

# Exploring spatial uncertainty of GPS coordinates and DEM interpolation in virtual environments

Dr Jing Li, Dr Claire H. Jarvis, Professor Chris Brunsdon  
*Department of Geography, University of Leicester, Leicester LE1 7RH UK.*

---

## Abstract:

In this paper, we present two such examples of visualisation exploring spatial uncertainty which benefit from the use of VR technology. First, we present a real-time simulation of Global Positioning Systems (GPS) satellite geometry is carried out in a virtual environment of the University campus. The number of satellites visible to the receiver is modelled in real time as a user walks through the university campus. In the mean time, Position Dilution of Precision (PDOP) is displayed on the screen as well as the satellite geometry being visualised in both 3D and aerial views. Since the factors affecting PDOP are inherently 3-dimensional, communicating the spatial uncertainty of GPS co-ordinates within an immersive stereo environment has been viewed as a particularly powerful communication tool by both Undergraduate and Postgraduate students. Secondly, we interpolate elevations of raw LiDAR (i.e. Light Detection And Ranging) points, irregularly spaced at around 1m resolution, to generate a continuous surface model using a variety of interpolation techniques. The resulting surfaces are rendered in a real-time visual simulation environment in which navigation and user interaction are enhanced. Cross-validation results are visualised on top of the surface, while 25 centimetre aerial photos are draped over the terrain in order to provide a spatial context for examining interpolation cross-validation error. The rationale for this interpolation tool is to improve our communication of inherently spatial processes to students finding difficulties in mapping mathematical representations of surfaces and parameter options, in the setting of an interactive and immersive group discussion within our VR theatre.

---

## 1. INTRODUCTION

Real-time computer graphics have come a long way over the last decade. As with the increased CPU/GPU processing power, Virtual Reality (VR) technology has been widely used as an efficient and powerful visualisation tool in exploring spatial characteristics of geographic data [1]. It is often the case that an improved understanding of the problem at hand can be achieved in a virtual environment in which users can gain 3D immersive experiences as well as interacting with the virtual objects with minimum efforts. In this paper, we present two such examples which benefit from the use of VR technology. Firstly, a real-time simulation of Global Positioning Systems (GPS) satellite geometry is carried out in a virtual environment of the University campus. Secondly, we demonstrate the results from interpolating elevations of raw LiDAR (i.e. Light Detection And Ranging) points to a continuous surface model using a variety of interpolation techniques. Both applications are used by Faculty in their teaching of MSc GIS postgraduate students, presented in the context of an immersive VR theatre.

## 2. REAL-TIME SIMULATION OF THE GEOMETRY OF GPS SATELLITES

It is well known that accuracy and availability of GPS in urban areas can be severely compromised by limited satellite visibility owing to signal occlusion caused by nearby buildings. A minimum of four satellites is required for 3D position fixing. When the number of satellites visible to a receiver drops down to 3 or less, the GPS is unable to give a position without the use of aiding techniques. Apart from satellite visibility, the receiver-satellite geometry also affects the accuracy of GPS. Generally, a wider spacing between satellites and a receiver produces smaller errors [2].

The contribution of receiver-satellite geometry to positional errors is quantified by various Dilution of Precision (DOP) values. Among the DOP values, PDOP is commonly used to assess the strength of satellites geometry in relation to positioning accuracy. Since the concept of PDOP is well known, only a brief explanation is given here.

The solution to the linearised pseudorange equations is depicted in Equation (1)

$$\hat{x} = (A^T A)^{-1} A^T b \quad (1)$$

Where:

- $\hat{x}$  are the corrections to priori estimates of the four unknowns (i.e. the receiver position  $(x, y, z)$  and the receiver clock bias  $\tau$ )
- The matrix  $A$  contains the line of sight unit vectors and 1s.
- $b$  is the difference between the measured pseudoranges and calculated ones.
- DOP values are a function of the diagonal elements of the covariance matrix (i.e.  $(A^T A)^{-1}$ ) in a coordinate system.

Position DOP is defined in Equation (2):

$$PDOP = \sqrt{\sigma_x^2 + \sigma_y^2 + \sigma_z^2} \quad (2)$$

The total position error can therefore be estimated by  $\sigma \times PDOP$ . For example, a standard deviation of 2m in pseudorange observations would give a standard deviation of  $2 \times PDOP$ m. Thus, the smaller the PDOP, the more accurate the position would

be. As a general rule, PDOP values larger than 5 are considered poor [3].

Figure 1 is a screenshot of a frame in a real-time simulation environment. As shown in Figure 1, a person holding a PDA equipped with a GPS receiver is standing in an urban canyon environment on campus. Green rays indicate satellites visible to the receiver, red rays point to satellites blocked by the buildings on either side of the road. The aerial view rendered on the upper right corner of screen visualises azimuth and elevation angle of the satellites. Shorter rays in the aerial view indicate satellites with higher elevation angles. For example, the shortest green ray in the aerial view in Figure 1 has an elevation angle of 80 degrees above the horizon, which means that this particular satellite is almost located at the zenith of the user. The text at the bottom of the screen, which updates as the user moves across the campus environment, indicates that while there are currently eight satellites in the sky, only three of them are visible to the receiver, and therefore a positional fix can not be obtained.

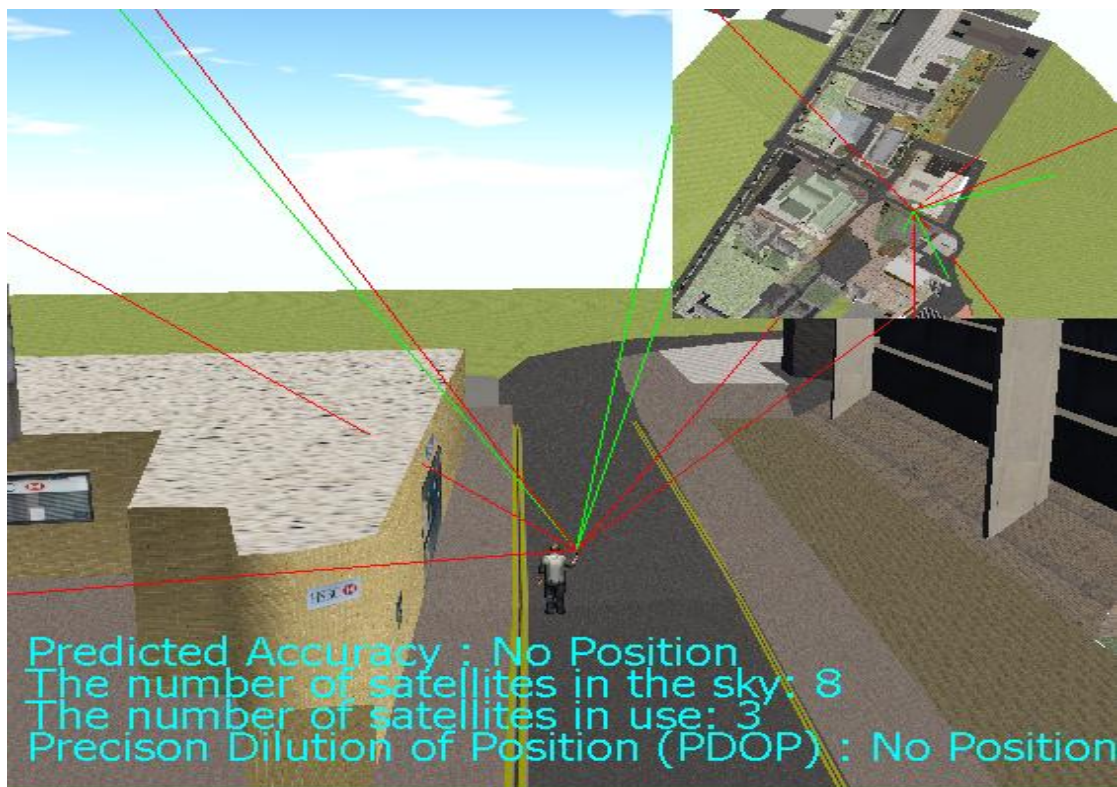


Figure 1. A screen shot of real-time simulation of satellites geometry with aerial view rendered on top of the 3D view

Figure 2 depicts a strong satellite geometry with a very small PDOP of 1.74, achieved when all the eight satellites are visible to the receiver while it is positioned in an open area. Assuming that the standard deviation in the pseudorange measurement error is 2 metres and this error is uncorrelated, the predicted position accuracy can be estimated at 2 multiplied by the PDOP, which is equal to 3.49 metres (as shown in Figure 2).

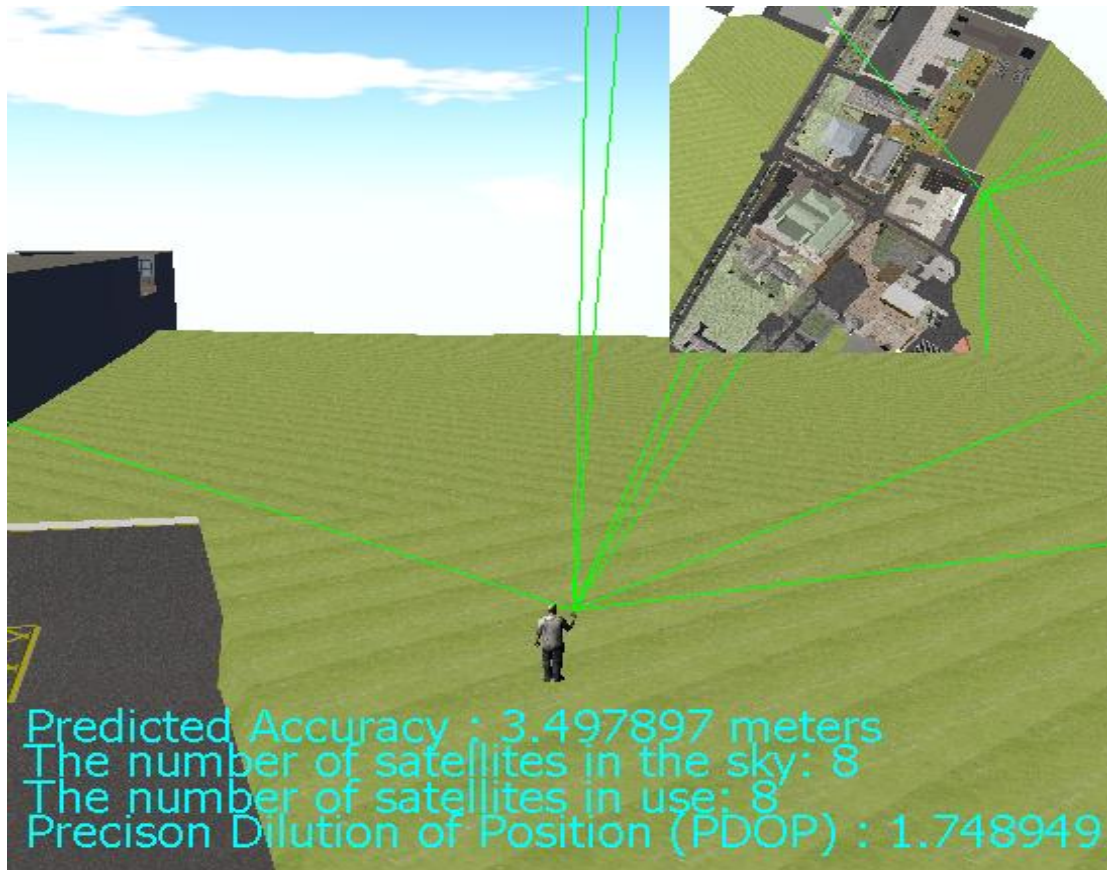


Figure 2. All the eight satellites visible from an open area, with a small PDOP of 1.74

As a rule of thumb, the more satellites that are visible, the smaller the PDOP. A smaller number of visible satellites is more likely to give a relatively larger DOP values. Figure 3 describes quite an extreme case of poor satellite geometry formed by seven visible satellites. As shown in the aerial view in Figure 3, the PDOP of 4.81 is quite large given that there are seven satellites visible to the receiver. The satellite blocked by the building (i.e. the ray in red colour) is located in north west direction relative to the user position in the aerial view, and plays a crucial role in the

Paper presented at VGE 2008, Hong Kong

calculation of PDOP at this particular moment. Without using this satellite, the receiver-satellite geometry formed by the seven satellites tends to cluster together and results in a relative larger PDOP in Figure 3, compared with the eight satellite scenario shown in Figure 2. Note that the length of the rays indicate the satellite elevation angle, and thus longer rays represent low elevation satellites, although all the satellites are around 20,000 kilometres above the earth.

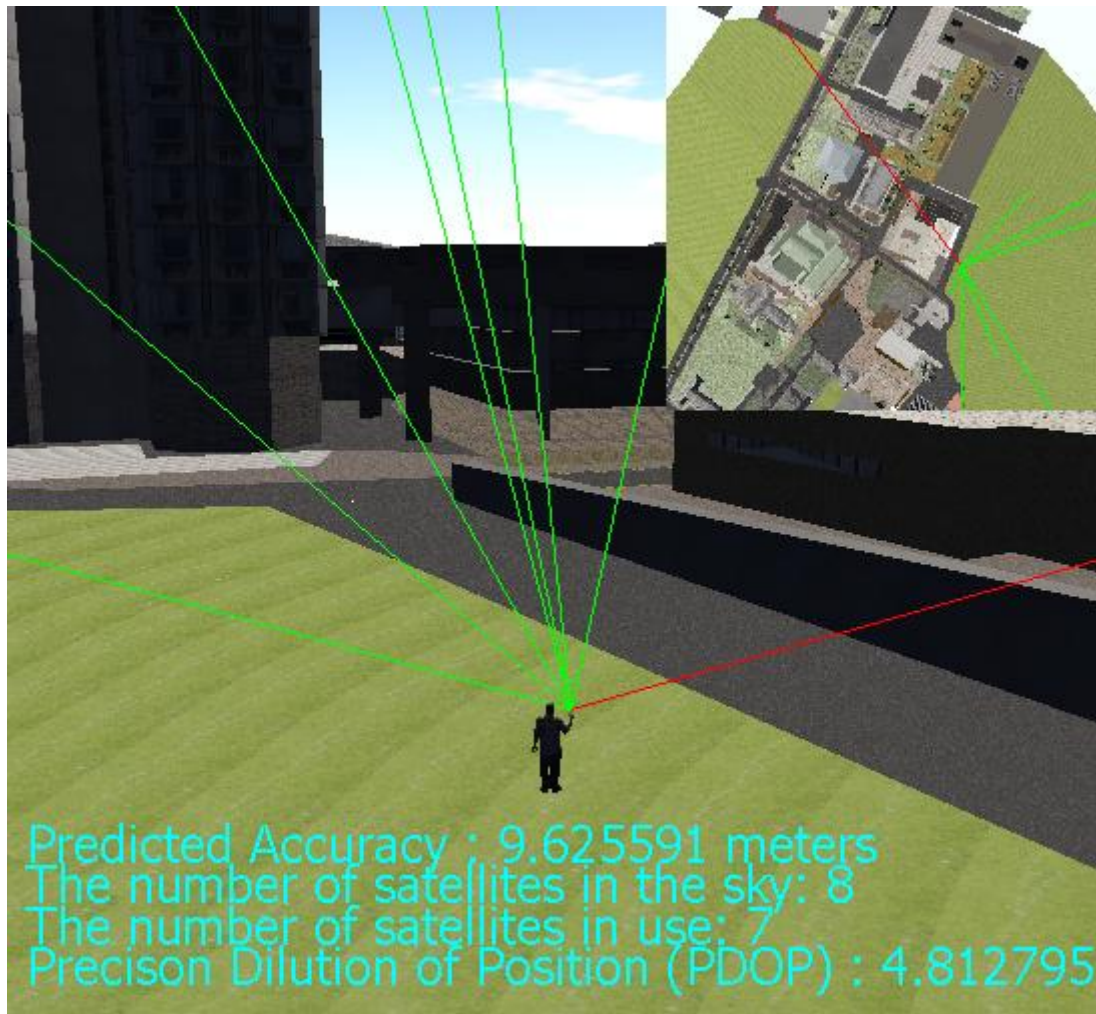


Figure 3. A relatively large PDOP, owing to the signal loss on the satellite (rendered in red).

Figure 4 demonstrates an opposing scenario, in that the loss of signal on the satellite located in the south east direction relative to the receiver position does not have much influence on the value of PDOP, as PDOP only slightly increases to 1.83 compared with a PDOP of 1.74 in Figure 2. Essentially, the lost signal is one in an area of sky

where other nearby signals are able to compensate.

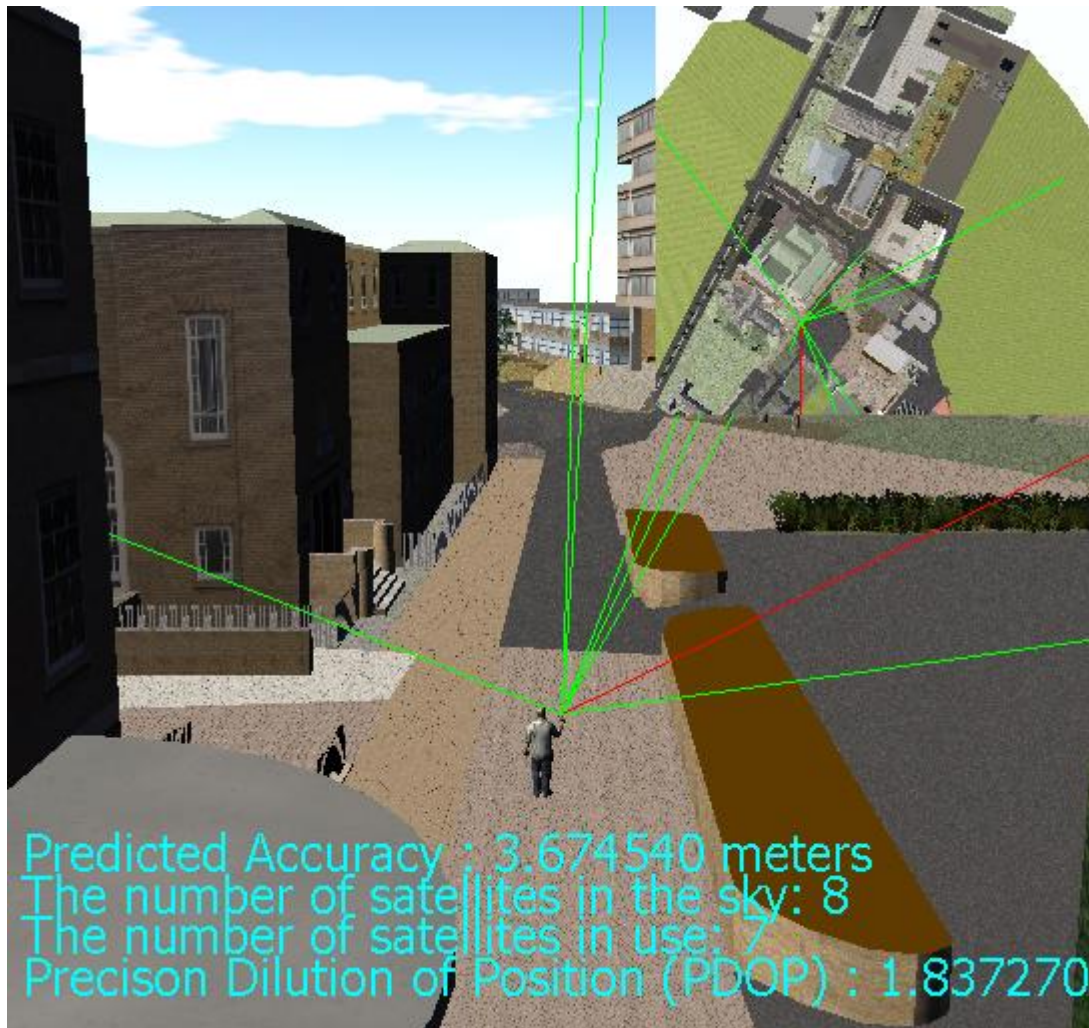


Figure 4: A relatively low PDOP, calculated from the signals of seven satellites

Figure 5 and Figure 6 demonstrate scenarios at locations where only four satellites are visible. The very large PDOP of 9.13 shown Figure 5 is obtained owing to both limited satellite visibility and poor satellite geometry. It is clear from Figure 5 that not only do three satellites have quite high elevation angles (i.e. shorter vector length), but also that the three satellites in the north-east direction have similar azimuth angles. In contrast, the satellite geometry visualised in the aerial view of Figure 6 is slightly better than that in Figure 5, and therefore the resultant PDOP is the smaller value of 5.49 compared with a PDOP of 9.13 of Figure 5. This is because the four satellites in Figure 6 are in general more scattered than those portrayed in Figure 5. This effect is

Paper presented at VGE 2008, Hong Kong

more evident by examining the top-down view of the scene, instead of the 3D view.

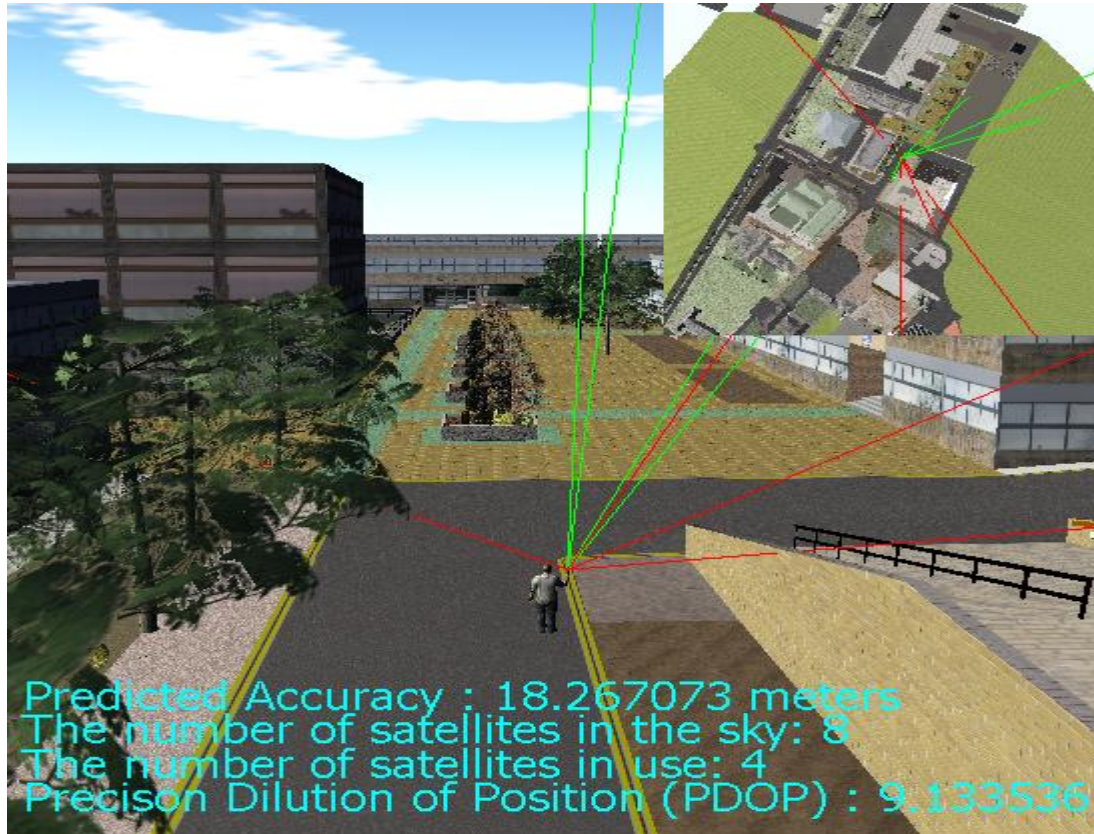
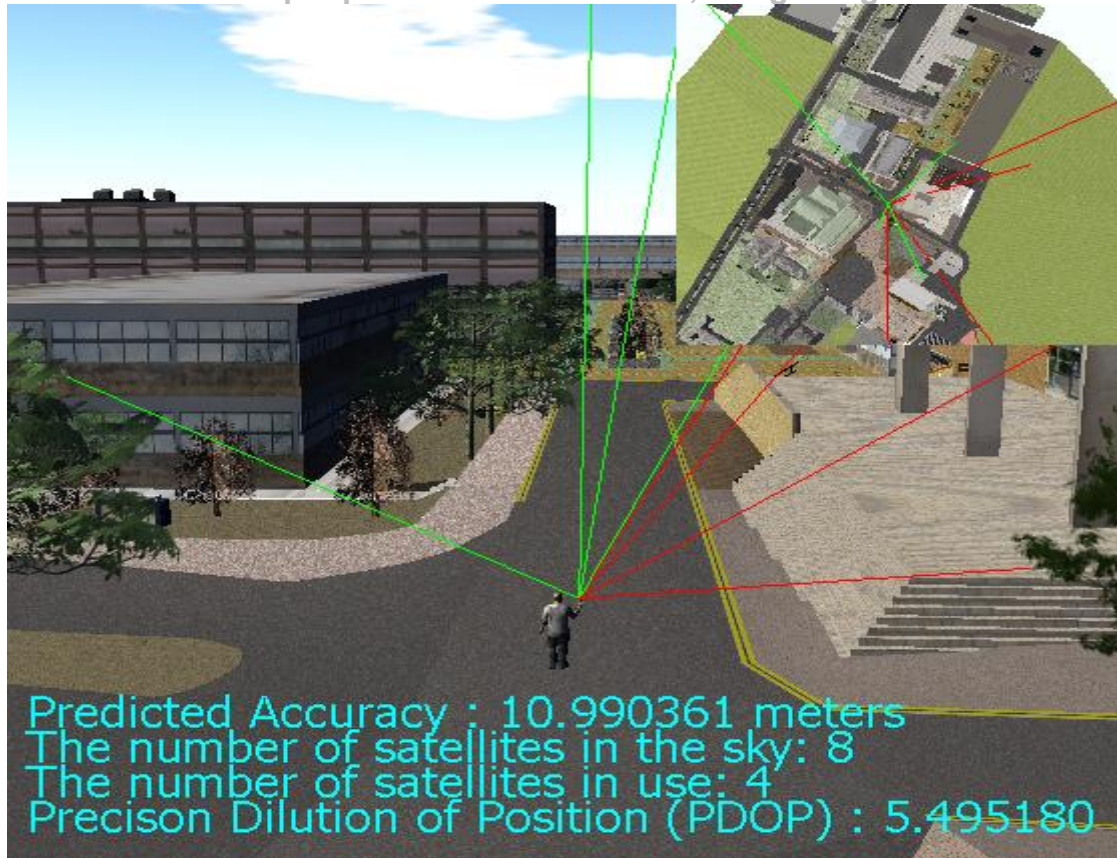


Figure 5. A very large PDOP, calculated from the signals from four highly clustered visible satellites



**Figure 6.** A relatively small PDOP, calculated from the signals from four more scattered satellites

The GPS tool was shown immediately prior to a set of practicals across campus using Bluetooth GPS and ArcMap PDA and/or rugged tablets with integrated GPS tablets for wayfinding and data recording. These exercises were embedded within the context of a two-day mobile computing skills workshop for a MSc GIS cohort of 25 students. Initial evaluations were supportive, and students commented that the GPS tool provided good instruction and definitely raised interest in GPS accuracies and geometries. The blended combination of VR demonstration and active learning facilities at the workshop appeared to allow the two teaching and learning techniques to reinforce each other well, Student A (MSc GIS) commenting that *“It’s good to know where like hotspots were, because if you did say where you are, looking at his model you could see where really bad spots, now that seemed to stay in my memory because I did go straight to [good] spots that I saw...”*. In other words, both the initial learning objectives and potential frustrations with loss of signal, frustrations that were encountered by previous students in this enclosed campus space prior to running the practical in conjunction with the VR tool as here, were avoided. Further, the



evidenced link students made between real and virtual space suggests that a sufficient degree of presence was captured in the VR tool design.

The tool was shown to the students by a demonstrator, rather than individual students interacting with the model directly in a directional, active capacity. Of this immersive but relatively passive approach, students commented that “*Actually having a go at driving it wouldn't have added to your knowledge in that kind of sense, but in a sense of being able to use the computer system to do it would have been interesting.*” Their apparent sense of appreciation that learning to navigate within the virtual environment could detract from the learning objective relating to GPS accuracy was interesting, suggesting that the inherent 3d nature of the application and its good fit to the 3d theatre environment, as opposed to the interactive argument for immersive VR, was sufficiently powerful in its own right as an argument for the approach taken. Whether the VR approach is *necessary* as an instructional method is a different question, the answer to which is likely to vary according to the ability of individual students to imagine spatial manifestations of mathematically described events.

In summary, an improved understanding of GPS availability and accuracy in urban areas can be gained by viewing a real-time simulation in an immersive virtual environment. A variety of receiver-satellite geometries can be visualised in real-time in relation to how PDOP changes with respect to surrounding geographic environments, as the human model walks through the campus. The textured human and campus models add realism to the 3D simulation and therefore can provide enhanced learning and teaching experiences in the context of higher education.

### **3. VISUALISATION OF DEM INTERPOLATION WITH LIDAR DATA**

Spatial interpolation of elevation data has long been the subject of debate in GIScience. Interpolation and surface fitting techniques such as kriging and splines have been thoroughly researched throughout the literature, and numerous experiments have been conducted in order to explore which method is most suitable in various spatial contexts (Fisher, 2007). Nevertheless, exploring the visual (rather than mathematical) differences in LiDAR Digital Surface Models (DSMs), derived from various interpolation techniques, in a virtual environment would allow one to gain an

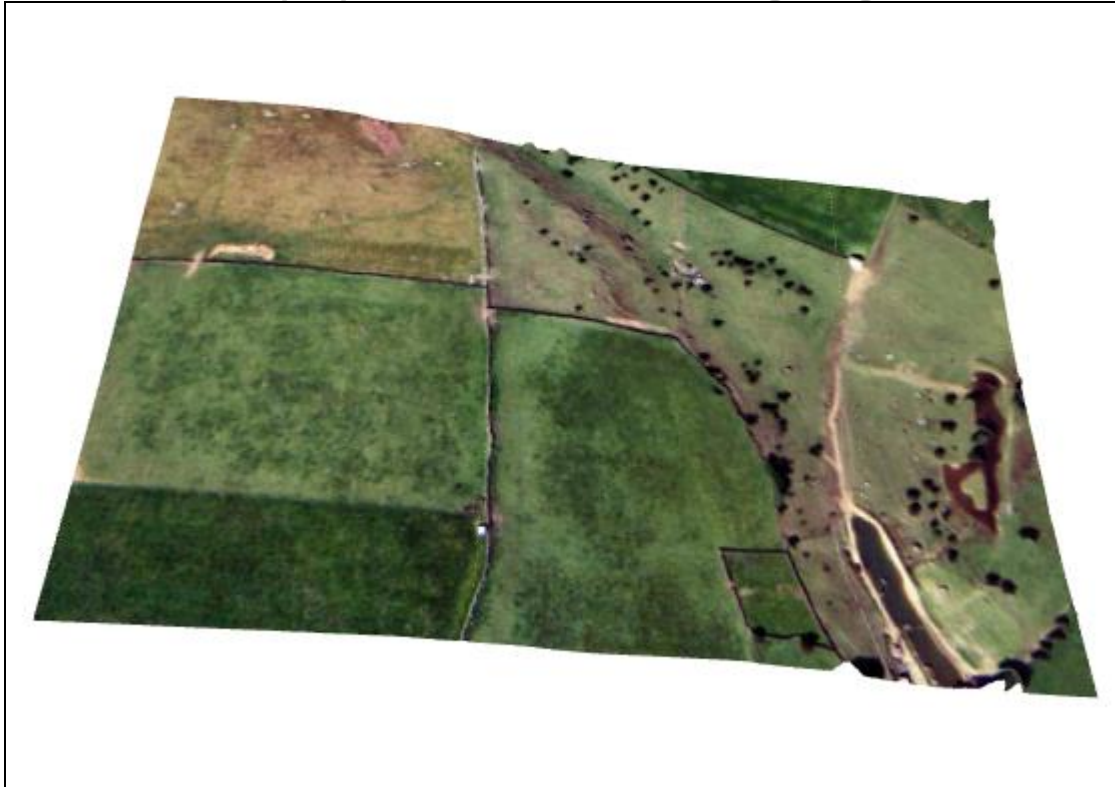
insight or improved understanding in the characteristics of these interpolation methods. This is especially important in the context of postgraduate education, as students often have difficulties in understanding the fundamental principles of spatial interpolation, as Student B comments below.

**Student B (MSc GIS), commenting on his/her learning experiences in spatial information science:**

*Spatial information science was challenging conceptually, but the practicals were not so difficult. It was very difficult conceptually, a lot of maths, a lot to get your head around, to imagine these interpolation techniques...*

The use of visualisation here is pitched at bridging the gap between mathematical formulae and spatial form, filling the gap that previously had to be ‘imagined’. With this interpolation tool, we “re-describe” academic knowledge in order to facilitate transfer at a level relating to ways of thinking (across space), particularly for those students with no geographical or mathematical background.

To provide a context, we interpolate elevations of raw LiDAR points sampled at 1m resolution to generate a number of continuous surfaces. As shown in Figure 7, the LiDAR raw points cover an area of 300m x 455m which comprises some field boundaries and vegetation. The underlying terrain has a gently rising trend from east to west. Since there are very few man-made features in the data (e.g. buildings), the expected Root Mean Square Error (RMSE) would be smaller than that in highly built-up areas, as dramatic local variations would occur around building footprints in dense urban areas, and therefore causes RMSE to increase. In this experiment, the RMSE is obtained through cross validation (CV) which estimates the heights of sample points using the remaining sample points in a 30 data point neighbourhood, and then calculates the differences between the observed and estimated heights as cross-validation error.



**Figure 7. The study area, over which the underlying data set of raw LiDAR points is irregularly sampled at around 1m resolution**

Figure 8, Figure 9 and Figure 10 visualise cross-validation results on top of the interpolated surfaces using the ‘spline with tension’ method with varying weights of the tension term. Note that only the sample points at which the cross-validation errors are larger than 1.5 standard deviation (i.e. around 0.44m) are rendered using 3D spheres of various colours in the scene. The vertical lines starting from the centre of the spheres indicate the magnitude of the cross-validation error. For example, overshoot is represented by the vertical lines shooting upwards from the spheres which are usually located at the bottom of hilly areas and therefore blocked by the terrain; these can also be viewed in wireframe mode. Undershoot occurs when the lines shoot downwards from the spheres. It is evident from Figure 8, Figure 9 and Figure 10 that the spline surfaces become progressively smoother, as the weights increase. The smallest RMSE of 0.292m is obtained with a weight of 10 as shown in Figure 9. The other two surfaces shown in Figure 8 and Figure 10 have similar RMSE values, but the appearances of the two spline surfaces are dramatically different as

Paper presented at VGE 2008, Hong Kong

shown in Figure 8 and Figure 10. This is because adopting the smoother spline surface with a weight of 10000 in Figure 10 caused large cross-validation errors to occur in more “bumpy” areas such as hills and deep slopes where local variations in height are significant, which further suggests that overshoot occurs at the bottom of more hilly areas and undershoot happens at the top of the hills across the study area. However, only undershoots are more likely to occur in less bumpy areas such as the gentle slopes of Figure 10. This visual observation agrees with theory in that a spline surface with a weight of 0.0018 in Figure 8 is more “stiff” than that in Figure 10 and therefore tends to produce both overshoot and undershoot even in less bumpy areas where only very gentle slopes are present. Overall, the RMSE can only be used a global measure of interpolation error, as it aggregates local variations in error across the study area. In Figure 8 and Figure 10, the large cross-validation errors in the two spline surfaces tend to even out owing to the fact that the two spline techniques perform differently according to the amount of local variations in height.

A second example of spatial interpolation is concerned with the sampling process after interpolation has taken place. As shown in Figure 11 and Figure 12, the two universal kriging surfaces, derived from the same geostatistical model without nugget, are sampled at 1m and 0.5m resolution respectively. It is very clear from the Figure 11 and Figure 12 that there is considerably more detail in the 0.5m grid than that in 1m grid, despite the fact that they are both derived from the same model and the 0.5 grid resolution is even finer than the original 1m raw LiDAR points. Also, the 0.5m grid is closer to the sample points. Although in theory both surfaces should go through the sample points, the final surface used for visualisation is based on the regular grid, which may not coincide with the original sample points. It can be seen that 3D visualisation can raise the awareness of the scale issues in DEM interpolation which is often lacking among postgraduates studying GIS.

To sum up, an improved understanding of the three different spline methods and scale issues in spatial interpolation have been gained through enhanced interactive real-time visualisation as apposed to the conventional desktop 3D GIS in that no windows GUIs

Paper presented at VGE 2008, Hong Kong

are required to switch on or off a particular surface and hence improve interactivity with the surfaces in real-time full screen mode. A simple keystroke allows users to flick through all the surfaces and cross-validation results currently present in the memory in a timely manner, and thus enhance memory retention as well as facilitating comparisons between different surfaces. The use of lighting, materials, colours, various types of geometries (e.g. lines and spheres), 2D texts (on the upper right hand corner of the screen) and a 'game motion' model enables user to inspect the surfaces from any viewing angles and further enhance learning experiences. The approach here differs from past work visualising interpolation results in its use of a theatre setting to tap a community approach to sense-making and data exploration and the explicit linkage by staff between mathematical descriptions and surface form in a blended manner. However, the degree to which this tool does indeed assist a wider range of students with specific spatial literacy related learning needs to transfer knowledge from theory and formula to spatially explicit application requires further testing.

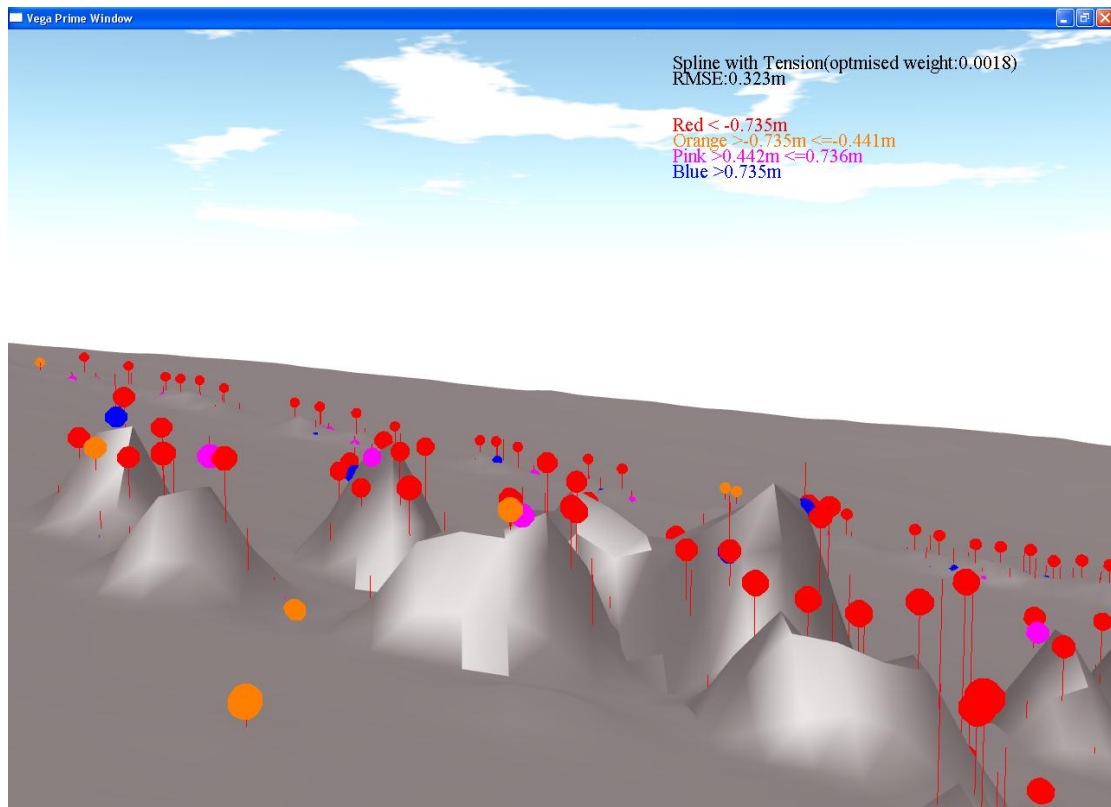


Figure 8. Cross-validation error calculated using spline with tension (RMSE: 0.323m, weight:0.0018)

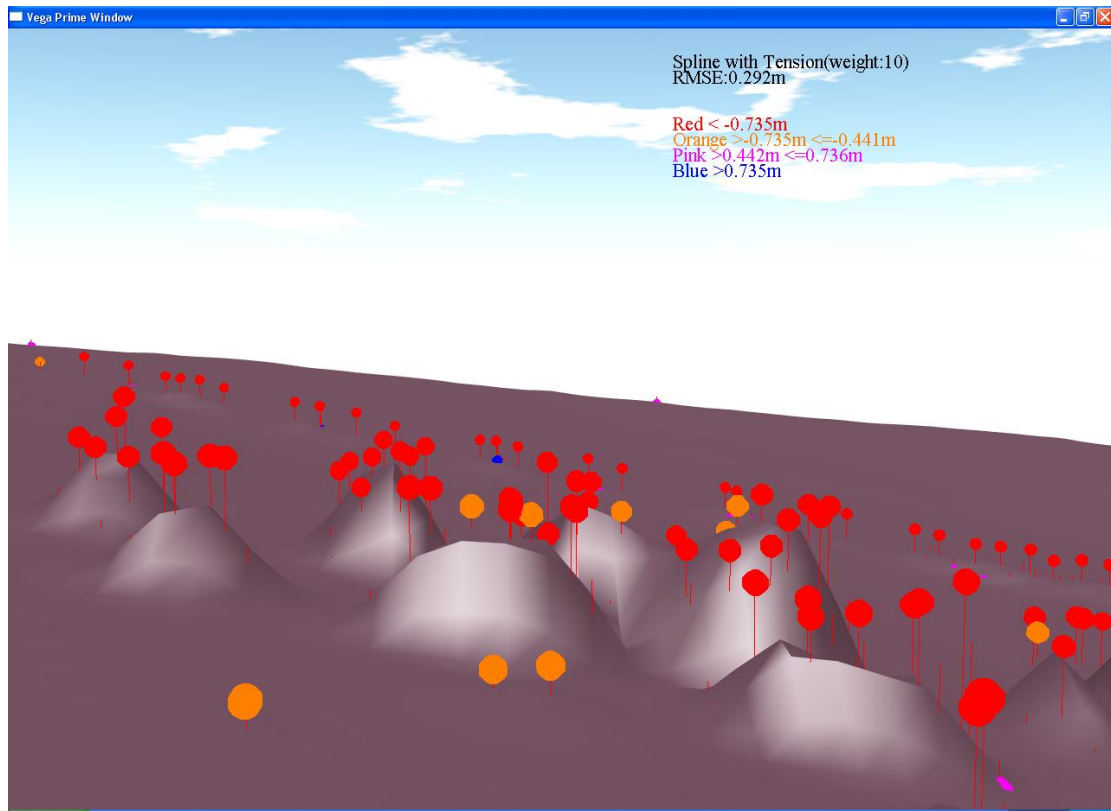


Figure 9. Cross-validation error calculated using spline with tension (RMSE: 0.292m, weight:10)

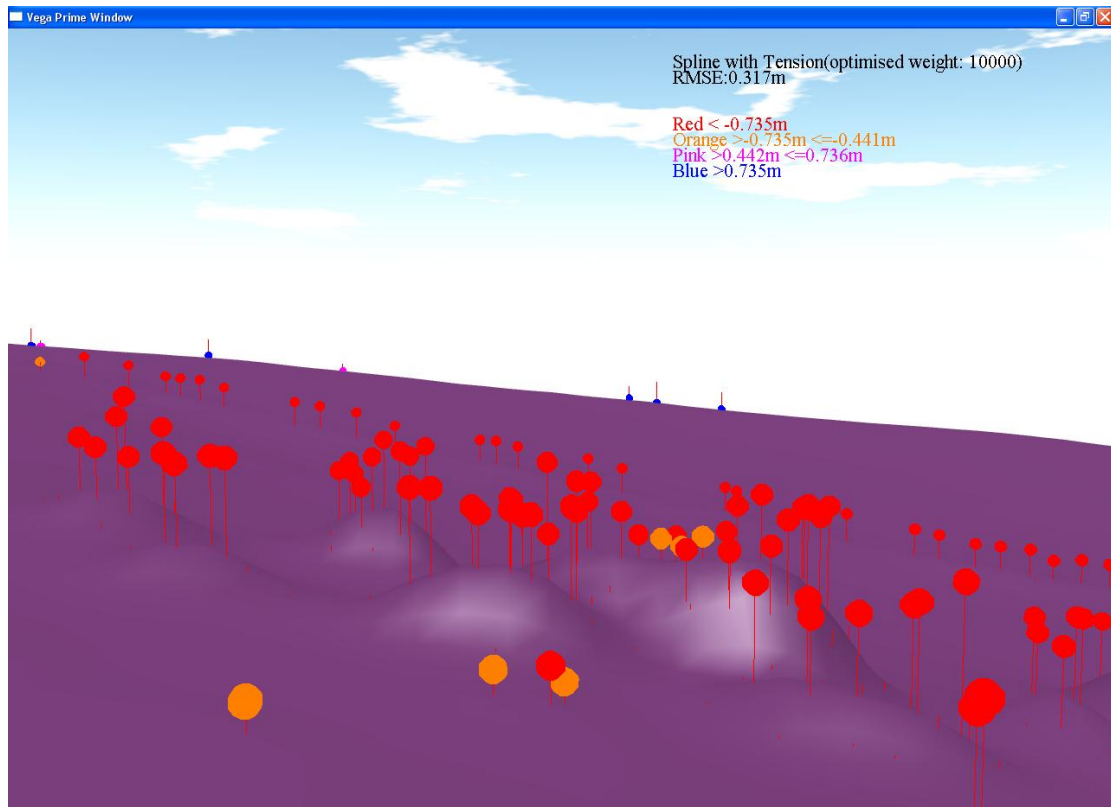


Figure 10. Cross-validation error calculated using spline with tension (RMSE: 0.317m, weight: 10000)

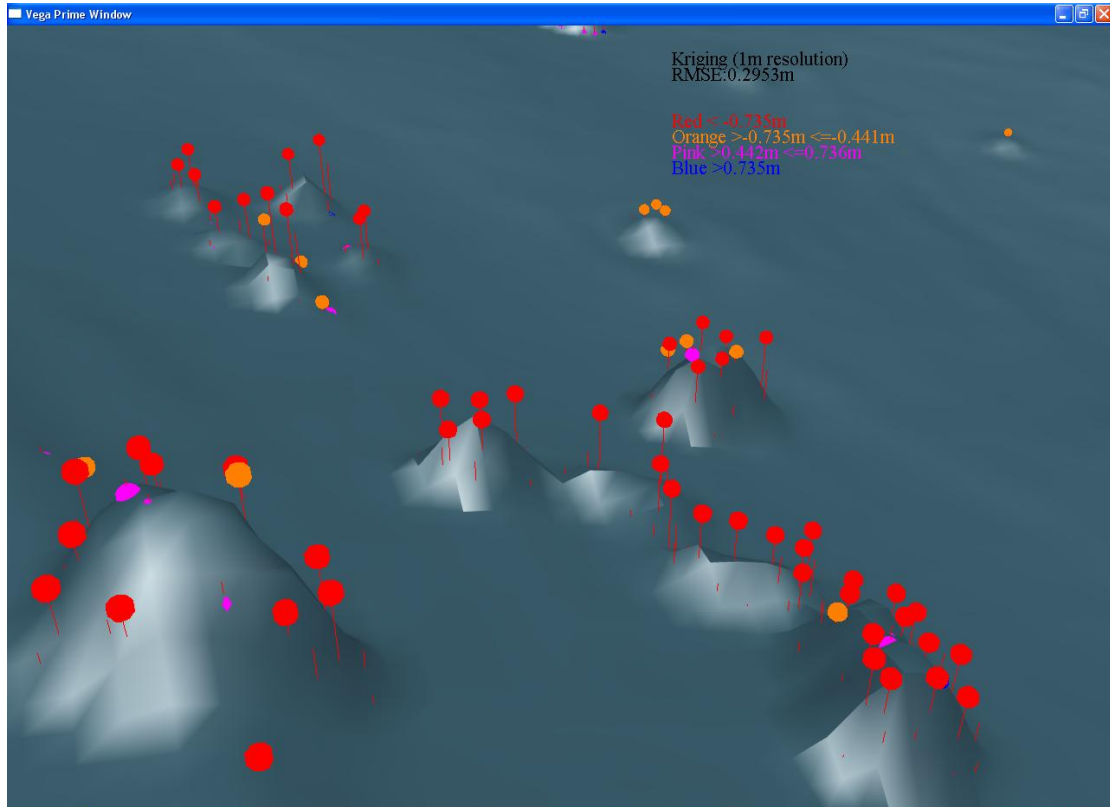


Figure 11. Cross-validation error calculated by kriging (RMSE: 0.2953m, 1m resolution)

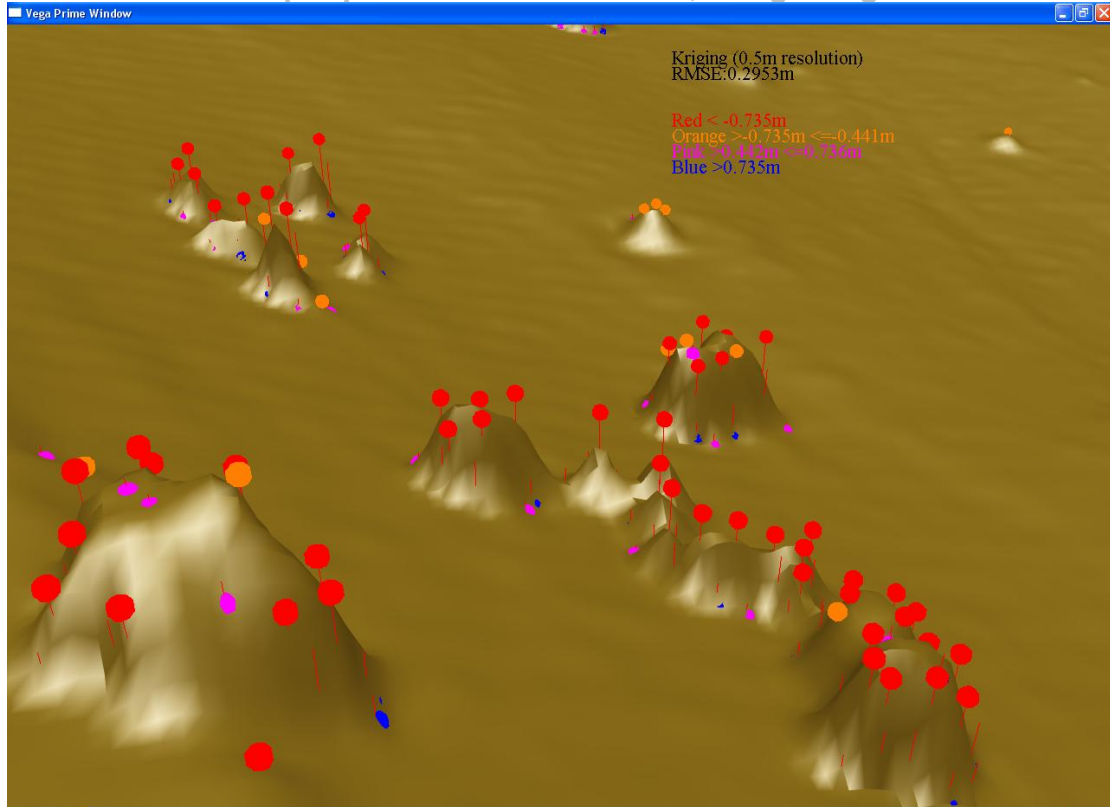


Figure 12. Cross-validation error calculated by kriging (RMSE: 0.2953m, 0.5m resolution)

#### 4. 3D IMMERSIVE VISUALISATION IN VR THEATRE

An added advantage of 3D visualisation is to create virtual environments in which users' feeling of immersion is enhanced and hence improve learning and teaching experiences. As illustrated in Figure 13, our VR setup is a 6-image generator/projector system utilising a curved semi-immersive screen which has a 160 degree field of view angle (FOV) that is equivalent to the FOV angle of human eyes. This distributed rendering system significantly reduces the computational overhead on the image generator/projector system since each PC only renders one image for either right or left eye. This is in contrast to a single machine stereo system in which one PC has to generate two separate images for the right and left eye respectively. A wider FOV angle has significant advantage over desktop 3D GIS in that users can explore a larger area of the terrain in 3D stereo and interact with virtual environments with minimum efforts.





GPS mission tool in VR theatre

Interpolation tool in VR theatre

Figure 13

## 5. CONCLUSIONS

This paper has described two examples of using 3D immersive VR technology to improve teaching and learning experiences in GIS education. It is evident that the power of 3D visualisation enables us to discover some hidden characteristics of geographic data and improve our understanding of some more abstract concepts and therefore bridge the gap between theories and human understanding. Two such examples have been presented. Firstly, the strength of arbitrary receiver-satellite geometry in urban areas is visualised both in 3D and aerial view while the value of PDOP updates. Secondly, the cross-validation errors are overlaid onto the interpolated surfaces, which provide visual cues to explore the behaviour of the different interpolation techniques across the study area. Future work will focus on further evaluation of the two teaching tools reported here and as well as developing more novel applications of 3D immersive VR in the context of geosciences.

## 6. REFERENCES

- [1] D. Unwin, P. Fisher, *Virtual Reality in Geography*, Taylor & Francis, 2001.
- [2] R.B. Langley, The mathematics of GPS. *GPS World*, Vol. 2: 45-50, 1991.
- [3] G. Taylor and G. Blewitt, *Intelligent Positioning: GIS-GPS Unification*, 2006.
- [4] P. Fisher, Valediction, *International Journal of Geographical Information Science*, Vol. 21: 1165 – 1170, 2007.