revising the mathematics syllabi. In 2008 they began implementing a trial in 24 pilot schools of a new programme known as Project Maths. The new syllabus has moved out of its pilot stage and in September 2010 began to be phased in across all secondary schools in Ireland over a 4 year period and will be completed by June 2015. The main aim of the curriculum in Project Maths is teaching for understanding. Also other objectives are to present mathematics to the student from the concrete to the abstract and to develop their problem solving skills. This new programme also refers to the history of mathematics.

“ Develop positive attitudes to mathematics as a result of being able to use mathematical methods successfully; acknowledge the beauty of form, structure and pattern in mathematics; and appreciate its history and its role in our lives.” (NCCA 2010 Project Maths Junior Certificate Syllabus, Objective F, Section 1.3.)

So why then is it not taught by many teachers? One may argue that it is never examined by the State Examinations Commission (SEC) who design the terminal examinations. However, some reports, in recent years, have highlighted the lack of pedagogical knowledge of some qualified mathematics teachers of all levels (Conway & Sloane, 2005). Ní Ríordáin and Hannigan (2009) argue that 48% of mathematics teachers surveyed in secondary schools in

Ireland were not qualified mathematics teachers according to the Teaching Council Standards. The U.S. Department of Education (2008) concluded that there is a strong correlation between students' achievement level and teachers' content knowledge.

In the Irish System there is a fourth year between the Junior and Leaving Certificate Courses. It is not a compulsory year set by the Department of Education and is often optional in many schools. This year is called Transition Year. Approximately 75% of schools provide this one year course (see the website NCCA.ie). This year is not examinable by the SEC and allows students to develop new skills and ideas about mathematics. It gives the teacher the freedom to show different aspects of mathematics and gives the perfect opportunity for students to engage with the history of mathematics. The majority of time is supposed to be spent on non syllabus material.

PROJECT OUTLINE AND METHODOLOGY

Our project had three parts:

(a) We designed a module of twenty lesson plans on a wide variety of topics from the history of mathematics for use by teachers in transition year. Each lesson plan lasted forty minutes. We also designed questionnaires and reflective journals for student feedback.

(b) The first author, who is a qualified second level mathematics teacher, implemented the twenty lesson plans to twenty-one transition year students in an all girls school. The lesson plans were implemented over a 3 month period in Spring 2010. The first author also gathered the questionnaires and the reflective journals that were filled out by the students. Each student was given a student number at the beginning of the module to protect their identity and allow them the freedom to express their thoughts and opinions without prejudice.

(c) We analysed the student feedback using Grounded Theory (Strauss & Corbin, 1990). We felt this qualitative approach, rather than a quantitative approach, was an appropriate one to obtain students' thoughts and feelings about mathematics and related issues. We had no preconceived ideas on what the feedback would produce. A motto for Grounded theory is "All is data", which means that the results in Grounded Theory should be fully justified and backed up by the data. We allowed concepts and categories to emerge. The first step in the so called coding process in Grounded Theory involves breaking down the data and then labeling the individual concepts. Then the individual categories are formed by grouping concepts that pertain to the same phenomena. Finally, the Core Category (if it exists) is the central phenomenon around which all the other categories are integrated.

In relation to (b) above, the first author used a multitude of teaching styles including discussion, chalk and talk, DVD and videos, group work, individual study and active learning methodologies. The classes were structured to bring students on a mathematical journey from a long time ago to current day applications. Most importantly the topics were chosen to show students the humanity in mathematics that seems devoid in the teaching and learning of mathematics (Rickey, 1996). The lesson plans had four main objectives:

1. Demonstrate mathematical topics as they developed through time.

2. Enable students to see the connections within mathematics and with other school subjects (music, art, science etc.) and also with the practical world.
3. Link the history of mathematics to the junior and leaving certificate curriculum.
4. Discuss the effect mathematics has on the cultural and social development of society (and vice versa).

Table 1 below gives a list of the lesson plans implemented after students completed the pre-questionnaire. Although most of the topics were chosen by the authors, it is interesting to note that some were chosen as a result of students asking to learn more on a particular area. In this way students had some ownership of the lesson plan topics and this made a very positive impression on the students.

Lesson Plan Number	Titles of Lessons
1	What does mathematics mean to you?
2	Practical power of mathematics
3	Beauty in mathematics
4	The history of number writing
5	Using old number systems and Egyptian doubling
6	Symbolism in algebra
7	Archimedes in his bath
8	Death in mathematics
9	The fly on the wall
10 & 11	Napier's bones and Donald duck in mathemagicland
12	The golden ratio
13	Fibonacci numbers
14	Pythagoras' Theorem
15	A lucky dip into mathematical history
16	The story of Pi
17	Women in mathematics
18	Two main pillars of mathematics
19 & 20	Secret gathering of mathematicians including special guest, William Rowan Hamilton

Table 1: Lesson plans implemented after students completed the pre-questionnaire

We will now give a very brief summary of two of the lesson plans. Lesson plan 14 was on Pythagoras' theorem. The class began with a discussion of Pythagoras' life, his secret society of followers and society at the time. The class then went on to look at various proofs of the theorem and the practical uses of Pythagoras' theorem. The students were then introduced to Pythagorean Triples which then led on to a discussion of Fermat's last theorem and Andrew Wiles who proved Fermat's last theorem in the 1990's.

Women in mathematics was the focus of lesson plan 17. It was a historical overview of some of the greatest female minds in the mathematical world, like Hypatia, Sophie Germain and Sonya Kovalevskaya. We discussed who they were as people, why they chose to study mathematics, the contributions they made to the field of mathematics and the impact that had on society.

Data Sources on Student Feedback

There were three main stages of data collection:

- i. Before implementation of the module.
- ii. During the module (on-going basis).
- iii. After completion of the module.

In relation to (i) a pre-questionnaire was distributed to the students without them having any prior knowledge regarding the module. Table 2 below is the list of questions each student answered when completing the pre-questionnaire. In relation to (ii) the students engaged in a continuous recording of their thoughts and opinions on the material being taught, with a reflective journal. Students recorded in their journal at the end of each lesson. The journal was used to analyse students' engagement with each individual topic and their enjoyment of the class. It also allowed students an opportunity to express their opinion on the teaching and learning of mathematics. Table 3 illustrates the questions in the student journal. In relation to (iii) the post-questionnaire (which is the exact same as the pre-questionnaire in table 2) was distributed to the students after the completion of the module.

1. What does mathematics mean to you?
2. Do you enjoy mathematics? Why?
3. Do you consider mathematics interesting? Why?
4. Do you think mathematics plays an important role in today's society? Why?
5. Would you like to study mathematics at third level?
6. Do you consider mathematics to be creative? Why?
7. Any further comments regarding mathematics?

Table 2: List of questions in pre-questionnaire

1. What did you think of this class?
2. What did you enjoy about today's class?
3. What did you learn in today's class?
4. What would you like to learn more about from today's class?
5. Were there any thought provoking moments in today's class?
6. Is there anything else you would like to write about today's class?

Table 3: Questions in the student journal

MAIN RESULTS

In our Grounded Theory approach, the Core Category that emerged was that all students' perception of mathematics changed for the better. In fact, an instant change of all students' perception of mathematics for the better, was evident from our analysis of the feedback from lesson plan 1. Also, a sustained change of all students' perception of mathematics for the better, was clear from our analysis of the responses in the post-questionnaires (compared to the pre-questionnaires).

Lesson plan 1 began with a discussion of the students' responses to the pre-questionnaires and then a discussion of what mathematics means to both authors. The students were then taken on a journey covering the evolution of number from when it was not thought of as an idea to the modern day where it is an incredibly powerful idea. The class also discussed how number can slot into infinitely many situations because it is an idea.

Instant Change

The instant change of perception mentioned above illustrates the powerful immediate impact that the history of mathematics can have on students' perception of mathematics, because after doing mathematics for about 12 years in school all the students changed their perception of mathematics for the better after just 40 minutes of lesson plan 1. Some students commented on the immediate impact after lesson plan 1 mentioned above:

Student 5: I thought the class was interesting. I learned some interesting facts that I never even thought about before even though I have been studying maths since I was 5. I never thought of the numbers like that.

Some changed their perception of mathematics so much after lesson plan 1 (LP1) that they looked forward to the next class:

Student 4: Today's class was really really interesting. It gave me a new outlook on maths as a whole and I found that I really appreciate it. History is one of my favourite subjects, and to be able to incorporate it into maths is fascinating. It makes maths more enjoyable and I'm looking forward to class next week.

Many students had previously thought numbers were just there and were not created by people:

Student 13: I thought it was interesting. I never really thought about maths in that way before. I changed how I think about maths being created. I never thought about how numbers started, I thought they were just there.

Some got quite emotional after LP 1:

Student 12: Wow! I really liked this class. It was very enlightening and I learned basic things I ever knew about maths. I loved this class.

Sustained Change

The sustained change mentioned in the Core Category above shows that all students' change of perception of mathematics for the better was sustained until after the 20 lesson plans. Quite a few students made a massive leap from thinking mathematics had no history to now realising it has a rich history:

Student 10: Mathematics means so much more to me now. I used to think that it was mainly numbers and equations and solving but it's not. There is a whole background to maths. Maths wasn't just made up on some random day. Maths was made up by amazing mathematicians over the years. (Question 1 - post-questionnaire)

Some students became more confident in their ability to do mathematics after their perception of mathematics changed for the better:

Student 17: I think that maths is a good subject and it's good to be able to work out things but I don't really understand it completely. I don't know why I'm answering the questions that I am and when I'll ever need to answer these again. Teachers don't really explain what the questions are, they just tell you how to do it and then we're expected to do this with everything in maths. I think maths is about working out difficult equations and understanding how and why you do it. (Question 1 - pre-questionnaire)

Maths means a lot more to me now than it did at the start of this course. I have a more open mind about maths. I have now realised how much we actually use maths everyday and how different our world would be without it. I'm now more aware of where maths came from and what it's all about. Maths means a lot more to me now and I'm more confident in doing honors maths next year. (Question 1 - post-questionnaire)

The following student not only changed her perception of mathematics from being a stressful subject to an interesting one but also (like some other students) greatly admired the role played by female mathematicians in history:

Student 21: STRESS!!!A compulsory subject for Leaving Cert. It's pretty boring, we've it everyday. It's a lot of hard work, it's very hard. Honors for the leaving cert. is supposed to be impossible even if you did well in your Junior Cert. I'll probably attempt honors for the start of fifth year and then drop to pass for the actual Leaving Cert. There's a lot of work involved. Teachers don't explain things properly (they usually only tell you once or twice), then if you ask them again because you don't fully understand it, they tell you that don't need to understand it - just learn it! But that's hard to do when you don't know what you're learning! (Question 1 - pre-questionnaire)

It's a subject we do in school. It can be really interesting. It's just not taught properly. There's beauty in maths and a history behind. There have been so many female mathematicians. They've fought so women can have the same education. They loved maths and thought it was fun. They did it by choice - because they had a passion for it not because they had to. (Question 1 - post-questionnaire)

Some Other Positive Impacts

The five main categories (that were all related to the Core Category above) were students' changes of attitude for the better in relation to mathematics and Humanity, History, Creativity, Beauty and Practical Power (meaning the powerful applications of mathematics to science, engineering, film making and many other areas). Here is a selection of responses related to the five categories mentioned above:

Humanity and History (quotes from the post-questionnaire):

A number of students made the big leap to now realising it is alright if they don't understand something immediately. This completely changed their approach to doing mathematics.

Student 17: I do enjoy maths now because I realised that it's okay if you don't understand something the first time it's explained to you because it took mathematicians thousands of years to understand some things that we know today.

Some students mentioned specific mathematicians and how related short stories would have enhanced their interest in mathematics in the Junior Certificate:

Student 14: I think it's good to learn about the background of maths before you learn a new topic. Take Pythagoras Theorem for example. If we learned about his life and his school and the secret society, then maybe students would be more interested in learning it. I know that it made me more interested.

A number of students now realise that mathematics is not a boring textbook anymore but it's a subject created by humans and new ideas and theories are still being created. This had a huge impact and resulted in a greater appreciation for mathematics by the students.

Student 21 : It's just an idea someone had but they turned it into a proof so now we study it. There's this amazing history behind maths that I never knew about. There's so many people behind maths. It's not just a boring textbook anymore.

Creativity (quotes from the post-questionnaire):

Student 21: It's creative because it's an abundance of ideas. People came up with maths in their heads. They then created it just like an art or music piece. It's an idea therefore it's creative.

Student 15: I do now consider maths to be creative as I now know that everything in maths was once formed by someone. It did not just appear from nowhere. It was thought up by someone.

Student 23: I consider maths extremely creative after the past few months because some of it came from nothing, it's just peoples ideas. Maths solves many problems and is created by peoples imagination.

The revelation that there can be beauty in mathematics was a very positive experience for many students. Here are some responses on LP 3 in that regard.

Beauty

Student 1: Maths can be as beautiful as a song or painting.

Student 11: I enjoyed today's class but at first I found it hard to understand the beauty maths has but as we got further into the class I began to understand it more.

Student 14: The thought provoking moment in today's class was how years ago people solved problems in maths not to find answers but purely for the beauty of maths.

Practical Power (all quotes after LP 2)

The history of mathematics gives plenty of nice examples of the powerful applications of mathematics in science and the wider society. As a result of this, many students became more aware of the important role that mathematics has had (and still has) in our society. Here are some responses by students in relation to the practical power of mathematics:

Student 17: I enjoyed finding out how cartoons need maths to be made. Without maths cartoons would still be in 2D instead of 3D. I never thought that maths would be involved in that.

Student 14 : I found this class very interesting as I learned how maths is used in different ways in everyday life. It was fun and not boring. I also found myself thinking about different things in maths and also found myself realising that areas in maths that I found boring before weren't so boring as I understood how they were used in everyday life.

Student 22: Maths is a hell of a lot more practical than it seems.

CONCLUSIONS

For all students there was an instant change of perception of mathematics for the better as a result of the first lesson plan. Also, there was a sustained change of all students' perception of mathematics for the better after completion of the twenty lesson plans on the history of mathematics. Other major positive impacts of the module were in relation to changes in students' attitude corresponding to mathematics and humanity, creativity, history, beauty and practical power. All these changes help to greatly enhance the students' awareness and appreciation of mathematics. Furthermore, teachers will now have some new resources and Teaching and Learning plans at hand for use in Project Maths. Some previous empirical studies on the use of the history of mathematics in other situations in mathematics education have also shown that it can have a positive impact on some students.

RECOMMENDATIONS

We feel that the history of mathematics would have a positive impact on students' attitudes towards mathematics in any second level year. Our resources are versatile in the sense that

they can be used as a full module or as individual lesson plans in transition year and also shorter pieces of appropriate lesson plans could be implemented separately, at the discretion of the teacher, in the Junior and Leaving Certificate Cycle. Future work could involve an analysis of student feedback to appropriate parts of our resources outside of transition year.

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