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Óllscoil na hÉireann Má Nuair

The Teaching of Mathematics at Post-Primary Level in Ireland
A Review of Traditional Patterns and an Exploration of Future
Practices

In Two Volumes

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ABSTRACT

This thesis appears in two parts. The first part consists of three chapters. The first two chapters consider and review Irish post-primary mathematics in an historical context and appraise the influential forces which and players who have contributed to and shaped mathematics teaching and inservice courses for mathematics teachers from 1924 to 2005. A review of the literature exploring the quality and shortcomings of teaching and learning at Junior Cycle level follows.

Part II contains the empirical study. The first chapter in this part describes the design and scope of the methodology used in the intensive study which was undertaken in three different Irish post-primary schools. This study was over a complete three year Junior Cycle period and attempted to analyse and gain insights into the teaching and learning of mathematics. Based on video analysis, the dominant teaching methodologies employed in the classes are described and examined. By analysing questionnaires and interviews with students and teachers, their attitudes and perspectives on teaching and learning mathematics are presented. The thesis concludes with a review of the main findings arising from this research. Finally, recommendations are put forward that seek to address issues such as the shortcomings in teaching and learning mathematics in Irish post-primary schools, the continuing professional development of mathematics teachers and the support structures necessary to cultivate richer teaching and learning environments for all.

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None of this research would have been possible without the co-operation of the three teachers and their students who shared their classroom experiences of mathematics with me. For reasons of anonymity the schools are not named and are referred to as Chestnut Hill, Kenmore and Riverside in this thesis.

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INTRODUCTION

This study aims to investigate some key aspects of mathematics teaching in the Junior Cycle of post-primary education in the Republic of Ireland. By international comparisons, there is no crisis in mathematics teaching in Ireland. In the year 2003, the OECD Programme for International Student Assessment (PISA) assessed the mathematics achievement of 3880 15-year olds in 145 Irish post-primary schools. The results show that the average scores achieved by Irish students were not significantly different from the OECD country average. However the mean mathematics scores of Irish students were significantly lower than those of students in a number of other European countries. Ireland ranked 17th of 29 OECD countries and 20th of 40 participating countries on combined mathematics (viz. based on the four content areas: Space and Shape, Change and Relationships, Quantity and Uncertainty). Yet Ireland's performance on students' reading literacy in PISA 2003 stands in stark contrast to that of its mathematics performance. Ireland ranked 6th out of 29 OECD countries and 7th out of 40 participating countries in the reading literacy domain. This suggests a need to question whether current standards in mathematics are adequate.

Another outcome of the PISA study showed Ireland to have fewer lower achievers in Mathematics (16.8%) than the OECD average which was 21.4%. The term lower achiever here means at or below proficiency Level 1 on the PISA scale. There is also a lower percentage of higher achievers in the Irish sample. 11.3% of the Irish sample are

at Levels 5 or 6, the two highest standards on the PISA scale, while the OECD average is 14.6%. The standard deviation of Irish students is 85.3, one of the lowest, which indicates that Irish scores have bunched around the mean. This relative absence of very high achievers and very low achievers provides further evidence worthy of investigation (Cosgrove et al., 2004).

The ESRI report *Coeducation and Gender Equality* (Hannan, Smyth et al., 1996, p. 141) found that: “Being in a co-educational school has significant and substantial negative effects on mathematics performance among girls, a difference of over half a grade from their single sex counterparts”. As a consequence, *Inside Classrooms* (Lyons et al., 2003), a research project funded by The Equality Committee of the Department of Education and Science (DES), was undertaken to contribute to the understanding of the social processes in classrooms. The study examined the gender relations in mathematics classes in both coeducational and single sex schools. The objective was to establish what differences, if any, exist between coeducational and single sex schools in terms of social and pedagogical relations. A further objective was to explore how such differences and similarities influence outcomes. Many of the concerns above came to light in *Inside Classrooms* but it also identified other areas:

1. The cultures that exist in post-primary Junior Cycle mathematics teaching and learning are very traditional;
2. Teachers place great emphasis on teaching to predictable mathematics examinations;

3. There is a strong tradition of mathematics teachers teaching from the textbook;
4. The uptake up of Higher Level mathematics is low by comparison with other Junior Certificate subjects;
5. There is much fear and anxiety expressed by pupils in the learning of mathematics;
6. Pupils' attitudes are picked up from teachers' practices;
7. Many teachers shared the view that if "if they (pupils) don't have that innate mathematical ability....they are not going to improve" (Lyons et al., 2003 p. 372).

These concerns will also be considered in the course of this current research. The main aims of the current research are:

1. To explore ways of enhancing the quality of mathematics teaching at Junior Cycle;
2. To explore ways of improving the quality of mathematics learning at Junior Cycle.

In pursuit of these aims the thesis will:

1. investigate secondary school mathematics from an historical perspective;
2. investigate the provision of inservice support for the Junior Certificate mathematics teacher;

3. investigate the dominant patterns in the attitudes and practices of secondary mathematics teachers at Junior Cycle and their pedagogical consequences;
4. investigate the effectiveness of the work done by Junior Certificate mathematics teachers as distinct from the examination results achieved by their students in state examinations;
5. investigate the relationship between teaching practices and students' attitudes to learning mathematics.

In the autumn of 2000 a revised syllabus for Junior Certificate mathematics was introduced into schools. The changes in content and assessment were relatively minor so mathematics inspectors in the Department of Education and Science (DES) initiated the Junior Certificate Mathematics Support Service (J.C.M.S.S.) with the aim that its inservice programme would focus on methodology on an unprecedented scale. Prior to this, revisions in mathematics syllabi were accompanied by inservice courses provided by the DES and the Irish Mathematics Teachers Association (IMTA), with the main emphasis on curriculum and assessment rather than on teaching methodology. Arising out of the author's experience as an experienced mathematics teacher and Regional Development Officer (RDO) for the J.C.M.S.S., a number of issues became clear to her. These include:

1. the isolation and insulation of the secondary school mathematics teacher;
2. the absence of engagement in professional dialogue among mathematics teachers;

3. the lack of exchange of ideas among mathematics teachers;
4. the tendency to feel that getting by is good enough among mathematics teachers;
5. the overlooked need for questioning of one's practices.

Having worked in a post-primary school for sixteen years and as a RDO for the J.C.M.S.S., the author has noticed that other subject departments, especially English and Science, meet regularly to discuss resources and difficulties they may be having in relation to the teaching and learning of particular topics. Through her experience she has noticed that this kind of discussion is particularly absent among mathematics teachers. There seems to be a lack of questioning and a very strong conformism in inherited patterns among mathematics teachers. This points to a situation which is neither healthy nor exciting for the secondary mathematics teacher or student. The OECD report *Education at a Glance* (OECD 2003, Table D5.4) tells us that 60% of Irish teenagers are regularly bored in school. This finding suggests that something is wrong with the learning environment.

The author has also worked as a Research Fellow with the Teaching and Learning for the Twenty-First century (TL21) collaborative professional development and research project between schools in the Leinster region and the Education Department of NUIM, National University of Ireland, Maynooth. The TL21 project was a four-year professional development programme with fifteen schools located in the Mid-Leinster

and East-Leinster regions, designed to enhance innovation in post-primary schools in Ireland. Its primary aims were to:

- Strengthen teachers' capacities as the authors of their own work;
- Encourage students to become more active and responsible participants in their own learning (www.nuim.ie/TL21/);
- The project developed existing practice in the classroom and promoted fresh thinking in the teaching and learning process (ref. to final or interim publication?). Workshops, seminars, conferences and research facilities were provided to the participating schools. The author, who was part of this project team, engaged with mathematics teachers in these schools on an ongoing basis for three years.

Internationally, there is growing concern about the low level of mathematical skills of students emerging from second-level education (Tickly and Wolf, 2000). A recent discussion paper from the National Council for Curriculum and Assessment (NCCA), *Review of Mathematics in Post-Primary Education* (2005, p. 12), indicates this is also an issue for Ireland. At Junior Certificate level examinations 2004, 41% of the candidates took the Higher level paper, 47% took the Ordinary level paper (a slight decrease on previous years), and less than 12% took the Foundation level paper (also a slight decrease). While the percentage taking the Higher course was a slight increase on previous years, nonetheless it is in contrast with other subjects (except Irish) where considerably more than half of the examination candidates take the higher level. The

current Leaving Certificate mathematics syllabi at Ordinary and Higher level were introduced in 1992 and first examined in 1994. The Ordinary Alternative syllabus, introduced in 1990 for examination in 1992, was re-named Foundation level (with only slight changes in content) in 1995. Figures from the NCCA *Review of Mathematics in Post-Primary Education* (2005, p. 5) show a worrying trend in Senior Cycle, with the proportion of students taking each of the three levels in mathematics approximately to be 11% at Foundation level, 72% at Ordinary level, and 17% at Higher level. While the percentage taking the Higher level is a considerable increase over that taking the course before the revised syllabuses were implemented, the proportions have not achieved the hoped-for uptake of these syllabi when they were being developed: 20-25%, 50-60% and 20-25% respectively (Oldham, n.d. [2007] and personal communication).

Ireland has not participated in mathematical international studies of achievement at Leaving Certificate level, but those for younger mathematical students provide worrying evidence that the performance of some Irish students at Junior Cycle gives cause for concern, as already outlined above in reporting the PISA study. The seeds for Senior Cycle are sown at Junior Cycle. *The Chief Examiners' Report for Junior Certificate Higher Level Mathematics* (1999) highlights the point that "the work of many candidates gave cause for concern in terms of overall competence in the basics of mathematics. As highlighted in the *1996 Report* and again in the *1998 Report*, overall performance tended to be adversely affected by weaknesses in techniques which are essential for a good command of the Higher level course" (1999, p. 6). This again provides evidence that the problems in Senior Cycle mathematics have their origins at an earlier stage. At present

the average failure rate at Ordinary level Leaving Certificate is slightly more than 13%. When combined with the number of students already mentioned taking the Foundation level mathematics examination, this represents a sizeable proportion of candidates who fail to qualify for entry to some third level education courses. Evidence is accumulating at third level that the expertise of students who achieved a grade D or higher in the Leaving Certificate Ordinary level examinations is insufficient for third level courses (Morgan Report, 2001, p. 100). This poses a real challenge to all involved in mathematics education.

To distinguish the research carried out for this thesis from the other studies being cited, the phrase “this current study” or “this thesis” will be used in references. The thesis is organised around the following chapters. Chapter 1 analyses secondary school mathematics from an historical context. Chapter 2 considers the influential forces and emergent questions in secondary school mathematics. Chapter 3 contains a literature review which sets the context for chapters 5, 6 and 7. The methodology for the thesis is examined in chapter 4. Chapters 5, 6 and 7 detail the findings and analysis of three case studies in the First, Second and Third Year of Junior Cycle mathematics. Extracts of classroom video recordings from Year 1, Year 2 and Year 3 accompany this thesis on DVD. The final chapter presents recommendations and conclusions and looks ahead to future possible practices.

CHAPTER 1

Secondary school mathematics in historical context 1920s – 2005

This chapter gives a chronological account of the changes and developments that have taken place in Irish post-primary school mathematics curriculum since the foundation of the state. Without this background, it is difficult to understand contemporary issues in mathematics education today in Ireland. Five periods will be identified in this historical survey and they will be dealt with in chronological order.

The forces of continuity 1920 – 1960

The programme of instruction for mathematics for post-primary schools adopted in 1924 remained in force with only a minor modification until it was replaced by the new mathematics curriculum in 1964. Pamphlets were designed to help mathematics teachers in the implementation of the 1924 mathematics programme. They outlined the aims of the course, criticised repetitive techniques, advised teachers on preferred teaching methods and encouraged teachers to explore different approaches.

In the 1924 programme, two courses “A” and “B” were offered at both Intermediate and Leaving Certificate level. A reduced course was available for girls but not boys. The Pass Leaving Certificate course was an extension of the Intermediate course, with additional elements in algebra, arithmetic and geometry. The Honours course at Leaving Certificate level represented an ambitious approach which challenged both teacher and student and formed the basis for the examination at this level for the next

forty years. Despite discontent and protestations from a number of sources, including teachers, the examinations of 1925 and subsequent years at both Intermediate and Leaving Certificate levels proceeded in a standard and uniform fashion.

Following an appraisal of the post-primary school curriculum by the Department of Education in 1937, no significant curricular changes were instituted except that set syllabi and prescribed textbooks were reintroduced for secondary school subjects between 1939 and 1941. Courses were shortened as it was felt that existing programmes were too extensive and vague (Coolahan, 1981 p. 80). Another review of the secondary school curriculum was undertaken by the Council of Education, a group appointed by the Government for that purpose in 1954. When its report was eventually published in 1962, it did not cause a stir among those who anticipated a reform of the system. It endorsed the *status quo*, essentially a curriculum taught by traditional methods.

The period from 1940 to 1960 represented a time of continuity in the history of mathematics at post-primary level. The promotion of an Irish culture was very much a priority in these early years. The first Minister of Education in the Dáil, November 1925 had said “The business and main function of the Department of Education in this country are to conserve and build up our nationality” (Dáil Eireann, Proceedings, 1925). During the period under consideration the post-primary mathematics teacher would typically have experienced a school ethos largely characterized by narrowness, insularity and a nationalistic orientation. As Akenson put it, this ethos was one

where church and state combined to confine the outlook of teachers and pupils, where the school curriculum was designed to serve a political purpose and where there was an apparent lack of concern for the development of individual children in the school as children, rather than as digits in the Irish revival (1975, p. 60)

It is difficult to imagine teachers of any subjects, restricted by such a conventional and inward-looking climate, presenting a proactive and powerful inspiration to their students. There was no participation by teachers in syllabus design or development. By the 1960's, however, leading practitioners were voicing complaints that rote learning and drill were over-emphasised. Dissatisfaction was experienced especially with the teaching and learning of algebra; for some the work had gone stale (Oldham, 1980a, p. 54). In some respects conditions were becoming ripe for change.

The introduction of the “new mathematics” courses: The 1960's

The reform measures introduced in the 1960's were closely linked to economic considerations. One indication of this was the launching by the Irish government of its *Second Programme for Economic Expansion* in 1963 to cover the period 1964 - 1970. In it the Government envisaged education as having a significant role to play. The Programme devoted a specific chapter to education which referred to the need for greater participation in education, the restructuring of post-primary school provision and curricular changes, particularly in mathematics, science and modern languages.

This overt linking of the education system with economic development was also apparent in the work of the survey team set up jointly by the Organization for Economic Cooperation and Development (OECD) and the government in 1962 to

carry out an analytical appraisal of the education system. When it reported in 1966, the *Investment in Education Report* contained a section dealing with the curricula in operation in the schools. Concern was expressed at the relative lack of success at honours level in mathematics and science. It was noted that only forty four per cent of the instruction in mathematics was given by teachers who had mathematics in their degree and sixty four per cent of science teaching was conducted by non-science graduates. From investigations made by the survey team, it was concluded that mathematics teaching in vocational schools was generally conducted by non-specialists.

Ireland's increasing links with international organizations also influenced changes of attitude. Ireland was represented at international conferences and symposia on educational affairs, which helped to reduce the isolation of previous decades. With the help of Irish Teacher projects, Irish teachers were afforded the opportunity to participate in organized educational visits to Europe and America. International influences thus began to affect educational developments in Ireland and it is against this background of wider social and attitudinal change that curriculum development in post-primary mathematics occurred.

Even before the publication of the *Second Programme for Economic Expansion* (1964) and the *Investment in Education Report* (1966) a number of events had already taken place, both in the United States and Europe, that were significantly shaping the nature of curricular change in secondary school mathematics in the early sixties. The European influence can be traced effectively to 1959, when the

Department of Education sent a representative to the OECD conference in Paris on the theme “New Thinking in School Mathematics” (OECD, 1961). Sweeping changes were recommended for secondary school mathematics curricula. The account that follows is compiled chiefly from a number of accounts (for example, Cronin et al., 1976; Madaus and Macnamara, 1970; Oldham, 1979; Oldham, 1980a; Oldham, 1980b) written or published in the period 1970-1980 when the implications of the changes were being analysed. In 1960, the Department's Chief Inspector, Mr. Seán O’Leary, visited North America, and whilst attending summer training sessions at New Brunswick and New Jersey, he perceived a need for the introduction of changes in Irish post-primary mathematics. In the U.S.A. the launch of Sputnik in October 4, 1957 also marked the start of new initiatives in mathematical education. On the advice of the Chief Inspector's report, Ireland responded by sending three teachers and an inspector to the United States in 1961 to attend training courses in the “new mathematics” i.e. mathematics from the world of academic mathematics, pure mathematics and the modern work associated with the Bourbakiste group in France. It was aimed at the more able senior student, but unfortunately some features of this work found their way into curricula for younger and less able students. Ironically, for the public, “new” meant rescuing pupils from the failure of antiquated mathematics.

The new material included modern algebra, sets, relations, functions, probability with statistics and also number theory with an emphasis on new approaches to old topics. On their return to Ireland, Inspector Nolan and Fred Holland (a post-primary mathematics teacher and later a teacher representative on the early mathematics syllabus committees) wrote to the Department of Education giving their observations

on new developments in post-primary mathematics. They made suggestions for new mathematics curricula and appropriate methods of teacher training. The suggestions were adopted. However, the Department of Education, in accepting their recommendations, decided that teachers in general were not ready for the new material and decided to start the new curricula at Leaving Certificate level only and to train the teachers concerned. The hope was that when these teachers had become acquainted with the new material they in turn would train Intermediate Certificate teachers in the new curricula. Also a smaller number of teachers would be involved at the Leaving Certificate level and they were likely to be more highly qualified mathematically. The first courses were eventually introduced at Leaving Certificate level in 1964 for examination in 1966. Changes in the Intermediate Certificate course were to follow later in 1966, for examination in 1969. The girls' course was abolished. The examination format changed with the introduction of the new courses. Instead of examining arithmetic, algebra and geometry separately, as was formerly the case, the new examination papers were 'integrated'. Students were required to answer six questions from a choice of ten in both mathematics papers, which promoted a 'question-spotting' approach to teaching. Most questions consisted of several subsections; there were 44 and 43 sub questions on the first and second pass papers respectively. There were 43 and 56 sub questions on the first and second higher level papers respectively (Madaus and MacNamara, 1970, p. 87). A review was promised in a few years time.

The new Intermediate course also contained some of the new material introduced into the Leaving Certificate course. This in turn forced changes in the Leaving Certificate course, which took effect in 1969. Following this, both courses were

subjected to revision in the 1970's. It is a matter of importance to note that while the first new courses were drawn up by the Department of Education in consultation with University representatives, all later changes referred to above were the result of the work of syllabus committees introduced for this purpose in 1965. These committees still exist, though in a modified form as Committees of the National Council for Curriculum and Assessment (NCCA), and comprise representatives from the Department of Education, school management bodies, teachers' unions, universities and a representative from the Irish Mathematics Teachers Association (IMTA). In practice, usually all the managerial and teacher union representatives are mathematics teachers; thus the committees do have an input from practitioners at school level. These committees also examine submissions sent in by bodies and individuals but in reality few submissions are made by the general body of mathematics teachers.

Although a certain state of readiness and excitement existed when the new mathematics courses were launched in the mid sixties (Madaus and Mc Namara, 1970, p. 85), this is not to say that the new courses met no resistance. When the Department of Education in early 1963 announced that they were ready to introduce major changes in the Leaving Certificate mathematics and science programmes in the following September for examination in 1965, it provoked objections from teacher unions. For example, the Central Executive Committee (CEC) of the Association of Secondary School Teachers of Ireland (ASTI) on 19th April formally objected to what it called inadequate notice and its mathematics sub-committee took a very strong line on this issue, insisting that before finalizing and introducing the proposed new courses, the ASTI be consulted and its views considered as to the content and

time of implementation (Coolahan, 1984, p. 254). A joint deputation met the Minister of Education on 29 May at which the teacher representatives pressed for the postponement of the introduction of the new courses for a year, for the satisfactory provision of textbooks and refresher courses. As Coolahan remarks however that “no real satisfaction on these issues was gained at this meeting but a series of other meetings and deputations took place which resulted in postponing the examinations in the new courses to 1966 instead of 1965” (1984 p. 254).

It is important to point out that, apart from this action by the ASTI, there followed a comparatively smooth transition to the "new mathematics" which was due entirely to the co-operation between the Department of Education and the Irish Mathematics Teachers' Association (IMTA). The joint endeavours of the Departmental staff and the IMTA proved to be very satisfactory. It is important to note that in the early 1960's comparatively small numbers of students took part in post-primary education and that the mathematics teachers were teaching well-established material. However in September 1967 the government inaugurated a scheme which made post-primary education free for all children. By the end of the 1960's teachers began to become disillusioned with the curriculum changes as the new material was being presented to a much more diverse range of pupils. The topic proving most difficult was geometry, which was now presented on the new syllabi in two contrasting approaches: “transformations” and “Euclidean (traditional)”. Teaching the more abstract new ideas, particularly to weaker students, was proving difficult for teachers. Many teachers also felt unfamiliar with some of the ideas and were not sure about certain aspects of the proofs (Oldham, 1980, p. 335-336). When the time came for revision in the 1970's there was pressure to change geometry (Oldham, 1980b, p. 335-356).

Change without development: The 1970's

With a commitment by the mathematics syllabus committee to revise courses on a regular basis, a slightly changed version of the Intermediate Certificate mathematics course was introduced in 1973 for examination in 1976. Subsequently, in order to accommodate these changes, an amended Leaving Certificate mathematics course was introduced in 1976 for examination in 1978. These syllabuses were expected to run for seven years, as their predecessors had done. The changes introduced were not as far reaching as those of the earlier reforms. Geometry for example, in the updated version of the Intermediate course, concentrated solely on transformation geometry whereas the earlier changes had combined the traditional Euclidean approach with a flexibility which included some transformation geometry in the Papy style. This streamlining of the geometry section, now based on explicitly stated axioms, proved to be a most controversial topic. The axioms were intended to be a starting point for teachers but because they were written into the syllabus, book companies wrote them into textbooks and they were thus regarded as part of the course to be taught to students.

The Department of Education *Rules and Programme* (1974/1975, pp. 64-67) divided the course into three sections, one section for each year of the Junior Cycle. Section One had some of the axioms appearing in what was effectively first year work. As a consequence, twelve and thirteen year olds were being introduced to geometry in an extremely formal way as opposed to a more intuitive fashion more appropriate to their age. With the preoccupation with geometry came a price. Other abstract aspects of the course in need of revision survived. The only other change was the inclusion of practical/social mathematics which was to be a compulsory question on the

examination papers. This first revision of the “new mathematics” syllabus “actually moved them further in the direction of modernity” (Oldham, 1992, p.138). The Department invited comments on drafts of the 1973 syllabuses. Few comments were received and in their absence the new modifications were implemented (Oldham, 1980a, p.53, 55).

Changes in the Leaving Certificate course in 1976 were less significant but the format of the examinations changed. Each of the two Leaving Certificate papers incorporated a compulsory short-answer section which spanned the entire course. The next section required a student to answer a compulsory "problem" followed by a choice of problem-type questions. The idea was to encourage coverage of the entire course (if only at a surface level) and to lay stress on certain basic skills. A similar lay-out existed at Intermediate Certificate level, except that the first section used a multiple-choice format spanning the content of the entire paper. These changes were accompanied by a gradual disenchantment on the part of teachers. There were many complaints among teachers about the introduction of the 1973 changes in the Intermediate course. As their comments were not effectively communicated to the Department of Education they had little effect. However this trend altered when the amended Leaving Certificate course was drawn up for introduction in 1976. Mathematics teachers in the IMTA engaged in a more vocal dialogue and their submissions had a sizeable impact on the final syllabus adopted. Thus the IMTA entered a new period of involvement with the Department of Education. Indeed the IMTA in 1973, after considering the unsuitability of the mathematics courses for the majority of pupils, began to formulate less abstract courses for the weaker pupils. A number of initiatives from the IMTA in the 1970's lend support to the idea that

mathematics teachers were becoming more closely involved in curriculum development work (see for example: Oldham, 1979; Oldham, 1992). These developments are reviewed at closer range in the next chapter.

The new courses of the 1970's were implemented on a national scale without any attempt to identify the different needs of the students who were expected to benefit from them. Admittedly, at the time the reform measures were introduced, neither curriculum development nor educational management principles were prominent concepts. It is hardly surprising that the attempts to reform the mathematics courses did not meet with total success. With all the benefits of hindsight, their introduction was badly managed, failing to take cognizance of the need for adequate attention to pedagogical matters and without due consideration to the pupil clientele.

Furthermore, the picture that emerges of the professional position of the mathematics teacher in the introduction of the new courses is not an encouraging one. Continuous in-service training for mathematics teachers did not take place, resources and teaching aids were not made available. These deficiencies, together with the regrettable absence of an appraisal mechanism to monitor the effects of the new courses, did not provide a recipe for genuine and sustained success.

The debate on the value of the modern approach to post-primary school mathematics continued throughout the decade of the 1970's. Despite the paucity of developmental support for teachers, the changes of the seventies were accompanied for the first time by the specification of objectives for the Intermediate and Leaving Certificate examinations. The set of objectives for the Intermediate course was published by the

Department of Education following the introduction of the new courses in 1973.

These new objectives stated that a student should

- acquire skill in computing with understanding, accuracy and efficiency;
- acquire an understanding of mathematical facts and concepts;
- understand the logical structure of mathematics and the nature of a proof;
- use mathematical concepts and processes to discover generalizations and applications;
- associate mathematics with applications from everyday life;
- discover attitudes that lead to application, confidence, initiative and independence;
- develop study habits, reading skill and vocabulary essential for independent progress in mathematics. (Department of Education, 1974, p. 64).

As a guide to help teachers achieve objectives, it was recommended that the teaching should be resourceful, inventive and creative and should examine the student's environment for the experiences, examples and analogies required to permit the formation, enrichment and refinement of the fundamental concepts. This guideline represents a considerable improvement on the imprecise statement which was offered by the Department of Education in 1956 in response to a questionnaire by the International Bureau of Education, Geneva 1956:

The aims of mathematics teaching are not formally set down on any official instruction, for the reason that it would be extremely difficult to specify them. It is generally understood however that the practical aims are the cultivation of greater reasoning power and of greater accuracy in thought and expression, and that the cultural aim is the rounding off of the pupils' general education as that education is

described in the Rules and Programme for Secondary Schools. (The teaching of Mathematics in Secondary School, 1956, XIX th. International Conference on Public Education, Geneva)

If the specification of objectives for the Intermediate Certificate course was perceived as an innovation, then sadly, it was not supplemented by the provision of adequate resources in the form of specifically designed texts or teaching aids. There is no evidence to suggest that mathematics teachers met in small groups to discuss the implications of the new objectives. Without feedback, weaknesses were not articulated or acknowledged on a national basis. Improvements in mathematics teaching could scarcely be expected to occur with such a haphazard curriculum planning approach.

In line with the trend already established, the Department of Education specified objectives for the amended Leaving Certificate course introduced in 1976. The claim was advanced that an attempt was being made "to combine in one unified structure topics which are traditional with those which are modern and relevant". Some of the objectives are listed below

- to develop conceptual and meaningful mathematics together with efficient computational skills;
- to emphasize key concepts and fundamental structures;
- to show mathematics both as an abstract, autonomous body of knowledge as well as a useful, operational tool;
- to enable students to attain knowledge and insight by means of classroom and independent study;

- to prepare students for further study in mathematics;
- to encourage logical thinking (Department of Education 1977, p. 206-207).

An analysis of these objectives will not be pursued except to say that, as in the case of the Intermediate Certificate objectives, they were not accompanied by extra resources, teaching aids or textbooks. Mathematics teachers were not provided with opportunities to either appreciate or help develop strategies to achieve these objectives by way of in-service education.

If the specification of objectives received attention, considerable dissatisfaction had also been expressed with the examination system in general. In 1974, the final Report of the Committee on the Form and Function of the Intermediate Certificate Examination (ICE Report, 1975, p. 5) was published. The report suggested that the Intermediate Certificate examination served no useful purpose. The Committee proposed a system of school-based assessment monitored by a central body which would take responsibility for all aspects of curriculum assessment, helping teachers to clarify educational objectives, providing external tests and opportunities for school-based assessments. However, the recommendations made in the report were not implemented and the Intermediate Certificate examination remained. The Madaus and MacNamara report on the Leaving Certificate (1970, p. 91, 92) was critical of the reliability and validity of the examination at this level. The study found that the examination at both Pass and Honours level might well be little more than largely a measure of memorized knowledge.

Although the Department did produce notes on the new courses of the sixties, explaining the content within the sections, guidance for teachers on pedagogical methods was absent. Apart from text-books, mathematics teachers in general had no additional teaching aids. The bulk of the reform measures were originally intended for the more able students. Little consideration was given to the needs of different groups of students. Furthermore, the introduction of technical terms and symbolism to junior pupils in first year aggravated the plight of weaker pupils. Frustration and confusion among this group of students followed “but it led, not to a withdrawal from commitment to major aspects of ‘modern’ mathematics, but rather to a second more thorough attempt to espouse key aspects of the work” (Oldham, 1992, p. 138). Competency in traditional computational skills, including arithmetic, ability to approximate and estimate, and skill in algebra suffered from neglect as teachers strove to introduce the many new modern topics.

Syllabus/Course committees become more influential: The 1980's

The establishment of the interim Curriculum and Examinations Board (CEB) in 1984 was the government's response to the widely held view that major changes were needed in curricular and assessment procedures. In 1985, the Minister for Education decided to transfer the functions of syllabus committees (which had been in existence since 1965) to course committees under the auspices of the CEB. However, the existing syllabus committees were allowed to complete work which was at an advanced stage. This included the Department of Education's mathematics syllabus committee, which had been convened in 1982 in an attempt to construct new syllabi at three levels - syllabus A for the more able pupils, syllabus B for those pupils of average ability and syllabus C for weaker students for the Intermediate Certificate.

The previous revision of the Intermediate Certificate mathematics programme was in 1973 and that of the Leaving Certificate programme in 1976. Thus, the process of appraisal and review was long overdue. From the outset, the determination to construct syllabi at three levels represented a significant change in syllabus and examination reform. The major consideration was the continuing dissatisfaction expressed at the ability of the current syllabi to meet the needs of all pupils especially the less able pupils. The high failure rate (approximately twenty per cent) on the Lower Intermediate Certificate mathematics examination provided ample evidence of the unsuitability of the syllabus for these weaker pupils. This had been a recurring theme of the mathematics debate during the 1970's.

A closer look at the work of the syllabus committee during this period of the early 1980's renewal generates insights into a number of factors which helped to shape the outcomes that emerged. Firstly, while the proceedings of Syllabus Committee meetings are not in the public domain, the anecdotal evidence is that mathematics teachers who actually sat on the syllabus committee contributed significantly to the debate. Throughout the period of consultation and discussion, draft syllabi were produced and all mathematics teachers were strenuously encouraged to comment on the proposed changes (Association of Secondary Teachers Review (Astir), January, 1984, p. 6). However, in general, the response from most practising mathematics teachers remained poor, and it was the mathematics syllabus committee who eventually determined the nature of the syllabi for national implementation.

In general, matters of content occupied the centre of the debate. Geometry predictably proved to be the most contentious topic, both for syllabus A and B. In relation to syllabus B, the committee was virtually unanimous in urging a departure from the prevailing approach. An approach based on congruence was suggested due to the power and simplicity of the concept and the early drafts of syllabus B included theorems to be proved in this fashion. Later, the question of whether the formal learning of geometrical proofs should be required of pupils at all was debated. The view was expressed that the recalling of formal theorem proofs for examination purposes was of little value. It was time-consuming for teachers, frustrating and largely meaningless for the majority of pupils and its contribution to their mathematical education was questionable. As an alternative, an intuitive approach based on measurement and distance was proposed as being more related to experience in reality. In this manner, it was claimed that all the geometrical facts, at present learnt as formal proofs, could be acquired by pupils without the accompanying drudgery and frustration (Astir, January, 1984, p. 6). When the final syllabi were sent to post-primary schools in September 1986 (a year in advance of their introduction), it was indicated that the formal proofs of theorems would not be expected of students taking syllabus B. In general syllabus B was less abstract than the Lower course which preceded it.

When syllabus A came up for discussion and debate, no great problems arose with the content of the algebra and arithmetic sections but there was serious disagreement over the method of proof to be used in geometry. In general, teacher representatives wished to see the concept of equipollence removed from the syllabus on the grounds that it was too abstract and that to ignore length and to postpone mention of distance

for as long as possible was unreal and contrary to experience. It was argued that the transformations of the plane were taught in metric terms and should, therefore, be defined in metric terms. This proposition was debated over three meetings and the outcome resulted in the formulation of an agreed set of axioms and proofs of theorems by congruence arguments. The inspectorate of the Department of Education insisted on the retention of the concept of equipollence to be used to prove the axioms on which the congruence proofs were based and as an alternative basis for the proofs of the theorems. They also continued to insist on defining the transformation of the plane in terms of equipollence rather than in metric terms (Astir, 1984). Shortly before syllabus A was due to be published, the syllabus committee received a letter from three Irish Professors of Mathematics expressing their dissatisfaction with the geometry content in syllabus A (Astir, March 1986, p. 2). In recommending a re-draft of the geometry section, they suggested the suitability of the Euclid-Hilbert context and emphasized the need for the inclusion of Pythagoras' theorem and ratio theorems for triangles as a basis with which to start trigonometry and co-ordinate geometry.

The implications of this approach meant the dropping of the concept of equipollence and terminology involving couples from the geometry section of the course. After a full discussion, the syllabus committee decided to adopt the position taken by the Professors and the geometry content of the syllabus was amended accordingly. No corresponding change was made in Syllabus B. The two courses were now at variance with each other. Thus, when syllabus A was eventually published in 1986, it contained no list of axioms and the transformations of the plane were defined not in metric terms but in terms of equipollence. Compared to its predecessor Higher level

course, Syllabus A was almost no different except for geometry. Such amendments indicate the high esteem which Professors of Mathematics enjoy in the Irish context and the influence which they can exert on the teaching of post-primary school mathematics. The revised courses were accompanied by sample papers but specific texts were not published by the Department of Education. Instead, it was left up to private enterprise to take up the initiative and the response was both enthusiastic and competitive with some texts catering specifically for syllabus C. Again however the lack of adequate in-service training for mathematics teachers taking on the new courses was disheartening. Those courses which were made available required that teachers give up some holiday time. Minimal expenses constituted a further disincentive. Once more, little or no attention was focused on the reality of the teacher-pupil classroom situation. The task of the mathematics teacher was simply to implement a central directive in the best possible fashion in the absence of proper in-service, resource aids, discussion and research.

The objectives of the new syllabi introduced in 1987 corresponded closely to those introduced for the first time in 1973. However a short “preamble” indicating the aims was included. According to Oldham (2006, p.3) “the preamble begins with an emphasis on meaning and process that was ahead of its time in terms of Irish public discussion: ‘The underlying philosophy of the syllabuses points to mathematics as a human activity rather than a ready made subject. It emphasises the practical experiences of the pupil...Above all, it works towards understanding...’ (Department of Education, 1994, p.28)”. The preamble to the revised mathematics syllabi of 1986 at Intermediate level contained no statement on teaching methods. Some of the summer in-service courses did look at strategies and textual material. However, there

seemed to be a basic assumption that teachers are competent to work out appropriate methods for themselves. The new syllabi contained notes on the various sections of the course. These were aimed at definition and explanations of the content rather than at pedagogical guidance. Materials other than textbooks were not very prominent. Altogether, matters of pedagogy received insufficient attention.

Implementation of the courses was delayed until 1987, as the CEB considered whether to accept the courses or to require further revision in the context of the then ongoing review of Junior Cycle education (Oldham, 2006 p.3). The Junior Certificate programme was introduced in 1989 and, as the revised Intermediate Certificate Mathematics syllabuses had been introduced only in two years previously, they were simply renamed Higher, Ordinary, and Foundation instead of Syllabus A, B, and C respectively. The course was never presented in the standard Junior Certificate format; it was still published as it had been as in successive editions of *Rules and Programmes for Secondary Schools*. Whilst most subjects had received a Junior Certificate rethink in the 1980's, mathematics did not enter these currents of thinking and an opportunity for more meaningful change was missed.

A separate development in 1985 resulted in pupils being allowed to use electronic calculators at the Leaving Certificate mathematics examination. Consequently, questions involving the use of logarithmic tables were no longer set. The latter type questions were also later omitted from the Intermediate Certificate examination, although calculators were not permitted. The CEB's course committee and the IMTA (which had advocated use of calculators at junior level since 1980) believed that the

calculator had an important function in mathematics education and that its introduction should be encouraged and facilitated at Junior Cycle level. It subsequently emerged that the CEB assented to the recommendations on the use of calculators for Junior Cycle. Accordingly, in February 1987, it was decided to proceed with the design of sample papers for the revised syllabi at Junior Cycle on the assumption that the use of calculators would be permitted in the Intermediate Certificate examination in 1990 (Astir, May, 1987 p. 5). Calculators were not in fact permitted until the revised Junior Certificate syllabus was introduced in 2000.

Three further initiatives were considered under the aegis of the CEB. The first concerned the structure of the examination papers. Originally, syllabus C was to be examined by only one paper. It was decided that two papers would provide a better sampling of the course, afford candidates a better opportunity to score, and facilitate comparability between syllabus C and syllabus B by making the second paper common to both. Similar arguments to ensure comparability between syllabus A and B resulted in the agreement by the meeting on Junior Cycle mathematics held on February 18th 1987 to the following structure:

Syllabus	A	B	C
Examination Papers	Paper 1 (same paper as syllabus B paper 2)	Paper 1 (same paper as syllabus C paper 2)	Paper 1
Examination Papers	Paper 2	Paper 2	Paper 2

Paper 2 of syllabus C would be the same paper as Paper 1 at syllabus B level. Paper 2 of syllabus B level would be the same paper as Paper 1 at syllabus A level. This was to establish comparability between the different syllabuses (Astir, May, 1987, p. 5).

Secondly, thorough in-service training for mathematics teachers and adequate monitoring were advocated as being crucial to ensure credibility (Astir, May, 1987, p. 5). Finally, an attempt was made to specify assessment objectives for syllabus A, B and C at Junior Cycle (Astir, May, 1987, p. 5). None of these recommendations came to fruition.

“Change in a new key: The nineties and the early years of the new century.”

The 1990's constituted a major decade of reform in education in Ireland. Throughout the decade a series of initiatives occurred on key issues of educational policy. The OECD Report of 1991 *Review of National Policies for Education* provided a first and crucial impetus to the emergence of an education policy debate in Ireland. The Green Paper *Education for a Changing World* (1992), was the first response to the OECD, 1991. In the Foreword to the Green Paper, the Minister invited a wide national debate from all who had a commitment to the quality of education. An unprecedented level of debate took place throughout the country at meetings, conferences, symposia and seminars. As well as this, almost 1000 written submissions were lodged with the Department of Education in response to the Green Paper. This bore testimony to the remarkably high level of interest by the Irish public in educational issues. Before the Government finalised its policy decisions in the

White Paper “Charting our Education Future” (1995), the Minister for Education convened the National Education Convention as an independent forum.

The National Education Convention 1993-1994, was an unprecedented, democratic event in the history of Irish education. It provided a forum for mature reflection and focussed debate by representatives of many agencies –including educational bodies, trade unions, Department of Education. Over the nine days of the convention there was a remarkable articulation of ideas, refining of ideas, analysing of ideas, challenging of ideas. At the Convention there was general recognition of the inadequacy of inservice courses for teachers, the need for time for curriculum planning. The need for styles of pedagogy which engage and involve all pupils more actively in the teaching-learning interaction than was traditional was also realised. Concerns were expressed that the current examinations were encouraging rote learning and a teacher-centred approach to teaching and learning in which a passive role was being assigned to pupils. (National Education Convention Secretariat (1994)). The Convention occurred at the penultimate stage before policy was formulated in the White Paper on Education “Charting our Education Future” (1995) and later in the Education Act of 1998. This heralded a major programme of legislation for the reform of education at primary and post-primary level, the teaching profession, parental involvement, and organisational framework. The Leaving Certificate changes happened before these reforms started. Consequently the major reforms of this decade left mathematics located out of context, with the emphasis on Leaving Certificate syllabuses changes solely on course content and assessment.

By now the National Council for Curriculum and Assessment (NCCA) had replaced the CEB and in 1988 it convened the Senior Cycle Course Committee. It was initially hoped that students introduced to the revised mathematics syllabi at Intermediate level would proceed to amended syllabi at Leaving Certificate level in 1990. This was not to be, however, as the structure of the entire Senior Cycle was under discussion and no decision had been made as to the number of levels at which Leaving Certificate courses might be offered. To cater for students who had taken Syllabus C in the Intermediate Certificate, an Ordinary Alternative syllabus was introduced on an interim basis in 1990 for examination in 1992 (Oldham, 1993). It was amended slightly and renamed Foundation level in 1995. The Higher and Ordinary syllabuses were introduced in 1992 for examination in 1994. Details of the context of and rationale for the changes are discussed in the next chapter. The 1992 courses were the first in Mathematics to be produced in the NCCA format, i.e. in a booklet with aims and objectives as well as course content (Oldham, 2007).

In the early 1990's support for the new curriculum had become the norm. As with other subjects, in-service training consisted of one-day sessions held during school term. Mathematics teachers were encouraged, though not required, to attend. As sessions were relatively brief, detailed exposition of mathematics was not possible and the focus of the in-service was on curriculum content and sample examination papers. A new feature however was an explanation of the rationale for the changes (Oldham and English, 2005). The in-service team members were picked from a group of teachers known to the organising team in the Department of Education. It was disappointing that it did not focus on pedagogy. Student and teacher needs had changed dramatically since the sixties. In 1977 when Ireland was completing

documentation for the Second International Mathematics Study, the Department of Education clearly defined its position in relation to teaching methodology and course content. Course content was the responsibility of the Department. Teaching methodology was the responsibility of the teacher (Cronin et al., 1977).

Under the NCCA, the mathematics Junior Cycle course committee was convened in November 1990; their brief was to analyse the impact of the Junior Cycle mathematics syllabuses that were introduced in 1987. After the first examinations of syllabuses in the Junior Certificate the committee were given a similar brief in 1992. The committee reported the following difficulties:

- The length of the Higher level syllabus;
- Aspects of the geometry syllabus, especially at Higher level;
- Proscription of calculators in examinations;
- Design of the Higher and Ordinary level examination papers (DES/NCCA, 2002, pp.5-6)

In the autumn of 1994, the course committee were allowed by the NCCA to examine the Junior Certificate mathematics syllabuses with a view to introducing some changes if needed. The committee were told that the syllabuses were to be reviewed, not redesigned. The brief did not allow for major revision of the course. Mathematics was still not to have the Junior Certificate rethink that other subjects had got back in the eighties. The syllabus committee were to take into account the work being done by the NCCA with respect to the Curriculum changes at Primary level. They were also to take into account the earlier reviews by the committee; the changing patterns of examination papers and the analysis of the examination results since 1990. The

committee were also asked to identify major issues of concern regarding the existing syllabuses in terms of design, implementation and assessment. They were also to draft a statement of aims and objectives for all three levels in line with Junior Certificate practice (DES / NCCA, 2002, p. 6).

In the event the course was rewritten to present it as a syllabus document, incorporating aims and objectives and a framework for assessment (DES / NCCA, 2000). The aims and objectives were specified in the introduction to the syllabus and applied to all three levels. Each syllabus was introduced in turn by a rationale, statement of aims specific to level and a list of assessment objectives for the different groups of pupils. The committee were requested to prepare Guidelines to assist in improving the teaching of mathematics. This was the first time that pedagogy was being introduced into mathematics curriculum design at Junior Certificate level. (Guidelines for teachers had been produced for the Leaving Certificate Foundation course, and the accompanying brief inservice sessions had focused on approaches to teaching the course.)

Amendments were made to content and assessment procedures to deal with the difficulties identified above. Calculators were assumed to be readily available for teaching and learning and for the first time were allowed to be used in the Junior Certificate mathematics examinations of 2003. The approach to geometry was one of the major areas of concern in revising the syllabuses. The revised syllabus of 2000 ring-fenced the transformational elements, returning to a more Euclidean approach based on congruency. The underlying ideas were consistent between the syllabuses,

unlike in the revision of 1987. To shorten the length of the geometry course at Higher level the number of proofs that pupils were required to supply was reduced from twenty one to ten. Another casualty from the Higher level course was logarithms. Other topics were pared down. At Ordinary level difficult areas of algebra and coordinate geometry were simplified and other areas were pruned. The intention was that by shortening the courses teachers would have more time to develop a greater conceptual understanding among students, develop suitable concrete approaches to abstract topics and use active learning methods. The Foundation level, whilst enjoying approval, was recognised to have difficulties with syllabus content and examination format. It was recognised that the standing of the syllabus needed to be improved and that students needed to be given a greater opportunity to show what they had learned. In the revised Foundation level introduced in 2000 there was less emphasis on fractions and more on decimals. The algebra section was slightly expanded as it was felt that the 1987 syllabus was minimal. An additional problem was that some pupils were taking this level when they were capable of doing Ordinary level. The course committee presented the draft syllabuses to the NCCA and Council approved them in May 1998. They were introduced in schools in autumn 2000 and examined for the first time in 2003 (DES / NCCA, 2002).

Elizabeth Oldham, the NCCA's Education Officer for second level mathematics from 1989, and Peter Tiernan, the IMTA representative on the Junior Cycle course committee (1991 to present day) had visited many IMTA branches to discuss drafts of the syllabuses and ask for feedback and submissions. The branch meetings with regard to Junior Cycle were badly attended (personal correspondence with Peter

Tiernan, 2005) and submissions were mainly anecdotal but did concur with the committee's findings (as outlined in the bullet points). Teachers could have taken a much more active part in submitting to the syllabus committee their ideas on existing and proposed syllabus content, implementation and assessment, but failed to do so.

Around this time, evidence was increasingly pointing to the need for consideration of pedagogy. The following discussion is drawn from English and Oldham (2004) [unpublished ESAI paper]. Analysis of the results of state examinations showed that the lower performing students were displaying very poor relational understanding (Skemp, 1976, p. 1-7). Chief Examiners' reports found that difficulties with elementary concepts were appearing right up to Senior Cycle level (Department of Education 1995, 1996, Department of Education and Science 2001). Evidence from national and international studies pointed to unsatisfactory levels of achievement in mathematics by Irish students. The Third International Mathematics and Science Study (TIMSS) in 1994-1995 highlighted that the prevailing styles of teaching might be contributing to the difficulties. Findings from TIMSS indicated that Irish teachers gave unusually high importance to procedural aspects of teaching and gave unusually low emphasis to logical thinking and applications of mathematics – aspects that might improve pupils' relational understanding and their ability to apply mathematics (Beaton et al., 1996, p.138-143). Other national research provided additional evidence that teaching styles were perhaps contributing to students' difficulties. O'Donoghue (2002) pointed out that mathematics teachers focused on getting pupils to pass examinations rather than learn mathematics. Lyons et al. (2003) highlighted the overriding emphasis Irish mathematics teachers placed on procedures rather than concepts and the tendency for Irish mathematics teachers to

be traditional rather than progressive in teaching approaches. A picture of a classroom culture not conducive to either teaching or learning mathematics had emerged (Nickson, 1992, p. 101-114).

The introduction of the revised syllabus for Junior Certificate mathematics in Autumn 2000 provided the Department of Education and Science (DES) with the opportunity to instigate an inservice programme focused on methodology, emphasising approaches through which understanding for pupils might be improved. Unlike most previous programmes (that for Foundation Leaving Certificate being the exception), this in-service initiative did not focus solely on content and assessment but on teaching methodologies that would facilitate achieving the aims and objectives of the revised syllabuses. The organisation of the in-service courses was different from previous state run in-service provision. Posts for presenters were advertised and were open to all, as opposed to being awarded to a group of teachers who were known to the organising team. Forty experienced practising teachers were chosen as part-time presenters on the basis of interviews. A National Coordinator was appointed to lead the programme. Presenters came together with mathematics inspectors for two residential training sessions. Presenters were given professional training on presentation and facilitation skills. Elizabeth Oldham, the NCCA's Education Officer for post-primary mathematics, explained the rationale and content of the changes in the revised junior Certificate courses. The main focus of the training sessions was on addressing student problems and developing materials and strategies that might enhance teaching and learning. Guidelines were published to accompany the revised syllabus and many "lesson ideas" were prepared to support

the inservice programme. The forty presenters had a major input into this (English and Oldham, 2004).

The first phase of the inservice courses for the revised Junior Certificate Mathematics syllabus ran for the academic years 2000-2001 and in an updated form in 2001-2002. Teachers attended during school time. Sessions were held all around the country. Although time was given to curriculum and assessment the main emphasis was on methodologies. In the second year, based on feedback from teachers who attended the first round of courses, five areas were concentrated on. Four of these were algebra, geometry, the use of ICT and calculators. The fifth area that teachers had identified as wanting more attention was the setting up and development of an active mathematics subject department in schools. Teachers then were showing a desire for a professional as well as a pedagogical emphasis in the courses. For the second round, presenters were all assigned to one of the five areas above and support materials were developed for the second year. At the sessions from round 1 and round 2, teachers who attended were actively involved with some of the methodologies being introduced. Feedback to the co-ordinator by way of teachers' evaluations was extremely positive (personal correspondence with National Co-ordinator Dr. Joe English).

On foot of this the DES introduced the second phase of the programme in autumn 2002 and this ran for two academic years 2002-2004. Five full-time Regional Development Officers (RDOs) posts were advertised and the five that were appointed and seconded to these positions were from the original team of forty. They

visited schools by invitation and worked with teachers in their own schools. The inservice courses again focused on pedagogy, curriculum and assessment (English and Oldham, 2004). The programme entered its third phase and ran for the academic year 2004-2005 with the five RDOs now working on a part-time basis presenting the courses. The National Coordinator organised the inservice sessions all around the country, focusing on algebra. Again the courses focused on pedagogy and assessment to a limited degree, actively involving the teachers present. At the time of writing, the inservice programme appears to be entering its fourth phase with the advertisement for five associate positions in the support service. These positions are on a full time basis for three months, with appointment taking effect from October to the end of December 2005. Of the five RDOs only three are now available for this work.

The content of the Junior Certificate inservice programme was from the start radically different from any inservice programme that accompanied changes in syllabuses before. Since the introduction of the revised syllabuses in Autumn 2000 the DES have advocated a change in teaching practices in Irish classrooms. However no specially designed teaching manuals incorporating advice on the use of teaching aids introduced at inservice courses have been produced.

Summary: Secondary Mathematics in Ireland 1924 - 2005

This historical survey has attempted to highlight chronologically the significant developments which occurred in post-primary mathematics in Ireland. From the foundation of the state until 1964 mathematics syllabuses saw only minor changes.

The 1960's saw the introduction of the "new mathematics" courses at both junior and senior level. In the 1970's changes in course content at Junior and Senior levels occurred with little development. The 1980's showed the vision and influence of the syllabus committee with the introduction of three levels for examination at junior cycle. Another wave of changes came about in the nineties at Leaving Certificate level, with amendments being made to syllabuses and a third level being introduced for examination. Finally change in a new key occurred with an emphasis on pedagogy in 2000, with the introduction of the revised Junior Certificate mathematics courses.

The next chapter will look at the key players and influential forces who contributed to secondary school mathematics from 1924-2005.

CHAPTER 2

Influential forces and emergent questions in Post-Primary

Mathematics 1924 -2005

This chapter reviews the roles of the key players who and forces which have contributed to and shaped mathematics teaching and in-service courses for teachers, in post-primary mathematics in Ireland. In particular it explores the significance of the Irish Mathematics Teachers Association (IMTA). It also examines teacher education, syllabus development, pilot initiatives and two international case studies of teacher involvement in curricular development. Finally it undertakes a review of questions and issues that have arisen from chapter 1 and chapter 2.

The IMTA as a support for teachers in the “new mathematics” courses

The initiative to introduce the “new mathematics” courses in the 1960’s came from the Department of Education, following in the long tradition of central determination of syllabuses. With the advent of the syllabus committees in 1965 teachers had the opportunity to have an input into syllabus changes. In conjunction with teacher organizations, and particularly the IMTA, the Department of Education directed and financed the in-service courses of teachers for the “new mathematics” courses. In order that the necessary reforms in mathematics education should have any chance of being carried through successfully, a large scale programme of teacher training and retraining was necessary. This need was in fact recognised by the Department of Education at the time the reforms were discussed and planned. Attention is now

turned to how this objective was realized in Ireland and here the role of the IMTA was very much in evidence.

In September 1961, immediately after coming from the United States, Fred Holland (a post-primary mathematics teacher and teacher representative on the early mathematics syllabus committees) organized a mathematical circle for post-primary teachers in Cork. In January 1964, a national society called the Irish Mathematics Teachers' Association (IMTA) was formed at a meeting in Newman House, Dublin. Mr. Denis Buckely of the ASTI became the first President of the IMTA. Among the distinguished mathematics teachers present at the inaugural meeting were President de Valera and Professor Lanczos of the Institute of Advanced Studies. The Cork circle amalgamated with the IMTA in 1965. The IMTA and its members were to play a major role in the implementation of teacher in-service courses in the "New Mathematics".

Universities in Dublin, Cork and Galway led the way in in-service courses for the new curricula. These curricula had already been published when the first Summer School Course was held in July 1963 in University College Dublin (UCD). This was followed that same summer by a course in University College Cork. Later in August 1963, the Department of Education organized a course in UCD which was attended by over five hundred teachers from all over Ireland. Six lecturers from UCD, Trinity College Dublin and University College Galway, together with Fred Holland, gave the lectures. As the new courses were to be introduced at Leaving Certificate level in 1964 it was mathematics teachers who taught the subject at this level who were

trained first. This was done in the hope that they in turn would pass on their knowledge and skills to Intermediate Certificate mathematics teachers. The winter months of 1963-1965 saw many in-service courses organized by the IMTA. In 1966, when much of the new Leaving Certificate material was transferred to Intermediate Certificate level, the teacher in-service courses were provided by both the Department and the IMTA. Teachers of Leaving Certificate mathematics gave the new courses to Intermediate Certificate teachers at the summer in-service courses. It is interesting to note that funding for such summer courses, which ran intermittently through the seventies, eighties and nineties, was withdrawn in 2001. The last summer course was held in Tipperary Institute of Technology 2001.

As noted earlier, a comparatively smooth transition accompanied the introduction of the new mathematics courses by the collaborative efforts of the Department of Education and the IMTA, a co-operation which was rare in the experience of post-primary teachers prior to 1960. Reforms initiated by centralized Ministries of education (such as exists in Ireland) commit those Ministries to an extensive programme of in-service courses. Merely changing the syllabus and textbooks is insufficient to bring about curriculum development, as the average teacher has a very great capacity for continuing to do the same thing under a different name. The toughest part of any development work is the in-service stage and in this respect it must be acknowledge the Department of Education, who in conjunction with the IMTA played a significant role, ensured a much more satisfactory reform of the mathematics curriculum than would otherwise have been possible (Coolahan, 1984, p.254).

The in-service courses which took place in the summer did not, however, emphasize adequately the philosophy behind the new courses. Insufficient attention to what the new courses were intended to achieve did not enhance the effectiveness of mathematics teaching. The emphasis at the in-service courses was mainly on content as opposed to methodological approaches to teaching the new courses. The courses given tended to be of a didactic nature. The efforts of the Department of Education, universities and the IMTA in organizing the initial in-service courses were commendable, but also fell short. The introduction in 1965 of a mathematics syllabus committee did give mathematics teachers a degree of involvement in syllabus design which had previously been the sole prerogative of the Department of Education. Notwithstanding their shortcomings, these moves constitute significant advances for their time. The IMTA showed vision in these pioneering initiatives. Nobody foresaw that the “new mathematics” would create problems when free education was introduced and would be unsuitable for pupils of average and below average ability. Another serious accompaniment to the “new mathematics”, which persists to the present day, was:

the further decision, a necessary consequence of the first, to change the examinations. To adapt the LC examinations to the new syllabus was to pledge the educational system to the new syllabus. In agreeing to a change in the examinations the teachers accepted the most stringent obligation to change their own teaching.

(Madaus and McNamara, 1970, p. 85)

Practitioners wanted the safety of the traditional shape and pattern of previous examination papers. This in turn dictated the shape of their teaching which brought about problems of “teaching to the examinations”.

The IMTA as a source of leadership in the period 1970s-1980s

Gradually, as teachers and students became more experienced with the new courses, an unfavourable attitude emerged towards the “new” pure mathematical approach to post-primary mathematics. There were increasing doubts as to the ability of the axiomatic approach in helping students understand and appreciate key geometrical concepts and fundamental structures. The “new mathematics” was proving unsuitable for students of average and below average ability. Evidence of a survey carried out by the IMTA in 1974 points to the unsuitability of the courses to cater for the weaker student. In particular it highlights:

- 70% of mathematics teachers surveyed considered that post-primary students would benefit from a choice of syllabi;
- 80% considered that present syllabi in mathematics were not suitable for all students;
- 82% thought that a significant number of students would opt for a more practical alternative syllabus (Kelly, 1974 p. 45-49).

Indeed the IMTA in 1973, after considering the unsuitability of the mathematics courses for the majority of students and the specific problem posed by the new geometry, began to formulate less abstract courses for the weaker students. At a delegate conference in 1977 a working party was set up to examine the needs of the weaker students for whom the present syllabi did not seem to suit. Under the chairmanship of Richard Coughlan, it reported to the delegate conference held in March the following year and a committee was established to draft a syllabus (Coughlan, 1978, p.3-9). A proposed course on social mathematics was published in

the IMTA Newsletter. A trend of teacher involvement can thus be identified, even if the large majority of mathematics teachers were not involved.

Consideration was also being given to the Leaving Certificate and a sub-committee of the IMTA had been set up to investigate the rationalisation of the Senior Cycle courses. It never met, but two of its members Seán Ashe and Professor Tony O'Farrell produced a report which set out a rationale and draft courses for all second level mathematics (personal correspondence with Seán Ashe). This was presented to a second delegate conference in November 1978 and was received enthusiastically. A partnership was now growing between the IMTA and third level institutions. The autumn Delegate Conference of 1978 established a syllabus committee of post-primary teachers and draft syllabuses were discussed and amended at the AGM, by branches and again at the 1979 Delegate Conference. The syllabuses were submitted to the Department of Education in 1980. The IMTA continued to press the Department for action and a consideration of the syllabuses. It also received support from the teacher unions and the main managerial bodies. The IMTA were now becoming proactive.

The Intermediate Certificate. syllabus committee was convened in Autumn 1982. The IMTA, participated in this committee via its representative Seán Ashe who reported back to Council meetings. The history of the role of the IMTA during this period is well documented by Oldham (1992a) drawing on the minutes of Council meetings. When the Leaving Certificate committee was convened in 1988 to work on a follow-up at Senior Cycle, the IMTA were again very active. Many Branch

meetings were held critiquing drafts of the syllabuses and strong views were expressed on the number of syllabuses to be provided, an issue which is discussed below. The IMTA representatives were the most committed of the constituencies to the practical contributions to the work of the syllabus committee (Oldham, personal communication). However from the mid-90s on the response by members to the meetings about the Review of Junior Certificate lessened.

New possibilities for professional development 1990s -2005

In 1994 a Summer course for teachers was organised by the Department of Education in Marino, Dublin. It was a residential one-week course and approximately two hundred teachers attended. The topic was the new material on the Higher Level Leaving Certificate course. The IMTA received funding from the Department of Education and organised the in-service, travel and subsistence payments to teachers who attended. Four lectures were given out in parallel tracks and teachers from experienced Higher Level Leaving Certificate teachers to novices were catered for. The in-service courses consisted of expert teachers giving an overview of how they were teaching the topics, followed by question and answer sessions. This was also the first time that marking schemes were discussed openly in a transparent fashion. Further residential courses were organised by the IMTA (again funded by the Department) for the Summers of 1995, 1996, 1997 and 1998 in Maynooth, Co. Kildare.

Towards the end of the decade, the emphasis was on the Junior Cycle. An innovative development of workshops approach to teacher in-service was incorporated for the

first time in the Galway Summer course 1999 organised by the IMTA again. The following observations from the workshops were made by Diane Birnie, Michael Brennan, Alan Monnelly and Ciaran O'Sullivan who were the workshop facilitators:

- Many teachers were apologetic when expressing their ideas;
- Workshops may be a way of improving teachers' self-esteem and enabling them to see the value of what they have to offer;
- Some teachers found working in groups encouraged them to use group work in their classroom;
- Teachers in general were willing to share ideas;
- Getting teachers to stand up, report back, and summarise on the groups' ideas at the end of the workshop was a challenge;
- The sight of teachers actively involved in a session as opposed to being passive at a lecture, which was the norm at summer courses, was encouraging (1999, Reports unpublished).

The concept of workshops was new and in the following Summer course held in Galway workshops were included again. The final Summer course was held in 2001 in Thurles (Tipperary Institute of Technology) and was organised by Seán O'Tuama. Funding for summer courses was then withdrawn. The IMTA however continued running courses throughout the school year typically to give at branch level, typically focusing on the content of Leaving Certificate Higher Level mathematics, with expert teachers sharing with teachers how they cover various topics. These courses continue to the present. The emphasis however is on teachers teaching rather

than students learning. The IMTA are also involved in “post mortems” held on examination papers each year and branch meetings on this topic are often the meetings that are best attended. These meetings, according to a contributor to the NCCA review, are often “obsessively devoted to what will get marks rather than what may improve students’ learning or what might be good mathematics education” (NCCA 2003, p. 13).

The IMTA has passed through many phases since its foundation in 1965. The phases can be summarised as (a) one of support and leadership to (b) involvement, negotiation and partnership to (c) one of relative passivity. It has returned to the tried and trusted ground of offering courses to teachers, in many cases based on the content of Leaving Certificate Higher level mathematics that the IMTA offered in the 1960s. Whatever benefits this practice may have for teachers’ immediate needs it can be seen to lack a professional developmental focus.

Teacher education – Lifelong learning

An adequate mathematical education for a nation's children at all levels ultimately depends upon the quality of its mathematics teachers. This in turn is dependent upon the education and training of such teachers, which must be adequate, appropriate and thorough. It is appropriate therefore to examine the tripartite “3Is” structure i.e. Initial, Induction and In-service (see on) of the present training system for the teaching career of post-primary mathematics teachers.

The Teachers' Registration Council, originally established in 1918, laid down requirements for post-primary school teachers. With regard to teaching mathematics in secondary schools, the essential requirements are a primary degree from a recognized university with at least one recognised teaching subject and a qualification in teacher education and training. There was no requirement to have studied mathematics at university, or to have taken the subject in a degree examination. For the most part, the initial training of post-primary school mathematics teachers is conducted within the universities which adopt the consecutive model i.e. where the professional/pedagogical element follows the undergraduate academic course. Theoretical and methodological issues are covered and students are required to engage in teaching practice in approved schools and under authorized supervision. Such a brief period of pedagogical training was acknowledged in the 1995 White Paper as inadequate for the very demanding profession of teaching: "Students are faced with so many and varied demands, that there is little time to reflect on their course content and their experience in a satisfactory manner" (Coolahan, 2002, p. 17). The newly-established Teaching Council (www.teachingcouncil.ie) is putting more stringent requirements in place.

Teacher educators acknowledge that second level pre-service education can at its best be a good start, cultivating basic pedagogical skills but also providing motivation to continuing professional development. On paper the Government have made a commitment to reform pre-service education in the 1995 White Paper (p.124-125). The government of the time in the White Paper pledged itself to the three Is: Initial, Induction and In-service training, an acknowledgement that the teaching profession should be one of continuous professional development. The

requirement of a minimum of one hundred hours of teaching practice is evidence of the inadequacy of the Higher Diploma course and consequently, a disproportionate part of the post-primary mathematics teacher's training has to be gained at the expense of pupils. Undoubtedly, there can be little real opportunity for acquiring an insight into such issues as effective mathematics teaching, class discipline, teaching aids and the merits of action-research for the improvement of classroom practice. In effect, the course in pedagogical training is much too brief for post-primary mathematics teachers to acquire a proactive and self-renewing perspective on their teaching which ought to be characteristics of the future lifelong learning mathematics teacher. The Teaching Council is changing the requirement of 100 hours of teaching to 200 hours of school experience – at the time of writing it remains to find out exactly what will count as school experience!

The second phase of the tripartite “3Is” is induction. The quality of a teacher’s experience in the early years of teaching is important for the development and application of knowledge and skills acquired during initial training and for the formation of positive attitudes towards teaching as a career (White Paper 1995 p.125). Beginning teachers are sometimes thrown in at the deep and this can leave them feeling isolated and unnecessarily stressed. Coolahan (2002, p. 25) and Darling-Hammond (1998, pp 5-15) reported that teachers who experience continuous support under a mentor are less likely to leave the profession and more likely to go beyond personal and classroom management concerns to focus on student learning sooner. The purpose of induction is the further development in newly qualified teachers of those skills, forms of knowledge, attitudes and values that are necessary to carry out their roles effectively. A comprehensive induction system would involve

planning, finance and school mentors but it is important to support and protect the beginning teacher who often starts off their career as a substitute or part time teacher. On paper the government has made a commitment to the development of the induction year for second level teachers: "A well developed and carefully managed induction programme, coinciding with the teacher's probationary year, will be introduced for second level teachers" (White Paper, 1995, p. 125). The National Education Convention 1993-1994 "gave a general welcome to the proposal for a structured induction year into teaching, following initial training" (*Report on the National Education Convention*, 1994, p. 86).

The third phase of the tripartite "3Is" is in-service education and training. Since 1995 the government in the White Paper has accepted the need for providing in-service education and training for teachers. The 1991 OECD Review concluded that the challenges that face the teaching profession in Ireland are:

How to address in a comprehensive way the needs and aspirations of talented and well-educated young teachers as they progress through their careers...we believe that the best returns from further investment in teacher education, will come from the careful planning and construction of a nationwide induction and in-service system using the concept of the teaching career as the foundation (OECD, 1991, p.98).

The *Report on the National Education Convention* (1994) concludes "It was agreed that *sine qua non* was provision for the in-career development of teachers, following very good initial education and teacher induction experiences" (p. 135).

It is widely recognised that an initial training of one year is inadequate for a career that can extend over forty years. In-service training is important for teachers to keep abreast of new developments in knowledge and pedagogy. Teacher isolation can be an occupational hazard. Best practice in in-service provision has been identified in the OECD Report, *Staying Ahead: In-service Training and Teacher Professional Development* (1998, pp 53-59), as one which incorporates on and off-site school dimensions, teachers setting the agenda, active engagement by teachers, use of innovative techniques and teachers themselves becoming facilitators and working with their peers. All of this gives rise to a sense of empowerment for teachers. Interactional techniques are increasingly in favour. This contrasts with the lecture type in-service to large groups which has been a feature of many in-service programmes provided by the DES. The OECD Report (1998) also commented that:

Improved planning, more involvement of teachers, better evaluation and dissemination will all strengthen the concept of professional development which must be seen to begin with pre-service and continue through a teacher's career. Professional development is not simply an "add-on" or a "quick fix" to be applied when a particular problem arises (OECD, 1998 p.56).

Syllabus development

Curriculum changes in mathematics have come in waves. For nearly forty years 1924 to 1964 there was little change. With such staleness in the system, small wonder the arrival of the "new mathematics" created great excitement. With the benefit of hindsight we can now say that many teachers did not understand the implication of the "new mathematics". Syllabus committees were not established until 1965 so teachers had no experience of involvement in them. The curricula were drawn up by the Department of Education and the Universities, who sometimes had an inadequate

understanding of the clientele who were going to receive these programmes. As noted earlier, academic material found its way onto mathematics curricula for younger junior cycle students in the sixties. The year 1967 saw the introduction of “Free Education” but few saw the unhappy consequences that the “new mathematics” would have for large numbers of students. Between 1961 and 1980 there was a 321 % increase in the numbers studying for the Leaving Certificate. Since then the numbers continued to rise from 36,539 in 1980; to more than 55,000 candidates (www.examinations.ie) in 2005. The “new mathematics” proved unsuitable for a large majority of students. Another problem with the “new mathematics” was that teachers were uncertain about some of the geometry aspects of the course (Oldham, 1980b, pp 335-336), and this inevitably had further negative effects on students.

Changes in the 1973 Intermediate Certificate course raise many questions. Syllabuses introduced in 1973 saw changes made to geometry which pushed it further down the road of new mathematics. If it was clear that this ‘hybrid’ aspect to geometry was seen as a problem, the question arises as to why it was embraced even further? A related question is: if teachers were having problems with teaching the geometry why then did they not communicate these more effectively to the syllabus committee? Teachers have representatives on these committees and were asked to make submissions. A conclusion that can be drawn from this is that most teachers are content to accept a syllabus passively and deliver it in turn to their students. The axioms on which geometry was based for the 1973 courses were meant for teachers only, and were printed in the syllabus, but found their way into textbooks, and teachers in turn taught them to students because they were in the textbook. It would

seem many teachers were ignorant of what was on the syllabus or alternatively does this point to the lack of ownership teachers have over their own teaching? The *Rules and Programme* for the 1973 courses divided the work into three years and as textbooks followed the syllabus what was in the *Rules and Programme* found its way into textbooks and teachers who were teaching from the textbooks were introducing first years to abstract and formal geometry instead of using a more intuitive approach. One wonders were teachers questioning this approach in their classrooms or simply following the book. It seems very clear that geometry was taking centre stage of syllabus reform, yet as early as 1960, leading practitioners had expressed concern that the teaching and learning of geometry was problematic (Oldham, 1992a, p.136).

Changes made to the Leaving Certificate courses in 1976 meant continuity for students and some developments were brought about by these changes for the less able student. Unlike the changes made at Junior Certificate the changes at Leaving Certificate saw the first turn away from increasing abstraction. Integration and groups were dropped from the Ordinary level courses. At Higher level some aspects of abstract algebra could be avoided by a choice of options. It was twelve years since the initial introduction of the “new mathematics” courses at Leaving Certificate level and Irish society had developed in the intervening years, as had the student cohort with the introduction of free education. The courses were being taught in classrooms very different to the ones “new mathematics” was designed to be taught in.

More changes were to follow in 1987 for the Intermediate level courses. Three courses were now introduced, taking into account the needs of the weaker students. This was a significant move. Once again geometry took centre stage in discussions about the courses to offer. The “new mathematics” courses were still wagging the tail of any meaningful curriculum reform. The changes left an incongruent situation between Syllabus A and Syllabus B in terms of their approach to geometry. The course committee had advocated an approach based on congruence for both levels but due mainly to the influence of academic figures (Astir, May 1986) syllabus A was redrafted in a way that left it at odds with Syllabus B and also left it abstract in nature. A more appropriate course at junior cycle level should be without the rigour and abstract nature that university “academic” mathematics has. Whilst three levels for examination in 1990 were established, and while there was recognition that there were students for whom the higher and ordinary courses were not suitable, the objectives remained unchanged. Logically a change in content and levels for examination would mean that objectives had changed.

The course committee had advocated the use of calculators for the Intermediate examination of 1990 and the CEB had assented and designed examination papers with this in mind. Yet nothing happened in the short to medium term and calculators were not introduced for Junior Certificate until 2000. In 1987 the CEB also approved providing in-service support, monitoring the new courses and providing specific objectives. None of these happened at the time however. Whatever the reasons were there seems to have been reluctance on behalf of the Department of Education to embrace new and progressive measures. Neither were the courses presented in the

NCCA Junior Certificate format, as had happened with other subjects as the courses had only been introduced in 1987.

The Leaving Certificate committee was convened late in 1988 to produce two or three courses to follow on from the Intermediate Certificate courses which were examined in 1990. For reasons outside the scope of thesis, the Department's decision on whether to offer two or three courses at Leaving Certificate level was not made for some time, so an Ordinary Alternative course was introduced (authorised on a year-by-year basis) for those who had followed the third (Syllabus C) of the Junior Certificate courses. The Ordinary Alternative course was introduced in 1990 for first examination in 1992. The students who sat the Intermediate Certificate examination in 1990 and took the Higher or Ordinary course (Syllabus A or B) had to go onto Leaving Certificate courses that had not been changed. Students who had taken the Syllabus B course and had been taught an approach to geometry based on congruence were now being taught geometry using the "new mathematics" approach. Mathematics is a subject that requires logical thinking but this transition does not appear logical. However the Leaving Certificate committee were not blameworthy in this regard as they did not know how many levels they were writing courses for. Eventually the package of three courses was sanctioned. The following principles influenced the design of all three courses:

They should provide continuation from and development of the courses offered in the Junior Cycle...They should be implementable in the present circumstances and flexible as regards future development. They should be teachable, learnable and adaptable.....They should be applicable, preparing students for further higher education as well as for the world of work and for leisure.The mathematics they contain should be sound, important and interesting.

(DES/NCCA, 1992, pp. 3-4)

The Higher and Ordinary courses introduced in 1992 were first examination in 1994. The Ordinary Alternative course was examined for the last time in June 1996 and was re-designated the Foundation Level course in 1995 for examination in 1997.

The changes included modernisation of the mathematics content (with regard for teacher knowledge and student learning), shortening of the Higher level course, the introduction of the three-part questions to allow for a gradient of difficulty in questions. These changes made the Higher level more accessible. At Ordinary level the changes provided a place for more challenge. There were several constraints operating on this revision: (i) maintaining acceptability of Higher and Ordinary level mathematics for third level courses (including those in the UK), (ii) gaining acceptability for Foundation level where possible, (iii) dealing with the problem of low numbers taking the "old" Higher level and (iv) the high failure rate at the "old" Ordinary level. These constraints are documented by Oldham (1992b, 1993, 2007). The numbers taking the Higher level did increase very considerably, and the Ordinary level failure rate halved its worst figure from the preceding period. While these two achievements were promising and boded well for the future the focus was still on explicit rationale, content and assessment not on actual approaches to teaching and learning.

The last wave of change documented here was the change made to the Junior Certificate Mathematics programme in 2000. The course committee drew up a syllabus and the most notable features were changes made to geometry and the introduction of calculators. Geometry saw a return to a presentation based on

congruence which had been sidelined when the “new mathematics” was introduced. Other areas of difficulty were pruned and teaching approaches to algebra, which was proving to be problematic back in the 1960s, were now being considered, albeit in a small measure. There have been no changes made to the Leaving Certificate syllabuses since 1992 (except the minor ones to the Ordinary Alternative syllabus when re-designated as a Found level). The absence of continuity for the pupils is regrettable.

Pilot Initiatives

A few pilot projects relating to the teaching and learning of mathematics, syllabus design and initial teacher training have been conducted in the state. For instance: The Individualiserad Matematik-Undervisning Project (IMU Project), An Alternative Mathematics Project for Leaving Certificate pupils by the Vocational Education Committee (VEC) of County Tipperary (North Riding) and The National Pilot Project on Teacher Induction. Each of these will now be considered in turn.

The I.M.U. Project:

IMU stands for *individualiserad matematik-undervisning*, which means individualized mathematical teaching and an independent learning system to teach mathematics to junior cycle students. It was developed in Sweden in the 1960's and the aim was to allow students to proceed at their own pace and level of difficulty, while in a mixed-ability classroom. The system involved students working individually on a series of booklets which contained all the necessary instruction, exercises and tests. These were available at a number of levels of difficulty. The teacher administered the

system and was available for small group tuition or individual consultation. Through the Centre for Educational Research and Innovation (CERI), the system was introduced on an experimental basis to a total of twenty-one post-primary Irish schools in the period September 1970 to June 1972 to teach junior cycle mathematics.

Mr. O'Keefe and Mr.Looney, two inspectors from the Department of Education travelled to Sweden in 1969 and recommended that the IMU system be introduced on a trial basis in Ireland. The primary reason for piloting the IMU project was the interest in devising means of dealing with mixed-ability classes (Report on the I.M.U. Project, 1972, p. 6). In the 1970s comprehensive and community schools were beginning to make an appearance in Ireland and it was felt that classroom techniques which had may have enjoyed some success with streamed classes would not be as successful with mixed-ability groups. The involvement of the Department of Education in the IMU Project was part of a larger feasibility study by the Centre for Educational Research and Innovation (CERI) of the international transfer of learning systems.

The IMU project has provided valuable insights into the operation of an innovation in the Irish post-primary context:

- Diverse styles of practice existed among IMU teachers in classrooms were observed, demonstrating that the mathematics teacher had a significant role to play;

- In the second year, teachers were more inclined to control student progress than had been the case in first year and to reduce the time spent working with individuals (Report on the I.M.U. Project 1972 p. 19);
- A majority of teachers felt that IMU facilitated good student-teacher relationships and general satisfaction was expressed with IMU materials (Report on the I.M.U. Project 1972 p29);
- The methodology of IMU was found to be at variance with the traditional approach to mathematics teaching which existed at the time. The IMU approach was of a *spiral* or cyclical nature while the tradition in Ireland at the time was to develop a smaller number of topics fully in the early stages - more of a *vertical* approach. The report suggested that this difference in approach deflected teachers' attention somewhat from more important features of the system and caused some practical difficulties for them (Report on the I.M.U. Project 1972 pp 27-28);
- The IMU system was competing with the constraining influence of the examination system in the sense that IMU was not part of the public examinations;
- Expertly guided group discussion and co-operative projects could greatly facilitate the integration and consolidation of mathematical knowledge (Report on the I.M.U. Project 1972 p. 33);
- IMU students were more favourably disposed to mathematics than their control group counterparts, although the level of enthusiasm dropped in the second year of the project. The main reason postulated for this decline was difficulties with the normal second year mathematics syllabus, resulting from

the discrepancy between the mathematical content of IMU and the usual first year course (Report on the I.M.U. Project 1972 p. 34).

Retrospectively, the IMU system must be seen in the context of innovations that were totally new to Irish mathematics teachers. Indeed Crooks & McKernan (1984 p. 32) have noted that in many respects, IMU was the precursor of much curriculum development work in Ireland. However, the impact of the innovation was restricted by the minimal training received by mathematics teachers in the use of the system. A further restriction was the influence of a strong allegiance to traditional patterns to teaching and learning which stood in contrast to the principles inherent in the IMU approach. These contextual constraints prevented IMU from functioning effectively. The IMU project had provided mathematics teachers an opportunity to adopt a new role and pedagogy for mathematics teaching. But the project came and went without a follow-through of work which would have capitalized on the experience gained.

The Tipperary VEC Alternative Mathematics Programme:

In 1979, the Vocational Education Committee (VEC) of County Tipperary (North Riding) reviewed the relevance of its educational provision at post-primary level to the needs of the economy. Recommendations for future policy in the administrative district were included in a report entitled *Post-Primary Education 1985-2000 and its relevance to the Economy: A Policy Document*. One recommendation called for the establishment of an alternative course in mathematics at senior cycle to be devised, examined and certified by the V.E.C. (p18). A number of factors had been identified as being influential in this area of concern (O'Donoghue & Murtagh, 1983, p. 42) :

1. failure rates in state examinations;
2. students' frustrations and lack of success with conventional mathematical syllabi;
3. parents' concern for their children's future employability;
4. employers' dissatisfaction with the preparation of school leavers for work;
5. an awareness of the international debate on the re-evaluation of the merits of the new modern approach to mathematics;
6. an awareness that educational goals and the needs of the economy were not necessarily incompatible aims.

O'Donoghue & Murtagh (1983, p. 43) articulate the prevailing attitude at the time:

Doubts persisted that existing school Mathematics courses fulfilled the aim of supplying pupils with the essentials for living. State syllabuses served the needs of only a minority of students, namely those aspiring to a career in mathematics. The practice of subjecting all the nation's children to a particular brand of Mathematics, which resulted in a large number of children failing to master even the basics after five years of second-level schooling, must be questioned.

A working party was set up to devise a new alternative mathematics programme. The aim of the programme was to meet the needs of senior cycle students who chose to go directly from school to employment by providing a mathematical programme designed to help them deal competently with the mathematical implications of living and making a living in a modern technological society. The project involved dialogue and cooperation between the VEC, mathematics teachers, a local third level lecturer in mathematical education together with the involvement of employers.

Certificates were awarded to successful students by the VEC. The credibility of the project received a boost when An Comhairle Oilúna (AnCO, the National Training Authority) recognized the certificate for the purposes of apprenticeship and for clerical appointments. However, the Department of Education refused to recognize the course as an alternative to the Leaving Certificate Ordinary mathematics programme. Although a number of post-primary schools outside the North Riding Tipperary VEC district did adopt the new course, dissemination on a national scale did not occur.

In an external evaluation of the project (Bajpai, 1983, p.15) noted the enthusiasm and commitment of students who pursued the course and the teachers who taught it. The evaluation suggested (p.17) that a primary reason for the small uptake of the project in schools was due to the non-recognition of the programme by the Department of Education in terms of equivalence to the Leaving Certificate Ordinary level examination. The project constituted an interesting project in local school/scheme-based curriculum development. For mathematics teachers to take up the challenge and adopt new teaching styles and the emphasis towards application and relationships to "real-life" situations were all commendable features which deserved wider support. The realization of the project was the culmination of collaboration and dialogue between educationalists and industrialists. The concept of this project is relevant to mathematics education today. Its failure highlights some key difficulties in implementing curricular change in Ireland. The aims of the Leaving Certificate 1992 revision, and the problems encountered with regard to the introduction of the

third course (Foundation Level) and its appropriate recognition, as discussed above echoes the aims and difficulties encountered here by this project.

National Pilot Project on Teacher Induction:

The National Pilot Project on Teacher Induction came into operation in 2002 with the aim of developing a national policy on teacher induction. It was funded by the Department of Education and Science (DES) and was based on a partnership model involving the DES, St. Patrick's College and the Education Department at University College Dublin, the teacher unions and the education centres. The project was piloted in primary and post-primary sectors. The general aim of the project was to develop proposals for an effective programme of induction for newly qualified teachers which would be tailored to their particular professional needs and sensitive to the strengths, requirements and challenges within the Irish education system. For the purpose of the project newly qualified teachers (NQTs) were defined as teachers in their first year of full-time teaching, either in a temporary or a permanent capacity.

Various induction models were devised, depending on the circumstances of the schools and the NQTs. Methods of delivering support ranged from mentor training, mentor support for inductees, time for observation and planning, professional development seminars, support for NQTs at a whole school level, involvement of education centres, website development, dissemination of information to the inspectorate, and the incorporation of expertise from teachers and teacher educators.

An analysis of the needs of NQTs was conducted at the beginning of the project. The needs analysis revealed that induction provision was required urgently if attrition from the teaching profession is to be prevented. Less than one third of all NQTs entering the teaching profession had access to any form of induction support. Most beginning teachers found that their first experience of teaching was enjoyable and rewarding and for the most part managed well. However, NQTs who had the opportunity to avail of an induction programme experienced a significantly higher level of professional support from all the teachers in the school and that it is clear that for induction to be successful there needs to be a supportive culture in the school.

The project took place in phases, involving schools in the Dublin area and later in neighbouring counties, in a variety of school types. Workshops and seminars were provided for both NQTs and mentors, and there was also a school-based element, which generally took the form of ongoing consultative advisory meetings between mentor and NQTs and arrangements for classroom observation. Findings and recommendations from the project have not to date been published by the DES. (National Pilot Project on Teacher Induction, 2005)

Teacher Involvement in Curriculum Development in Mathematics: Two International Case Studies

In this section we will explore two international case studies that involve teacher involvement in curriculum reform: (i) Lesson Study and (ii) The Research Institutes on Mathematical Education (IREMs).

Lesson Study

Lesson Study is a form of teacher development used in Japan to improve the teaching and learning of mathematics. In all of the cross-national studies of mathematics achievement, from the First International Mathematics Study (FIMS, 1964), the Second International Mathematics Study (SIMS, 1976-1991), TIMSS (1995) to PISA (2000, 2003), Japanese students have consistently performed extremely well and have had low variance nationally in scores on these studies. The TIMSS 1995 and 1999 video studies reveal that the quality of teaching and learning found in Japanese classrooms is particularly high. According to Conway & Sloane the teaching and learning practices in Japanese classrooms should be emulated where possible worldwide (Conway and Sloane 2006). Researchers such as Lewis (2002), Stigler & Hiebert (1999), Willis (2002), and Chokski & Fernandez (2004) have credited Japan's steady improvement in mathematics teaching to teacher-led Lesson Study. Lesson Study is essentially part of the culture of teaching in Japan.

Japanese Lesson Study, as described by Yoshida (2002), involves teachers working in small collaborative groups to examine and develop their practice systematically. The teachers meet in groups from a single school, or from across various schools. The meetings are conducted by teachers in clubs, special interest groups, professional organisations and in-service teacher training programmes. The most popular format is for teachers to meet in school. The meetings discuss in a sustained and focused manner how to improve the effectiveness and quality of the learning experience that they, as teachers, provide for their students, based on an examination of how their students learn in different content areas. They think about the kind of student they want to see develop and identify gaps between their aspirations and reality. To do

this they take a closer look at students' test performances, interview students and engage in peer observation. They identify a goal that aims to narrow the gap between their hopes and the current state of affairs.

Once a particular goal is agreed upon teachers co-plan a classroom lesson. This planning involves the actual activities students will do and the teachers anticipate students' responses. One teacher from the group teaches the actual lesson while the others from the group observe and collect data (transcribe teacher/student interactions) to study the impact of the lesson on student thinking, learning and understanding. The teachers then debrief the lesson and share observations. The lesson may then be revised and taught to another class by another teacher from the group. After further reflections and discussions and revisions, critically involving a mathematics advisor, the Lesson Study group reports the results of the lesson so that other teachers can learn from it. These research lesson reports are often distributed nationwide. Chokshi & Fernandez, (2004, p.524) point out that it is essential to recognise that:

The central idea of Lesson Study is that it is meant to be a generative process through which teachers continually improve and redirect their teaching as needs arise from their students and classrooms. Lesson Study is therefore not meant to be a vehicle for teachers to assume an entire set of static teaching practices.

In general, the type of practices that teachers engage in with Lesson Study, are not a standard practice among Irish mathematics teachers.

The Research Institutes on Mathematical Education (IREMs)

The Research Institutes on Mathematical Education (IREMs) were established in France in 1969 to accompany the reform of the “new mathematics” curriculum that was introduced to post-primary schools at that time. While the provision of in-service education to mathematics teachers for the “new mathematics” was minimal in Ireland, as we have already seen from chapters 1 and 2, the French embarked at that time on what they saw as the dual necessity of retraining (primary and post-primary teachers) and researching new teaching methodologies. The IREMs detected and analysed the serious difficulties (discussed in chapter 1 and 2) to which the reforms in the sixties led. In that context IREM members realised that a study of the relationship between teaching and learning mathematics was urgently needed. This type of study would represent a crucial venture in Irish mathematics education, yielding, as in France, fruitful pedagogies for teachers at all levels. The IREMs carry out research on mathematics education, take part in the pre-service and in-service training of teachers and publish and disseminate relevant research documents for teachers and teacher trainers. There are now 26 IREMs in France which coordinate this work. In Ireland there are 21 full-time and 9 part-time Education Centres which are potentially well located to take up at least some of the kinds of practices that the IREMs engage in. Of particular interest in this regard is the work of the IREMs together with practising teachers. This includes: the dissemination of reports on teaching experiments, and of studies of mathematical topics considered from a teaching point of view; the furnishing of tools for the classroom, including handbooks, software, audiovisual material; the conduct of research on the history, epistemology and didactics of mathematics in liaison with university departments and the participation in a direct way in organising CPD for teachers.

Some Emergent Questions and Issues

Chapter 1 and 2 have raised a host of questions and it is important to consider what key issues have emerged from taking a chronological historical survey (chapter 1), after considering the influential forces in mathematics education in Ireland since 1924-2005 and the two international case studies above. Some interesting questions emerge and they fall into eight areas:

1. Pedagogical issues
2. Levels of achievement in Mathematics
3. The degree of teacher participation in curriculum change in mathematics
4. Preferences in the provision of inservice support courses
5. Emphasis within inservice provision
6. Initial teacher education and induction
7. Syllabus development
8. Pilot initiatives

1. Pedagogical issues

As far back as 1924 the Department of Education openly criticised teachers using repetitive techniques in teaching mathematics, encouraged teachers to explore different approaches to teaching, and advised teachers on preferred teaching methods. Four decades later, commenting on teaching and learning of the “new mathematics” introduced in 1964, leading practitioners felt that rote learning and drill in the previous courses were over-emphasised (Oldham , 1980 (a), p. 54). In the 1970s teachers were again reminded to be innovative. *The Rules and Programme* for

1973 course changes at Junior Cycle, and the 1976 course changes and preamble to the syllabuses for Senior Cycle, asked teachers to be resourceful, inventive and creative (*Rules and Programmes* 1974/75; 1976/77, p. 64).

More recently in the 1990s the TIMSS 1995 international study indicates that Irish mathematics teachers' teaching styles are problematic. Beaton et al. (1996), made a worrying finding that Irish mathematics teachers generally rated lower-order abilities of remembering formulae and procedures more highly than higher order abilities. The latter would include providing reasons to support conclusions, thinking creatively and using mathematics in the real world. This is a revealing criticism of practices in Irish mathematics classrooms (Beaton et. al., 1996, pp. 138-143). This is borne out by other studies. National studies show Irish teachers to be more traditional in their methodologies than progressive (Lyons et al., 2003). It would appear that in some key aspects the teaching of mathematics in Irish post-primary schools has remained largely unchanged since the foundation of the state. It would be interesting to find out why this is the case and why Irish mathematics teachers have clung to traditional methodologies. It would be interesting to find out how this might be changed. The students in mathematics classrooms today are potentially the teachers of the future. If traditional practices are what a student experiences in a mathematics classroom then it is possible that these are the practices that a student may use when they become a teacher. In this way a cycle is created. Teacher education, in pre-service and later stages of professional development, has a key role to play in breaking this cycle.

2. Levels of achievement in Mathematics

Chief Examiner's reports on state examinations at all levels (Department of Education 1995, 1996, Department of Education and Science, 2001) point to a high proportion of students having difficulty with elementary concepts. Recent statistics based on 2004 examination results provided by The State Examination Commission (SEC) (www.examinations.ie) indicate a worrying trend for students at both Senior and Junior Cycle. At Leaving Certificate 18,080 students achieved a grade D or less at Ordinary and Foundation level. This represents 30% of the total number of students who sat the Leaving Certificate in 2004 at these levels. A total of 2,130 students, which represents 25% of the Higher level mathematics students, achieved grade D or less in the examination. The foundations for this unfortunate situation begin at Junior Cycle level. A total of 7,091 students achieved a grade D or less at Ordinary and Foundation level Junior Certificate mathematics in 2004. This represents 22% of the total number of students who sat the Junior Certificate at these levels. A total of 6,143 students achieved a grade D or less at higher level. This represents 27% of the students who sat higher level. Not only do these percentages represent a worrying "tail"; they also indicate that the seeds sown badly at Junior Certificate level continue to bring trouble at Leaving Certificate level. The results from the Leaving Certificate 2007 give cause for serious concern. Out of a cohort of 53,926 students, only 15.6% sat the Higher Level Leaving Certificate mathematics paper and just 12.5% of these secured an honours grade. This figure contrasts with 44% achieving a similar grade in English Higher Level. The development of a knowledge economy depends on a strong supply of scientists, engineers and technologists and that this will be seriously undermined if we do not address the requirement of having a strong foundation in mathematics in secondary level. The

current Minister for Education, Mary Hanafin, stated after this year's (2007) Leaving Certificate results that she was concerned about both the decrease in numbers taking mathematics at Higher Level and the high failure rate at Ordinary Level. The results of 2007 revealed that almost 5,000 students i.e. 10% failed either Higher, Ordinary or Foundation level, making many students ineligible for many third-level courses.

Irish students' performances in International studies in the eighties and nineties have been decidedly moderate (International Assessments of Educational Progress (IAEP) 1988, IAEP 1991, TIMSS 1995, PISA 2000, PISA 2003). As teachers are central to improvements in the mathematical performance and competencies, this raises a serious question as to the reforms that are needed to improve the current initial, induction and in-service programmes that are now on offer.

3. Teacher participation in curriculum change

The degree of teacher involvement and participation in syllabus changes in mathematics has been disheartening. Admittedly teacher involvement in curriculum design did not exist until 1965. Since then teachers have had ongoing opportunities to make submissions to the syllabus committee for changes to Junior and Senior courses. Responses from teachers for changes in the junior courses in the key years of 1973, 1987 and 2000 were poor. This was different however when changes were being made to the Leaving Certificate courses in 1976 and 1992. Teacher submissions were coordinated and submitted in this instance through the IMTA and had a significant impact on the Department of Education. A pattern would appear to have emerged of teachers showing little interest in making submissions when

changes were being made to Junior Cycle mathematics courses, but when changes were being made to Leaving Certificate courses submissions were properly coordinated by the IMTA. Many questions are worth asking here: Are teachers who teach mathematics in Irish secondary schools more concerned about Leaving Certificate mathematics than Junior Certificate mathematics? Are teachers of Leaving Certificate mathematics more qualified than Junior Certificate teachers? If so, does having a qualification in mathematics mean that teachers have a greater interest in the subject than those who do not?

4. Preferences in the provision of in-service support courses

It is opportune to consider that teachers of Leaving Certificate mathematics were the first to be trained in the “new mathematics” as the revisions started at Leaving Certificate level as they were deemed to be more qualified than the teachers at Junior Cycle. Does this say that the Department of Education held Leaving Certificate mathematics teachers in higher esteem than their Junior Cycle colleagues? Anecdotal evidence suggests that these teachers were then expected to deliver to Junior Cycle teachers the “new mathematics”. Whilst there is no proof to suggest otherwise, it is difficult to believe that this happened in schools. Should this have been monitored? The changes to syllabuses in the seventies and eighties at Junior and Senior Cycle had little in service provided. Even though the *Rules and Programme* called on teachers to be resourceful, inventive and creative (*Rules and Programme* 1974/75 p.64), there was no provision made for extra resources or teaching aids for teachers. How did the Department of Education expect these expectations to be met without support? In 1985 calculators were introduced for Senior Cycle and no in-service was

provided to advise on rationale, school issues or implication for the Leaving Certificate examination. Surely such an innovative measure merited support.

5. Emphasis within in-service provision

In-service was provided for the Leaving Certificate changes of 1992 and the in-service concentrated on content changes, examinations and rationale. Unfortunately no mention of pedagogy was made (Oldham & English, 2005). In 2000 the Junior Certificate Revised syllabus was accompanied by in-service of a different nature to that of 1965 and 1992. Pedagogical matters were introduced, teaching aids and resources were introduced to teachers. How did the DES expect these innovative methodologies to be used without making adequate provision to make them available to schools? Surely resources need to keep apace to meet the needs of a new course. Apart from Summer courses as this was the first in-service provided for teachers of Junior Cycle mathematics during school time since the foundation of the state clearly in-service education in the area of Junior Cycle mathematics was not a priority throughout the previous decades. Furthermore such in-service as was provided at Leaving Certificate level concentrated predominantly on course content and examination papers to the relative neglect of pedagogical considerations.

6. Initial teacher education and induction

Several ideas are worth considering for initial teacher training. These include that the CAO for the selection of Higher Diploma (now Postgraduate Diploma) students might incorporate elements based on a consideration of the competencies needed for effective teaching and providing improved field

experience. These may be forthcoming with new Teaching Council requirements.

Teacher induction lacks systematic provision. The DES need to consider strengthening the support service available for beginner teachers. It could concentrate on improving their knowledge base and creating an awareness of mathematical research on teaching and learning mathematics. The pattern of in-service courses provided for Irish mathematics teachers seems to be that of a “quick fix”. State in-service courses for Leaving Certificate mathematics have not been provided since the changes to the syllabuses in 1992 (except for Summer courses). At present there is a lack of incentives for practising Irish teachers to involve themselves in continuous professional development. There are limited rewards and recognitions of teachers’ work. There is also limited and appropriate support available to teachers who wish to improve the quality of their teaching. The DES could consider means whereby teachers could integrate professional development throughout their career and gain accreditation or monetary rewards for same. In addition the DES could focus on the need for various forms of continuing professional development for teachers of mathematics and the need to change radically the culture of expectations in relation to professional development.

7. Syllabus development

Mathematics curricula in Irish post-primary education were re-shaped back in the sixties. Since then it is clear that efforts at reform have also been determined by the

courses of the sixties and that new ground and developments have been hindered by a concentration on geometry and neglect of other areas. No radical rethink has taken place. The “new mathematics” courses and subsequent courses failed to communicate effectively to students the usefulness and relevance of mathematics outside the classroom. There was also a lack of attention to the experience of the learner. Students find it difficult to understand what use algebra, coordinate geometry and trigonometry are to them after their examinations are over. Many teachers are not familiar with the practical applications of mathematics because they themselves had not been shown by their teachers when they were in school. Plans for Project Maths have been under way for some time and stem from the NCCA rather than the DES, though consent from the latter was needed in order to engage in the project. The forthcoming changes will see “a much greater emphasis being placed on student understanding of mathematical concepts, with increased use of contexts and applications that will enable students to relate mathematics to everyday experience. The initiative will also focus on developing students’ problem solving skills” (www.ncca.ie).

8 Pilot Initiatives

An interest in pedagogy, curriculum development and pre-service training was aroused by a range of projects in recent decades. However, the projects point to the difficulty of achieving change when the projects’ fruits do not gain acceptance at national level by the DES. Notwithstanding this, the experience and insights gained from all these initiatives highlight the possibilities and the problems that are inherent in achieving worthwhile change. They remain as case studies which can be utilized as a source of material in future attempts to resolve fundamental issues.

This chapter has taken a chronological account of the phases that the IMTA has passed through since its foundation. It has also looked at key questions and issues that chapter 1 and 2 have raised. These include questions relating to the teaching and learning of mathematics in Irish post-primary schools, the level of involvement of teachers in curriculum design. Questions have also arisen about the style of in-service being offered to teachers at Junior and Senior Cycle and the need to examine reforming the syllabus. Open to debate is the present system of induction for Irish mathematics teachers and the experiences to be gained from several pilot initiatives that have been undertaken in relation to the teaching and learning of mathematics, syllabus design and teacher initiation.

The next chapter will undertake a review of the research literature in the context of exploring the quality of teaching and learning in Junior Cycle mathematics.

CHAPTER 3

Exploring the Quality of Teaching and Learning in Junior Cycle Mathematics: A Review of the Literature

This chapter seeks to undertake a critical exploration of mathematics teaching at Junior Cycle with a view to placing in sharper context some of the research aims set out in the introduction. In particular we will be looking at:

1. the dominant patterns in the attitudes and practices of post-primary mathematics teachers at Junior Cycle and their pedagogical consequences;
2. the relationship between teaching practices and students' attitudes to learning mathematics;
3. the effectiveness of the work done by Junior Certificate mathematics teachers as distinct from examination results achieved by their students in state examinations;
4. some key instances of international developments in teaching and learning mathematics.

Different Conceptions of Mathematics and their Pedagogical Consequences

From an educational point of view conceptualizations of what mathematics is varies widely. At one end of the spectrum there is the absolutist perspective and at the other end of the spectrum there is the fallibilist perspective. In this section we will show the distinction between these two perspectives. We shall also show how these two

perspectives of mathematics influence teachers' and students' attitudes towards the subject and the relationship of both to the kind of teaching and learning that goes on in classrooms. As described by a number of thinkers, the absolutist philosophy of mathematics views the subject as difficult, abstract, a rigid logical structure, inaccessible, for the super intelligent, for mathematical minds (Ernest, 2004; Thompson, 1992). Henningsen and Stein give a particularly memorable description of the absolutist philosophy as "A static, structured system of facts, procedures and concepts" (Henningsen and Stein, 1997, p. 524).

Burton (1992), Thompson (1992), and Ernest (1988) associate a didactic approach, "transmission style" pedagogy with the absolutist philosophy of mathematics. According to Ernest (2004) and Thompson (1992), the absolutist view of mathematics is communicated in terms of teaching in school, by giving students mainly unrelated tasks which involve the application of learnt procedures, right versus wrong answers and single approaches to the solutions of problems, coupled with disapproval and criticism of any failure (Ernest, 2004, p. 13; Thompson, 1992, p. 133). Dossey points out that if teachers view their teaching approach to be one where they present a fixed body of knowledge to students then students learn "how" rather than the "why" of mathematics (Dossey, 1992, p. 43). According to Burton, students taught in this way "must accept, understand and manipulate" (Burton, 1994, p. 207). And the learning that takes place is: "by transmission from knowers to novices who search for certainty, singularity and clear definition from their teachers" (ibid.). It is generally assessed "by unseen pen and pencil tests requiring knowledge and skill reproduction" (ibid.). Ernest (2004) maintains in fact that the worldwide consensus among mathematics educators (Howson and Wilson, 1986; Skovsmose,

1994) is that school mathematics must counter this image and offer something personally engaging, evidently useful and motivating to students in our schools (Ernest, 2004, p. 13). According to Thompson (1992) and Kuhs and Ball (1986, p.27), for many teachers, mathematics is the mathematics of the school curriculum instead of viewing mathematics as a larger world of enquiry and questioning (Thompson, 1992, p. 137). Thompson (1992) adds that this narrow static view of the discipline, based on school mathematics, may help explain the preponderance of the absolutist view of the discipline (Thompson, 1992, p. 134).

A different view of mathematics is put forward by the fallibilist perspective, which emphasises the human side of mathematics. Ernest (2004) describes it as an approach that emphasises that mathematics education should be accessible, personally relevant and creative (Ernest, 2004, p. 14). According to Burton a fallibilist perspective admits processes and products and “emphasises the interaction between individuals, society and knowledge out of which mathematical meaning is created” (Burton, 1995, p. 277). Hersch gives a good example of the kind of thinking that informs the fallibilist view:

Anyone who has even been in the least interested in maths, or has even observed other people who were interested in it, is aware that mathematical work is work with ideas. Symbols are used as aids to thinking just as musical scores are used as aids to music. The music comes first the score comes later. Moreover, the score can never be a full embodiment of the musical thoughts of the composer. Just so, we know that a set of axioms and definitions is an attempt to describe the main properties of a mathematical idea. But there may always remain an aspect of the idea which we use implicitly, which we have not formalized because we have not yet seen the counterexample that would make us aware of the possibility of doubting it (Hersch, 1986, p. 18-19).

A key distinction between the absolutist and fallibilist perspective (which is not too common in Irish schools) is that the emphasis is on students making mathematics, not solely a transmission of information from the teacher. As Nickson (1992) so aptly puts it: “The pupils are actively engaged in doing mathematics and in the process, they are receiving messages that mathematics is about questioning, conjecturing, and trial and error” (Nickson, 1992, p. 110). According to Dossey (1992) mathematics viewed by teachers from a fallibilist perspective encourages students “to construct their own mathematical ideas and procedures by attempting to mathematise meaningful problem situations” (Dossey, 1992, p. 45). Yet according to Thompson (1992) the traditional emphases in classrooms has been on the mastery of symbols and procedures (the absolutist perspective) largely ignoring the processes of mathematics and the fact that mathematics knowledge often emerges from dealing with problem situations (the fallibilist perspective) (Thompson, 1992, p. 128).

Inside Classrooms (Lyons et al., 2003) suggests that Irish mathematics teaching and learning would appear to sit in the absolutist tradition (Lyons et al., 2003, p. 4) and that in Junior Cycle mathematics classrooms “the subject is presented as static rather than dynamic, abstract, formal and remote rather than relevant and accessible” (Lyons et al., p. 363) and that this is accompanied by instrumental as opposed to relational understanding. It is now worth considering the crucial difference between instrumental and relational understanding in some detail.

Skemp (1978) distinguishes between two different kinds of understanding in relation to mathematics accounting for sharp differences in instructional approaches and

emphases. Skemp (1978) described relational understanding as “knowing both what to do and why”. With respect to instrumental understanding which is knowing “how to do sums” he noted:

Instrumental understanding I would until recently not have regarded as understanding at all. It is what I have described in the past as “rules without reasons”, without realising that for many pupils *and their teachers* the possession of such a rule and ability to use it, was what they meant by “understanding”. (Skemp, 1978, p. 9)

Skemp (1978) proposed a corresponding distinction between instrumental knowledge of mathematics and relational knowledge of mathematics, the distinction reflecting the type of knowledge each contains. According to Skemp (1978), instrumental knowledge contains “fixed plans” for performing mathematical tasks. The plans prescribe a step by step procedure to be followed, each step determining the next, “The kind of learning that leads to instrumental knowledge of maths consists of the learning of an increased number of fixed plans, by which pupils can find their way from particular starting points to required finishing points” (Skemp, 1978, p. 14).

On the other hand relational knowledge of mathematics is characterized by the possession of conceptual structures that enable the possessor to approach a task in various ways. As Ma, L. (1999) puts it, in learning relational mathematics, the learner acquires knowledge that is not fragmentary but is a coherent whole and allows for flexible understanding (Ma, L. 1999, p. 122). Skemp (1978) makes it clear that:

..we are not talking about better or worse teaching of the same kind of maths...It has taken me a long time to realise that this is not the case. I used to think that maths teachers were all teaching the same subject, some doing it better than others. I now believe that there are two effectively different subjects being taught under the same name “mathematics”. (Skemp, 1978, p. 11)

Hiebert and Carpenter (1992) distinguish between two different kinds of knowledge in relation to mathematics: procedural and conceptual knowledge which parallel Skemp's relational and instrumental understanding. For Hiebert and Carpenter (1992) procedural and conceptual knowledge are both crucial for mathematics expertise. According to Hiebert and Carpenter (1992) the advantage of procedural procedures that are practised and memorised is that they "can be executed quickly and efficiently with relatively little mental effort" (Hiebert and Carpenter, 1992, p. 78). However it is not procedural knowledge but conceptual knowledge that promotes understanding (Hatano, 1988; Hiebert and Lefevre, 1986). For Hiebert and Carpenter (1992) when encountering problems that differ from those for which a procedure was initially learned, conceptual knowledge extends the range of applicability of mathematical procedures (Hiebert & Carpenter, 1992, p. 78) and as Ma L., (1999) points out gives the student the capability to pass through all parts of the field-to weave them together (Ma, 1999, p. 121).

Given that conceptual and procedural knowledge are both crucial for mathematics expertise, which should come first when teaching mathematics, procedural knowledge or conceptual knowledge? Hiebert & Carpenter (1992) amongst others (Goldin, 1987; Hiebert, 1988; Kaput, 1987) would say meaning (conceptual knowledge) should come before practising the rules (procedural knowledge) for effectual execution. They suggest that "learners who possess well practised automatized rules are reluctant to connect the rules with other representations that might give them meaning. There is a tendency for students to persist in using procedures that are well rehearsed without reflecting on them. Gestalt psychologists call this "functional fixedness". When a particular approach or procedure is practised

so many times it can become fixed, making it difficult to think of the problem situation in another way” (Hiebert and Carpenter, 1992, p. 78). As a procedure is practised repeatedly the individual pieces of knowledge lose their identity and become part of a single procedure, making it difficult to reflect on the individual steps. Hatano (1988) sums up the consequences of instrumental teaching and learning of mathematics as: “This process of acceleration of calculation speeds results in a sacrifice of understanding and of the construction of conceptual knowledge. It is hard to unpack a merged specific rule to find the meaning of any given step” (Hatano, 1988, p. 64).

Hiebert and Carpenter (1992) conclude that the development of relational understanding is the basic goal of mathematical teaching. They argue that teaching should be designed so that students build meaningful relations/connections and that procedures and concepts should therefore not be taught as isolated bits of information. The differences in these two conceptions of what constitutes mathematical understanding and mathematical knowledge is at the root of many of the difficulties experienced in Irish mathematics education, which is pointed out in the Chief Examiner’s Report for mathematics 2001:

The evidence suggests, therefore, that fundamental objective B of the syllabus, (*instrumental understanding*) is being achieved quite well.....There is a significant weakness regarding sound conceptual understanding of much of the material, with corresponding weaknesses in its application in contexts which, though familiar, do not mimic well-rehearsed examples precisely, or do not contain the standard “trigger phrase”. It is particularly noticeable when more than one idea or skill is involved. This indicates that objectives C and D, *relational understanding* and *application*, are not being as well achieved as might be hoped. This is the case not only with advanced material, but also with quite fundamental concepts and skills. Worthy of particular note here is the extent to which basic algebraic skills manifest as isolated mechanical procedures without underlying understanding or synthesis. Whereas this is often sufficient for survival with very familiar routine exercises, it is a serious

disadvantage when any degree of higher application is required. (Chief Examiner's Report, 2001, p. 21).

What the Chief Examiner has described is a worrying situation where students are capable in examinations of reproducing well practised procedures (procedural knowledge) but experience difficulty when faced with any deviation from the latter. What is also worrying is that students are not only having problems in applying these procedures (relational understanding) to difficult material but also to basic concepts. This lack of understanding of relational concepts is preventing students from engaging in higher – order thinking.

There is not a necessary connection between the absolutist and fallibilist views of mathematics and the relational/instrumental approaches to teaching. But it is outside the scope of this thesis to investigate this relationship. Indeed the converse of Hersh's statement can be used to characterize school mathematics – first comes the score, but the music never follows. *Inside Classrooms* (Lyons et al., 2003) and The Chief Examiner's report (2001) show us that Irish mathematics teaching belongs mainly to an absolutist philosophy of mathematics and that the skills being taught in schools are largely instrumental skills. For Povey et al. (2004), one of the consequences of such a philosophy is that the transferral of the teacher's knowledge becomes the goal and is implied in the pedagogy the teacher uses (Povey et al., 2004, p. 45). Students experience for most of mathematics classes a "drill and practice pedagogy" (Povey et al., 2004, p. 48), reflecting the commands of the curriculum and made audible by the teacher (Skovsmose, 1994, p. 185). The pattern of teaching becomes one which practises "a system of oppression which draws its strength from the acquiescence of its victims who have accepted the dominant image of themselves

and are paralysed by a sense of helplessness” (Murray, quoted in Collins, 1991, p. 93) instead of a classroom based on communication.

The teacher in a classroom where the absolutist/instrumental view of teaching has become the norm maintains control over the structure and content of the lesson and over the behaviour of the students as well. The paradox is that while the teacher is powerful in deciding what is done or not done from the curriculum in any particular lesson they seem powerless as regards engaging in imaginative and innovative activities (that they may feel instinctively are appropriate) because these activities might challenge the essence of the absolutist view of mathematics. Consequently the pattern of teaching becomes unimaginative and predictable and, for students, limited understanding and engagement in lower order activities are the result. A worrying finding from a study by the ESRI Smyth et al., 2007, is that students perceive mathematics to be the most difficult of school subjects. In the next section we will consider the impact and the effects that the absolutist/instrumental view of mathematics has on students’ experiences of learning mathematics in school.

Students’ Experiences of Learning Explored at Closer Range

We will now look in more detail at what happens when the absolutist view of mathematics and an instrumental understanding of mathematics becomes the cultural norm in the classroom. The work of Brousseau et al. sheds illuminating light on this and will be our main point of reference at the start of this section.

Brousseau et al. (1991) identified an implicit contract (didactic contract) between the teachers and the learners of mathematics in a classroom based on the absolutist view. The didactic contract is that: “The teacher is obliged to teach and the pupil to learn” (Brousseau and Otte, 1991, p. 18) or at least to pass the assessment. The teacher shows the pupils “how to do” mathematics and sets tasks; the learners accept the methods and carry them out; the contract is that by doing the tasks the learners will do enough to pass. Implicit in this contract is the absolutist view of mathematics teaching and learning. According to Brousseau et al. (1991) “the contract must be honoured at all costs, for otherwise there will be no education. Yet to be obeyed, the contract must be broken, because knowledge cannot be transmitted ready-made and hence nobody – neither the teacher or the pupil – can be really in command” (Brousseau and Otte, 1991, p. 180).

In other words the mathematics teacher feels that education is transmission of information (absolutist view) and learnt procedures (instrumental understanding) yet Brousseau et al. (1991) points out that knowledge cannot be simply transmitted from expert to novice. For Brousseau et al. (1991) next comes a paradox when absolutist teaching and instrumental learning has become the norm for the learner and the teacher:

the paradox of the didactic contract between the teacher and the learner. If both the problem and the information about its solution are communicated by the teacher this deprives the pupil of the conditions necessary for learning and understanding. The pupil will only be able to reproduce the methods of handling and solving the problem communicated to her...mathematics is not just a method (Brousseau and Otte, 1991, p. 121).

The teacher in showing the pupil “how to do” the problems is only promoting instrumental understanding and depriving the pupils of relational understanding which are both crucial to mathematics expertise (Hiebert and Carpenter 1992). We will now consider other negative consequences in terms of students’ experience of learning mathematics in an absolutist classroom where students are not expected to figure out methods for themselves but passively accept and learn what is handed down to them by their teachers. According to Belenky et al. (1986) students in such a mathematics classroom as described above experience:

1. silence
2. external authority (Belenky, et al., 1986)

The two perspectives outlined above do not cover all classroom experiences but these distinctions help us understand how the pedagogical practices of the absolutist/instrumental view of teaching and learning mathematics are experienced by learners. Belenky et al. (1986) maintain that for students the silence perspective offers an experience where the learner sees themselves as: “mindless and voiceless and subject to the whims of external authority” (Belenky et al., 1986, p. 15).

For Povey and Burton (2004) this means that students do not see themselves as developing, acting, learning, planning or choosing (Povey and Burton, 2004, p. 44). The students feel dumb in the classroom, Buxton gives a memorable description of how students feel: “the wall comes up...down comes the blanket like a green baize cover over a parrot’s cage” (Buxton, 1981, p. 4). When students feel cut off from the learning experience in this way it is linked by Isaacson (1990) and others (Buerk,1985; Buxton,1981; McLeod, 1992) with anxiety and fear on behalf of

students, captured by Isaacson's comment: "If unable to answer some fate worse than death would be waiting" (Isaacson, 1990, p. 23).

For the second perspective above, i.e. external authority Povey and Burton (2004) draw attention to the way in which students arrive at the conviction that authority on the subject belongs to the experts i.e. the teachers. They also claim that this experience is probably the most common classroom experience of mathematics than any other (p.44). This is the classroom with an absolutist view of mathematics where mathematical knowledge is associated with certainty and doing mathematics means following rules laid down by the teacher; knowing mathematics means being able to get the right answer, reproducing learnt techniques quickly and mathematical truth is determined when the answer is ratified by the teacher (Skemp, 1978; Brousseau et al., 1991; Schoenfeld, 1985; Stodolsky, 1985 and Lampert, 1990). For Povey et al. (2004) a consequence of this is that students become deeply dependent on the teacher when taught in an absolutist/instrumental fashion. Nickson (1992) claims that students are not only dependent on their teachers but see their own criteria for learning mathematics dominated by a concern for the "right or wrong" answer (Nickson, 1992, p. 104). She further claims that this focus on the "right or wrong" answer is not conducive to teacher-pupil exchange or to interaction among students. Nickson (1992) concludes that where mathematics is viewed in absolutist terms, students make little connection between their work and real life and that mathematics, when taught and experienced in this fashion, has on the whole remained inaccessible to students (Nickson, 1992, p. 104).

Schoenfeld (1992) found, from observing high-school geometry classes over a year that students who hold such beliefs about mathematics as described in the previous paragraph come to believe that mathematics problems should be completed in five minutes or less. They will give up on a problem after a few minutes of unsuccessful attempts, even though they might have solved it had they persevered (Schoenfeld, 1992, p. 359) Wagner, Rachlin, and Jensen (1984) observed students who were taught in a traditional way who, when stuck on a problem, would sometimes get upset and grope wildly for any response that would get them past the blockage, no matter how irrational (Wagner, Rachlin, and Jensen, 1984, referred to by McLeod, 1992, p. 582).

Goos et al. (2004) point out that, however reasonable the absolutist view of teaching mathematics appears to be, numerous research studies (e.g. Schoenfeld, 1988) have shown that such mathematics instruction can leave students with an imperfect understanding of the concept (Goos, 2004, p. 92). Cobb and Bauersfeld (1995) point out that teachers and students coming to associate mathematical competence with being able to memorise and reproduce techniques is another undesirable consequence of the absolutist view of teaching mathematics (Cobb et al., 1995, p. 92). Goos et al. (2004) point out that this in turn leads students to think that there must be a readily available technique for every problem they encounter (Goos et al., 2004, p. 92). Schoenfeld (1992) tells us that this leads to helplessness when students meet a genuine problem for which a solution is not immediately obvious (Schoenfeld, 1992, p. 343). At this level mathematics becomes a meaningless practice of routine procedures and routine exercises (instrumental understanding).

Effectiveness of Work done by Junior Certificate Mathematics teachers

It is important now to examine briefly these issues in the context of the effectiveness of the work being done by Irish Junior Certificate mathematics teachers as distinct from the results achieved by their students in state examinations. National data on Irish teachers' conceptions of mathematics from the TIMSS 1994-1995 study show that Irish teachers have an absolutist view of mathematics. Lyons et al. (2003) in their study of 10 Junior Certificate mathematics classrooms concur with these findings (Lyons et al., 2003, p. 254). Lyons et al. (2003) also found that the absolutist view of mathematics held by the Irish teachers had an impact on their pedagogical practices. Lyons et al. (2003) found that the focus of teachers was largely on learnt procedures and formulae, on the "how" (instrumental understanding) rather than the "why" (relational understanding) of mathematics.

The effectiveness of this type of teaching is questionable in terms of student learning. Brousseau and Otte (1991) tell us it deprives students of the conditions necessary for learning, lends itself generally to lower-order thinking and instrumental rather than relational understanding. Davis (1992) tells us that the teacher sends out the message to students that nobody can solve a problem unless the teacher shows them how. Consequently students stop thinking for themselves and become dependent on the teacher (Povey et al., 2004). Davis (1992) points out that this leads to students adopting the strategy of trying to learn off by heart what the teacher has said (Davis, 1992, p. 725) and Cobb et al. (1995) tell us that as a result students feel that success in mathematics is being able to memorise and reproduce the examples from the blackboard. Goos et al. (2004) and Schoenfeld (1992) amongst others, point out that this type of teaching leaves students with an imperfect understanding of concepts,

inability to deal with genuine problems and a tendency to give up easily because they believe mathematical problems should be completed quickly and effortlessly. With the best of intentions, the effectiveness of this type of teaching in terms of student learning is worrying. Lyons et al. (2003) point out that for Irish students at Junior Certificate level, this experience of learning mathematics as a skill-oriented subject leads to feelings of anxiety about mathematics. Prevalent among students in the study conducted by Lyons et al. (2003) were feelings of insecurity, vulnerability and even fear, which were all a source of a negative attitude towards the subject. The consequences of such practices for Irish pupils in terms of learning and their classroom experience of the subject can hardly be described as being either effective or positive in nature. What alternatives then exist? What would they look like in a classroom? These two questions will be examined now in the next section.

International Research on Mathematics Teaching

Internationally, current trends in mathematics education include emphasis on *problem solving*, *modelling*, and *realistic mathematics education*. Generally it is accepted that the main goal of mathematical teaching is that students become competent problem solvers. In the last thirty years problem-solving has been given a special focus in some national curricula. For example problem solving was the theme of the 1980s in the U.S.A.. *Everybody Counts* (National Research Council, 1989) and the *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989) both emphasised problem-solving. The *Standards* document placed problem-solving as the first of its list of “standards”. The newer version, *Principles and Standards* (NCTM, 2000), “may be a little more realistic in its assessment of what can be

achieved and what works in the classroom, but fundamentally advocates a similar approach” (NCCA, 2005 p. 5, www.ncca.ie).

In England, Wales and Northern Ireland, emphasis was placed on “*investigation*” more than on problem solving. While both of these concepts are relevant to each other, investigation is more geared activity based than problem solving. In general investigation, involves model creation, measurement and induction.

Modelling practices are diverse, ranging from the construction of physical models to the development of abstract symbol systems. It involves determining the essential or significant features of a problem, translating these into a mathematical model, validating the model, interpreting the results. The results must then be reinterpreted in terms of the initial problem. Modelling is most suited to spatial visualization, and geometry and measures of uncertainty.

Realistic mathematics education (RME) stems from the Netherlands. Developing from a reaction to the “new mathematics” movement, it emphasises the solution of problems set in contexts engage students’ interests. It combines elements of problem-solving and modelling approaches. (NCCA, 2005 p.6, www.ncca.ie). It is probably the most fashionable approach among mathematics educators at present, and underpins the OECD PISA study (NCCA, 2006, p. 5-6). Irish post-primary mathematics syllabuses do not make reference to the modelling or RME approaches. In comparison with international trends, Irish post-primary mathematics syllabuses (considered in chapter 1) have remained largely abstract, formalist and comparatively

conservative. Internationally the “new mathematics” of the sixties has come to be perceived as a failure (Schoenfeld, 1992, p. 336). Since the sixties much more is known about mathematical thinking, teaching and learning. Internationally, reconceptualizations of mathematics curricula have taken place based in part on advances made in the understanding of thinking and learning in mathematics and on an evolving shift in the conception of mathematics as: “the science of patterns and of doing mathematics as an act of sense making” (Schoenfeld, 1992, p. 337).

This shift in relation to the adoption of a different view of the nature of mathematics that mathematics educators are now drawn to, is to consider a view of the subject in fallibilist terms. We shall now consider briefly how a fallibilist view of mathematics may manifest itself in the culture of a mathematics classroom and then take a closer look at its pedagogical consequences and students’ experiences of learning mathematics.

A fallibilist view of mathematics characterises the subject in terms of its openness, of its questioning and of its testing of ideas and problems, and as a result it has been pointed out, by Nickson (1992, p. 104) amongst others, that the implementation of the fallibilist view of mathematics affects the social context of the mathematics classroom by implicitly encouraging the active participation of all concerned (Nickson, 1992; Kuhs and Ball, 1986; Jaworski, 1989; Pirie, 1988 and Goos et al., 2004). For Nickson (1992) the fallibilist view of mathematics gives rise to the possibility for increased discussion in the classroom of ideas among students and between students and teachers (Nickson, 1992, p. 104). This discussion we have

already seen is more often than not lacking in a classroom where the absolutist view of mathematics exists. Nickson (1992) says it is likely that when mathematics is taken beyond establishing facts and practising skills (absolutist pedagogy) to a fallibilist approach with more openness, investigation, problem-solving and critical discussion, there will be more social interaction, more negotiation and more emphasis upon shared interpretation and evaluation of what goes on in mathematics classrooms. A fallibilist view of mathematics then precipitates a different kind of teaching and learning to the absolutist view.

The pedagogical consequences of a fallibilist view of mathematics for the teacher is that the teacher becomes, as Thompson (1992) puts it :“facilitator and stimulator of student learning, posing interesting questions and challenging students to think, judge their own ideas, support and defend their conclusions” (Thompson; 1992, p.136). To see what this might look like in a classroom we will consider a two-year project by Goos, Galbraith and Renshaw (2004) who investigated senior school mathematics classrooms where the fallibilist view of mathematics was practised. In this research programme the teachers took on a pivotal role in the classroom to support students and be an expert partner, providing opportunities for students to become more active participants in classroom activities to improve, as Goos et al. put it, students’ mathematical understanding (Goos et al., 2004, p. 98).

We will now look at an illustrative episode from one of the classrooms involved in the project by Goos et al. (2004) which illuminates how the fallibilist view of mathematics manifests itself in the classroom. It is important to look at this

manifestation as many of us are only familiar with the absolutist view of mathematics classroom teaching. In terms of the teacher's pedagogy the teacher in the project by Goos et al. did not merely demonstrate "how to do" mathematics (absolutist pedagogy); instead he involved the students in the process by presenting a problem for them to work on which necessitated the students to use existing knowledge of another topic. The teacher then elicited students' conjectures based on the problem posed. The students' initial conjectures (some of which were incorrect were not rejected by the teacher) were treated by the teacher as a starting point rather than an error. Students then tested their conjectures and justified them to their peers. The teacher asked questions that encouraged the students to question their own conjectures and locate their errors for themselves. The teacher helped the students make sense of the mathematics by asking questions that prompted the students to clarify, elaborate, justify and critique their own and each other's assertions. The teacher made interventions that moved students forward to new ideas, backwards towards previous knowledge and consolidated students' thinking by drawing together ideas developed during the lesson (Goos et al., 2004, p. 101-105).

The above episode helps to shed light on the pedagogical consequences when the fallibilist view of mathematics is practised by a teacher in the classroom. Goos et al. (2004) point out that the teacher has to move out of his/her traditional position as the dispenser of knowledge (absolutist view): "To resist the urge to do the mathematics for the students and to let them grapple directly with ideas in what might appear to be a messy and inefficient fashion" (Goos et al., 2004, p. 113). Teachers thus become more responsible for structuring the cognitive and social opportunities for students to experience mathematics in a meaningful and interesting way (Goos et al., 2004). The

teacher explores and experiments along with the students on mathematical situations as opposed to the teacher acting as the expert (absolutist view). Goos et al. identified a number of resulting positive pedagogical consequences which are clear from the classroom episode outlined above (from the two-year project by Goos et al. 2004). Firstly the teacher creates a classroom atmosphere in which all students feel comfortable trying out new ideas instead of focusing on rapidly producing answers. Secondly, the teacher reassures the students that errors are just expected way stations on the road to solutions, and errors are analyzed in order to increase understanding, whereas from the absolutist view making an error is viewed as lacking competence. Thirdly, the teacher invites students to explain their thinking at all stages of problem-solving and allows for the fact that more than one strategy may be needed to solve a given problem. Thus, alternative solutions are accepted whereas in the absolutist classroom students are often only aware of one strategy which is the one the teacher has demonstrated.

We will now see how these pedagogical consequences, when a fallibilist view of mathematics has been adopted in a classroom, will affect students' experiences of learning mathematics and refer to the positive benefits gained by students.

Schoenfeld (1992) points out that the transformation in instructional style described above means that students are involved in seeking solutions, not just memorizing procedures; exploring patterns not just memorizing formulas; and formulating conjectures not just doing exercises. As a result they are less dependent on their teachers than students taught in a traditional way (Schoenfeld, 1992, p. 334-335?).

When students are less dependent on their teachers, Schoenfeld points out that

learning mathematics is empowering for students and that “mathematically powerful students are quantitatively literate” (Schoenfeld, 1992, p. 335).

Povey et al. (2004) point out that students gain greater autonomy (Povey et al., 2004, p. 46) and that involving students in constructing their own mathematics may move students away from “fearful mathematical silence” (Povey et al., 2004, p. 48). Ma, L. (1999) tells us that because students are given time to reflect on the procedures for solutions and are afforded opportunities to express their own ideas and participate and contribute to their own learning process, they develop relational understanding (Ma, L., 1999, p. 151). Ma, L. (1999) also points out that when students learn to solve a problem in multiple ways, ideas become connected and students can build a “road system” which allows them to go anywhere in the domain and as a result are relationally enriched (Ma, L., 1999, p. 111-113). In turn Schoenfeld (1992, p. 335) points out that such learners are flexible thinkers with a broad repertoire of techniques and perspectives for dealing with novel problems and situations :“They are analytical, both in thinking through issues themselves and in examining the arguments put forth by others” (Schoenfeld, 1992, p. 335).

In conclusion, mathematics then is a subject which, when viewed in absolutist terms, is severed from the real world. When mathematics is taught from the absolutist view the consequence for a teacher’s pedagogy is that it can easily become dry and pedantic and may fail to give much inspiration or motivation to students. The basic pedagogical practices underlying this view of mathematics are as follows. The teacher demonstrates techniques. Emphasis is placed by the teacher on students’

ability to master a corpus of mathematical facts and procedures. Student activity is largely confined to working rote exercises using the teachers' techniques and getting the right answer. When all of the techniques are mastered by the students this comprises their mathematical knowledge and understanding. For students it consists of often meaningless bits and pieces, and teachers expect pupils to learn it as a large collection of bits and pieces (instrumental understanding). Students can learn a few bits, pass a test on them, forget them, learn a few more small bits, pass the test on them, forget them and so on. There is no compelling interest in what they are being asked to do; students often see very little reason for doing it. The outcome of such a state of affairs, among other things, is that students are often left with a poor understanding of mathematics. Consequently students develop a frustration in their beliefs about what mathematics is, about their own mathematical ability, about what mathematics teaching is and about their attitude to the subject.

Various educators have suggested that one way to enhance mathematics instruction and learning is by promoting the fallibilist view of mathematics. This view has gained increasing currency in recent years; its emphasis is on process rather than on content (absolutist view). When mathematics is taught from a fallibilist view the teacher's pedagogy centres around the students' active involvement in doing mathematics-in exploring and formalizing ideas. The teacher is viewed as a facilitator and stimulator of student learning. The teacher's pedagogy models mathematical thinking processes, providing scaffolding to support students' appropriation of mathematical strategies, and in doing so mathematics is seen as a social and collaborative act. A fallibilist view of mathematics offers an increased potential for variation within the context of the mathematics classroom with respect

to what is taught, as well as how it is taught .Consequently there comes greater potential for teachers to be innovative and use skills of a higher order that might be empowering for the teacher and motivating for the student. These new methodologies allow teaching and learning situations where the student is encouraged to be an active learner, challenging and questioning the teacher as well as other students. Implicit in this situation is the need to acknowledge and value what the student offers. The importance of the fallibilist view of mathematics teaching and learning is implicit in the statement by Popkewitz (1988): “School mathematics involves not only acquiring content; it involves participating in a social world that contains standards of reason, rules of practice and conceptions of knowledge” (Popkewitz, 1988, p.221).

This investigation of different conceptions of mathematics and mathematics education brings to a close the first part of this thesis. The issues reviewed in this opening part will now be investigated at closer range, as the focus shifts to a three-year exploration of mathematics in Irish post-primary schools.

Chapter 4

Methodology

Introduction

In this thesis intensive studies in three different Irish post-primary schools were undertaken over a complete three year Junior Cycle period to analyse and gain insights into the teaching and learning of mathematics. The first part of this chapter describes how the schools, the teachers and the classes were chosen and identifies the reasons why they were chosen. The main body of the chapter is devoted to the research instruments used. This describes fully both the design and scope of the methodology and the reasons why these particular instruments were used. In this context Lyons et al. (2003) point out that a multi-faceted methodological approach “enables us to study the complex interface between the teaching and learning of mathematics from a range of different standpoints” (Lyons et al., 2003, p. 66). We will now briefly outline the main features of the methodology, both quantitative and qualitative in nature, and the purpose of each feature.

Student questionnaires and teacher questionnaires were the main quantitative instruments used in the three studies. Also following the advice of Stigler et al., “questionnaires are relatively simple to administer and can be easily transformed into data files that are ready for statistical analysis” (Stigler et al., 1999 b p. 2). The purpose of the student questionnaire was to ascertain the students’ views on their

experience of learning mathematics in Junior Cycle. The purpose of the teacher questionnaire was to elicit their views on issues of classroom practice in teaching mathematics at Junior Cycle. Another quantitative instrument used was a forty-item mathematics test based on TIMSS 1994-1995 released items from their website (appendix 1 to chapter 4). The purpose of the test was to determine if there is any connection between students' mathematical ability and their attitude to the subject.

One of the qualitative methods used in this study was structured interviews-cum discussions with students. The purpose of the discussions was to determine what the students' perspectives were on their classroom experiences of mathematics. Another purpose for the focus group discussions was to determine if their attitude to mathematics was positive or negative and try to establish the reasons why their attitude would be either one of these. The teachers involved in the study were also interviewed. The purpose of these interviews was mainly to elicit their own theories on teaching and to ascertain what practices they actually employed in the classroom and a description of what classroom instruction practices they employed. Another qualitative method used was the use of video recording of the classes in the study. (Extracts from the videotaped classes accompany the thesis on DVD). The purpose of these recordings was to see how the mathematics lessons were structured and developed, what kind of mathematics was being presented and the kind of mathematical thinking students were engaged in.

The final part of this chapter outlines the variety of measures taken to give due care and consideration to the Junior Cycle students in the study in the three schools. We

will now consider how the teachers, students and schools for the case studies were selected.

Entering the classrooms to be studied

Bishop highlights the distinctive yet significant problems and relationships in classroom research which can make "...the teacher an object – not a subject – in the research. The individuality and humanness of the teacher can become at best an irrelevance and at worst a confounding influence" (Bishop, 1992, p. 117). For Bishop the roles of the researcher and teacher can be incompatible, "The teacher must act and must interact with the learners but the researcher is concerned with data collection and analysis" (Bishop, 1992, p. 117).

Bishop advises that in mathematics classroom observations, to avoid this "hierarchy" with the researcher appearing to have the role of an "expert", it is sensible if the teacher and researcher are engaged in joint research. In the three classroom studies in this thesis the researcher is involved in joint research with the three teachers in the case studies and is familiar with them, having been involved in the TL21 research project with them from September 2004 to June 2007. This allowed the researcher the opportunity to engage with the teachers on a regular basis and build trust over a sustained period, hence breaking down what Bishop (1992) refers to as "separateness" and "hierarchy".

Choosing the three schools for this study took several months. The schools were strategically chosen, not only to represent different school types but also on the basis

of their geographical location in the context of the overall research project: one school being urban (Riverside), one being rural (Chestnut Hill) and the third being semi-urban or “dormitory” (Kenmore). The names of the schools used in this study are fictitious and are the names of suburbs of Boston in the United States of America.

- (i) Kenmore: A large co-educational secondary school in a small dormitory town
 - (ii) Chestnut Hill: A large co-educational community school in a small rural town
 - (iii) Riverside: A large single-sex girls’ secondary school situated in a city
- (Large means between 600-800 students).

Riverside is a voluntary secondary school for girls run by Trustees and the Board of Management. The vision for the school emphasises the importance of imparting a full and balanced education to its students, focusing on nurturing creativity and embracing diversity. The school also stresses its Christian values and the importance of equipping its students for the world of work as well as for leisure.

Chestnut Hill is a coeducational community school managed by a Board of Management. It is a large school and is situated in a small town in the midlands. Chestnut Hill was built as a community school arising from the amalgamation of two schools. The school’s mission statement emphasises that Chestnut Hill is a community of learning in an environment of respect and commitment. It also emphasises that all who live in the area are welcome to participate in the learning experiences in an atmosphere of concern and openness.

Kenmore is a coeducational secondary school in a town within 30 kilometres of Dublin city centre and is managed by a Board of Management. Kenmore was established as a secondary school arising from the amalgamation of two schools in the voluntary secondary sector. The school's mission statement promotes the school as part of a community to ensure that each student gradually becomes responsible for themselves, society and the world. The school also stresses that while educational excellence is promoted, it is the needs of the learner that direct the development of the school.

According to Lyons et al. (2003) it is necessary to choose schools and classes where teachers and students are interested and willing to be part of the research project (Lyons et al., 2003, p. 69). For the purposes of the current research it was necessary for the participating teachers to be teaching a first year mathematics class in their Junior Cycle. The reason for starting with first year students was that the design of the study was to be longitudinal. Walberg et al. (1986) point to a shortcoming in the design of much survey work in the field of mathematics education. This is that the surveys are characteristically cross-sectional, with data being gathered on one occasion only. This, Walberg et al. maintain, makes it impossible to make attributions or comparisons about the extent or rate of change (Walberg et al., 1986, p. 238). Robitaille and Travers (1992) also comment on the value of several data-collection points over a period of time. One value they cite is that information about changes in students' achievement and attitudes and the relationship of these changes to the teaching practices employed by teachers can be investigated. For Robitaille et al., these relationships are crucial to the development of an adequate understanding of what happens in mathematics classrooms (Robitaille and Travers, 1992, p. 707).

The researcher was keen to ensure that teaching and learning in mathematics was observed throughout the Junior Certificate cycle, and at different levels of ability. In first year all the case study classes were of mixed ability. In the second and third years of the study the mathematics classes in all three schools were streamed according to students' mathematical ability. Table 1 below summarises the grouping profiles of the three case-study classes:

Table 1: Case study schools: Grouping profiles

Class Name	Year 1	Year 2	Year 3
Kenmore	Mixed	Bottom Stream	Bottom Stream
Chestnut Hill	Mixed	Middle/Top Stream	Middle/Top
Riverside	Mixed	Top Stream	Top Stream

We will now consider the scope of the quantitative procedures used in this study together with the reasons for their use.

Quantitative Data

Quantitative data in this study was obtained from attainment tests and questionnaires. A thirty-four item questionnaire (some with sub-items) was designed and distributed to students in each of the three years of the study and is contained in Appendix 2. As stated in the introduction the questionnaires focused on issues concerned largely with their attitude and experiences of learning mathematics. The questionnaires were conducted each year to explore how students' engagement with the subject may change as they progress through the Junior Cycle. Another reason why the

questionnaires were conducted each year was the student body did not remain the same throughout the three years. As with all aspects of the study students were informed in advance that they need not fill out the questionnaires if they had any objections to doing so. Seventy one students were involved in the first year of the study and, happily for the researcher, all chose to complete the questionnaire. In the second year fifty four students were involved in the study and again, all chose to complete the questionnaire. The total number of students involved in the study decreased in number by seventeen students from the first year to the second year of the study. This was due to the fact that in Year 2 of the study Kenmore was a foundation level mathematics class. Such classes tend to have smaller numbers than higher or ordinary level classes. In the third year of the study fifty one students were involved in the study and again all chose to complete the questionnaire.

The three teachers were also issued with a questionnaire in each of three years of the study. The reason for this is that, as Stigler et al. point out, "a limitation of questionnaires is their static nature. Teachers can only answer the questions we as researchers thought to ask" (Stigler et al., 1999 b p. 3). As the study progressed the researcher noticed some important and perhaps unanticipated aspects emerging each year. Consequently the questionnaires were amended by the researcher until it was felt they were more effective. In the case of the teachers the questionnaire was shorter. It was a thirteen item questionnaire with most sections having sub-sections and it appears in Appendix 3. The questionnaire focused on questions relating to the classes videotaped and, as noted in the introduction, also covered a range of issues on the teaching and learning of mathematics at Junior Certificate level.

Both questionnaires were based on related questionnaires in *Inside Classrooms* (Lyons et al., 2003) and *Schooling and Sex Roles: Sex Differences in Subject Provision and Student Choice in Irish Post-Primary Schools* (Hannan et al., 1983, p. 332-354). The language used was age appropriate and the opportunity for more qualitative data was provided through respondents being asked to elaborate on item statements during focus group discussions with students and interviews with teachers. Questionnaires were completed in April 2005, March 2006 and February 2007 and the process was administered by the researcher and the classroom teacher in each of the three schools. The questionnaire data was analysed using Statistical Package for the Social Sciences (SPSS) software programme while the interviews were transcribed in full and rigorously analysed.

Peligrini (1995) argues that the questionnaire model, when used alone, is of limited value (Peligrini, 1995, p. 14). However quantitative data can validate or debunk behavioural observations. Quantitative research, particularly in the form of questionnaires, ignores the emotions, which qualitative research methodologies characteristically include. Stigler et al. also point out that:

A problem with relying on questionnaire-based indicators of instruction concerns their (teachers') accuracy in reporting processes that may, at least in part, be outside of their awareness. Teachers may be accurate reporters of what they planned for a lesson but inaccurate when asked to report on the aspects of teaching that can happen too quickly to be under the teacher's conscious control (Stigler et al., 1999 b p 3).

Bearing in mind the limitations of questionnaires, the researcher also used qualitative instruments. We will now consider the scope and the combination of qualitative procedures used in this study together with the reasons for their use.

Qualitative Research

Seale (1998) advises using triangulation when a subject is complex and not easily understood by employing a single research methodology (Seale, 1998, p. 231). For this reason and the limitations associated with questionnaires described in the previous sections, a range of complementary instruments, including interviews, was employed in the current study. Structured interviews-cum discussions were conducted with students and teachers from the classes videotaped. In the current research semi-structured interviews were used so that the researcher could explore in depth aspects of students' experience of learning mathematics and teachers' experiences of teaching mathematics. The advantages of interviews are that they afford the researcher the opportunity to probe students and teachers on their experiences and views of the mathematics classroom and to follow up emerging issues from questionnaires. As Bell (2004) so aptly puts it "A major advantage of the interview is its adaptability" (Bell, 2004, p. 135). On the other hand Bell also points out that the disadvantage of interviews is that students' and teachers' answers are subjective and the researcher must be cautious in making generalisations from the answers given: "Even so, the interview can yield rich material and can often put flesh on the bones of questionnaire responses" (Bell, 2004, p. 135). Complementing this is the advice given by Robitaille and Travers (1992), "interviews with teachers and students help develop a more complete depiction of the important variables at work in mathematics classrooms" (Robitaille and Travers, 1992, p. 698).

Choosing students for focus group discussions cum interviews was based on three factors:

- a. a mix of students who did and didn't participate well in class;

- b. a mix of students who were getting high grades and lower grades;
- c. where applicable, an equal gender mix.

Six students from the participating class in each of the three schools were selected for Year 1, Year 2 and Year 3 of the study. In total 9 interviews were conducted with groups of students with an average of six students per interview over a three year period. Each focus group discussion lasted approximately forty minutes. The focus group discussions focused on their experiences and the opportunities provided for them in learning mathematics. Focus group discussions were conducted each year. The discussions were conducted to explore how engagement with the subject might change as they progress through the Junior Cycle. Another reason for conducting the focus group discussions each year was because the body of students did not remain the same for the three years of the study. At Chestnut Hill there was some progression of students (11 out of 23) from the First Year class to the Second Year class. At Kenmore and Riverside none of the students remained the same from First Year to Second Year due to changes in class make up. In the case of the students from Chestnut Hill it was possible for three of those who were among the focus group as First Years to be in it also as Second Years. In the case of the students from the Kenmore and Riverside schools the students who were in the focus group in First Year were completely different from the students who participated in the focus group in Second Year. The objective of the interviews with the students was to explore in some depth their views on the teaching and learning of mathematics. The questions in the interviews with students related to their liking for mathematics, the progress they have or have not made in the subject, their expectations in terms of Junior Certificate results and their overall view of the subject.

At Chestnut Hill there was some progression of students (20 out of 23) from the Second Year class to the Third year class. At Riverside all of the students progressed from the Second Year to the Third Year of the study. At Kenmore there was some progression of students (6 out of 8) from the Second Year to the Third Year of this study. In the case of the students from Chestnut Hill it was possible for five of those who were among the focus group as Second Years to be in it also as Third Years. In the case of the students from the Riverside school it was possible for all the students who were in the focus group in Second Year to be in the focus group as Third Years. In the case of the Kenmore school it was possible for four of those who were among the focus group as Second Years to be also in it as Third Years. The objective of the interviews with the students was to explore in some depth their views on the teaching and learning of mathematics. The questions in the interviews with students related to their liking for mathematics, the progress they had or had not made in the subject, their expectations in terms of Junior Certificate results and their overall view of the subject.

How students approach the learning of mathematics and how they learn is not just influenced by their views and interests in the subject. It is also strongly determined by the attitudes of their teachers (Lyons et al., 2003, p. 73). Because of this, the researcher undertook intensive interviews with the three mathematics teachers for each year of the study. The teacher at Riverside did not continue with the study after the first year due to illness. The researcher was fortunate that a teacher in the same school, also involved in the action research project, agreed to step in for her. For teachers the interview questions focused on the class observed in the video recordings. Other issues dealt with in the interviews related to their professional

activities and their own theories and practices of mathematics teaching at Junior Certificate level. The interviews with the students and teachers were structured interviews –cum discussions and are included in Appendix 4 and Appendix 5 respectively. All the interviews were audio tape recorded. Verbatim transcripts of interviews with teachers and students were made by the researcher from the audio recordings over the three years.

Bearing in mind the limitations of questionnaires and the subjectivity of interviews it was decided to add a further dimension to the gathering of data – namely the videotaping of classroom sessions. Each of the three classes was videotaped on two occasions in First Year, two occasions in Second Year and two occasions in Third Year. Each recording lasted approximately thirty minutes. Robitaille and Travers. (1992) tell us that “to know what really goes on in maths classrooms, we need to observe what teachers actually do and not rely solely on what they say they do” (Robitaille et al., 1992, p. 708). Verbatim transcripts of each video tape were made for each year of the study and were analyzed by the researcher, based on how lessons are structured, how lessons are delivered, what kind of mathematical thinking students are engaged in during mathematics class and teacher-student interaction patterns. The analysis of the tapes was guided by the TIMSS 1994-1995 videotape classroom study (Stigler et al., 1999, b). Appendix 6 outlines what was coded from studying the videotapes, some of the codes being developed by the researcher herself to suit the needs of this study. In coding the transcripts a distinction was made between private and public talk. Public talk was defined by Stigler et al. (1999, b, p. 32) as talk intended for everyone to hear; private talk was intended only for the teacher or an individual student. When the teacher stopped at a student’s desk to

make comments on the student's work this was coded as private talk, regardless of whether others could hear what the teacher was saying or not. The important thing is that the talk was primarily intended for this individual student. All further coding of discourse was done on public talk only. The researcher feels justified in doing this, because, as Stigler et al. point out, "public talk is accessible to everyone [and] we assumed it would provide the most valid representation of the discourse environment experienced by students in the classroom" (Stigler et al., 1999, b p. 32).

Guided by insights gained from the TIMSS videotape study of mathematics classroom (Kawanka et al., 1999), it was appreciated that videotaping can be intrusive. Lyons' study had suggested that videotaping was a very valuable research tool for understanding how mathematics was actually taught in classrooms. Contrary to expectations, the researcher did not find that the videotape interfered greatly with the flow of the lesson. Following an initial curiosity at the presence of the video, the classes settled down to a normal lesson routine. The focus group interviews with students after the video recorded classes suggested that being videotaped was not unusual for students, as many of them had been videotaped at home.

To minimise the effects of a video camera on the teachers' behaviour in the classroom, the researcher informed teachers that the goal was to videotape typical lessons. The teachers were explicitly asked to prepare for the lessons to be videotaped as they would for a typical lesson. After an analysis in the first year of the videotape recordings and student questionnaires relating to the classes videotaped the researcher became aware that she was seeing, for some of the time at least, a slightly

idealized version of what the teachers and students normally do in the classroom. To minimize this “special event” effect in the second and third year of the study teachers were asked to fill out a questionnaire based on the class being videotaped (Appendix 7) in which they rated for example, the typicality of what was seen on video and describe in writing any aspect of the lesson they felt was not typical. Whilst Stigler et al. (1999, a), note that the camera may have an effect on what happens in a classroom it is important to note that questionnaires and interviews have a similar potential for bias. Stigler et al. (1999, a) conclude that one must use common sense in interpreting the results of a video study:

It seems likely, for example that students will try to be on their best behaviour with a videographer present, and so we may not get a valid measure from the video of the frequency with which teachers must discipline students. On the other hand it is probably less likely that teachers use a different style while being videotaped than they would when the camera is not present. Some behaviours, such as changing the way a teacher teaches, is notoriously difficult to do, and that the routines of classroom discourse, are so highly socialised as to be automatic and thus difficult to change” (Stigler et al., 1999 a p. 7).

Having looked in some detail at the qualitative aspect of the methodologies used we will outline in the next section the measures taken to ensure that the students in this study were not treated, as (Denscombe et al., 1992, p. 121) put it, like “research dopes”.

Consideration of students

Bishop (1992, p. 117) was concerned that students might be seen by researchers as “..mere examples of stages or of particular types, possessors of certain abilities and attitudes”. In the light of Bishop’s comments considerable thought had to be put into

the consideration of the students in the study, so as to involve them actively, as distinct from being merely research subjects.

Prior to the research each year, the researcher and the class teacher spoke to the students who were being asked to participate. It was explained that the researcher was writing about mathematics education for the University and students were informed what the writing would include. It was made clear that any information they gave, whether it be in the questionnaire or focus interview, would be treated in confidence; that they would not be named or identifiable and that the video footage would not be made public. The researcher visited each of the classrooms and explained the function of the questionnaire. It was stressed to the students that completing the questionnaires was a voluntary exercise. It was explained that their identity was unknown as the researcher did not put any place for students' names on the cover sheet. It was also explained that the results of the questionnaire would be put into a computer to process the answers. For the focus group interviews the researcher made up badges with fictitious names and gave these to the students to wear during the interview. This was one way of trying to make the students feel at ease during the interviews. The students were also told that the answers they would give might help in improving the way mathematics is taught and in developing new ways to help young people like themselves with their mathematical problems.

It is envisaged that the use of various qualitative and quantitative methodologies in a longitudinal study and the triangulation of these will increase the validity of the research. The longitudinal design of the study gives a fuller picture of the teaching and learning of mathematics and is more than a mere snapshot in time. In the next

chapter we will look at the findings from the first year of the study. The chapter will focus on an in-depth analysis of the classroom interaction from the video tape recordings. It will also look at the teachers' perspectives on mathematics and their views on learning mathematics. Students' perspectives on learning mathematics are described and finally the core themes that emerge in the teaching and learning of mathematics in first year mathematics will be outlined.

This thesis now progresses to a presentation of the findings of Year 1, Year 2 and Year 3 of this study respectively. Each chapter follows a common structure. The first part of each chapter is based on an analysis of the videotape transcripts from each of the three schools' (Chestnut Hill, Kenmore and Riverside). (Videotaped classes accompany the thesis on DVD for each of the three years). The focus is on addressing and analysing three broad questions (1) what kind of mathematics content was studied? (2) how was the lesson environment organised? and (3) what were the dominant teaching methodologies employed across the mathematics classes? These three questions form the basic organizing principle of the first part of each chapter. The implications regarding students' learning opportunities in mathematics are discussed at the end of the first part of each chapter.

The video analysis of each section begins with an examination of the mathematical content of the videotaped classes and gives a general description of the content of the class for each year and then moves beyond the intended curriculum to reveal the implemented curricula that students actually encountered in the classroom. The organization of teaching and learning is examined next in terms of the type of

activities students and teachers were engaged in during the lessons. The way in which the mathematics content was worked on during the lessons is then explored, namely how mathematics problems were presented and worked on, resources used and the classroom discourse. The section closes with a discussion of the findings and the implications regarding students' learning opportunities. The analysis that follows does not include private exchanges between teachers and students or those among students themselves.

Each chapter then moves to examine the views of the three teachers involved in the study. This analysis is based on data from questionnaires issued to the three teachers and also from semi-structured interviews-cum discussions with the three teachers. The analysis focuses on their responses to a number of attitudinal statements, such as their views on mathematics as a subject, their approach to teaching mathematics and their experience with the observed class groups. Each chapter then engages in a similar analysis of the perspectives of each Year's students' attitudes to and experiences of the teaching and learning of mathematics. The analysis is based on questionnaires issued to all students in the three classes. The students' general attitude to mathematics and their self-image in relation to mathematics were examined in the questionnaires. These issues were explored in depth in semi-structured interviews-cum discussions in the focus groups with groups of six students from each of the three classes. Each chapter closes with an analysis of the major findings and reviews the main implications for teaching and learning mathematics.

Chapter 5

Findings from First Year

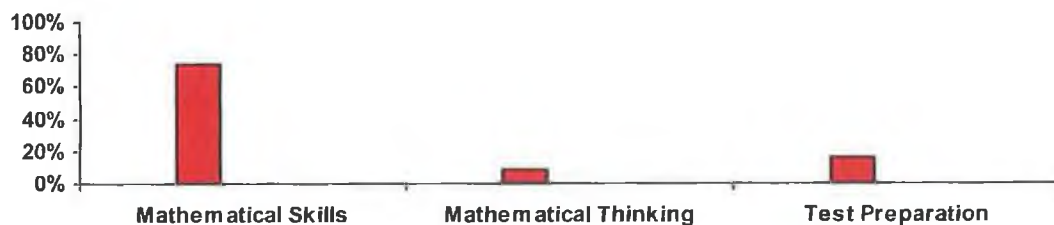
Section 1: Videotape Analysis: Pedagogical Practices and Classroom

Interactions.

It is useful to examine mathematical content apart from the lesson in which it is embedded. The rationale for this is that no matter how good the teaching is if a lesson does not include rich mathematical content it is unlikely that many students will construct a deep understanding of mathematics from the lesson. A general description of the mathematical content of each of the mathematics lessons that were videotaped in the first year of the study is contained in Appendix A Table 1 to chapter 5. It also includes a description of how teachers described the content of the videotaped lessons on a continuum from “all new” to “all review”.

Based on the videotaped classes the three teachers were asked to describe the “main thing” they wanted students to learn in the lessons. A study of the answers shows that the main thing the teachers wanted the students to learn in the lessons that were videotaped was mathematical skills (how to solve specific kinds of problems) which leave students with an instrumental understanding of mathematics (Skemp, 1978) (Figure 5.1).

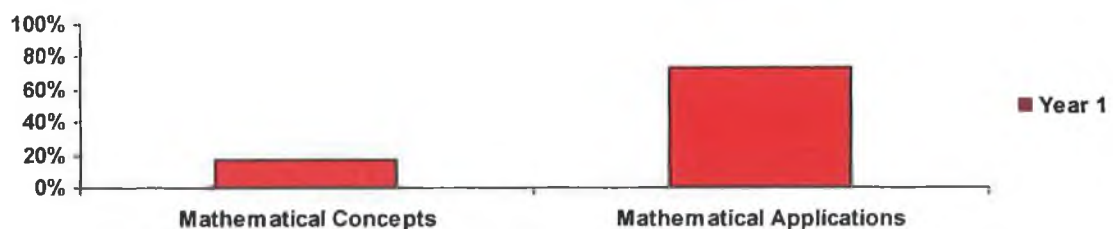
Figure 5.1: Comparative emphasis on mathematical skills and mathematical thinking in the six recorded lessons in Year 1



Having determined what appeared to be the goals for the six recorded lessons (two from each of the three classrooms Chestnut Hill, Kenmore and Riverside), the lessons were divided into two categories: mathematical concepts and mathematical applications. These categories were defined to broadly catch those instances in which students might have constructed concepts or learned how to apply them. For this purpose the mathematics presented in the six lessons was divided into two categories: mathematical “concepts” and mathematical “applications”. “Concept” in this instance means when a mathematical property, formula or theorem is explicitly referred to, defined provided or explained to the class e.g. “Here we are using vertically opposite angles” or “The formula for the area of a circle is πr^2 ” or “The three angles of a triangle add up to 180 degrees”. “Application” means the explanation of the concept was not included but that the concept itself was used without explicitly referring by name to it to develop skills for specific types of mathematical problems. An analogy occurs with regard to language learning: just as in using grammar in a foreign language we would have to say for example “Now we will use the future tense”. However, when we are already well versed in a language we do not have to refer explicitly to the grammar. Application then is something we do when we are fluent without referring to the concept/grammar we are using. But

for learners we need to refer explicitly to the concept we use whether it be the concept of vertically opposite angles or the future tense, otherwise a presumption is being made in the application that the concept is understood. While the writer is not making a criticism here good practice would be for the teacher to intervene and not presume that all students know which concept is being used or why they are using it. These presumptions occur as a matter of course in customary practise. The average percentage of topics in each lesson that include mathematical “concepts” or “application” is summarised in Figure 5.2.

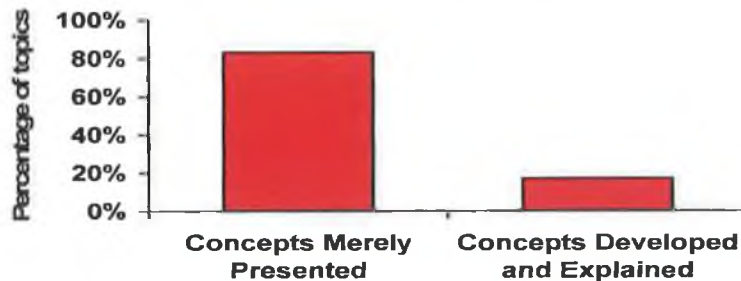
Figure 5.2: Percentage of topics in each lesson that include mathematical “concepts” or mathematical “applications”



It was of interest to see if “concepts” that were introduced in the lessons were (a) simply referred to and provided by the teacher or students in order to guide the solution of a problem. The focus here is on mathematical information and mechanical skills rather than the process of development. On the other hand (b) explaining and developing a “concept” collaboratively by experimentation, by conjectures, or by both, increases a students’ relational understanding of it (Skemp, 1978; Nickson, 1992; Dossey, 1992). In the six classes observed 80% of topics contained concepts that were merely presented without further explanation in order

to solve problems, as opposed to 20% that were developed and explained (Figure 5.3).

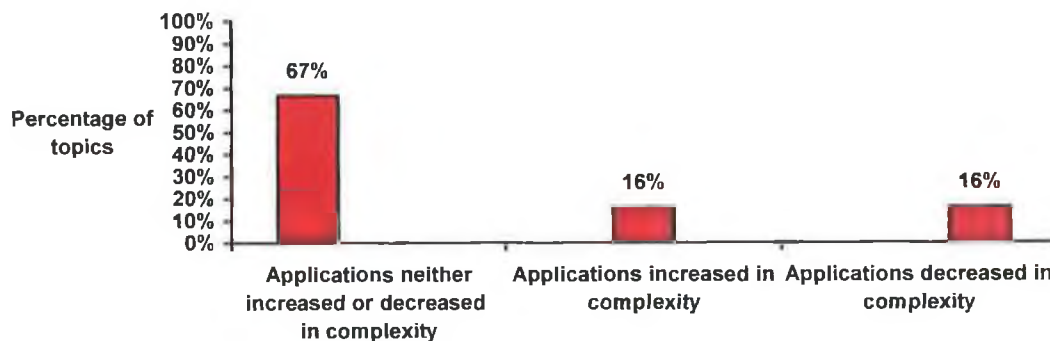
Figure 5.3: Average percentage of topics in mathematics lessons that contained concepts that were merely presented or developed, Year 1



There was only one example from the six classes videotaped where a concept was developed using experimentation by the teacher and the students collaboratively. The collaboration tended to be conducted privately and the students were not asked to present their findings. This occurred in Riverside where the teacher did not tell the students that the three angles in a triangle added up to 180° . Instead the students drew a series of triangles and by measuring the three angles and adding them up came to the conclusion themselves that the three angles added up to 180° . (See Appendix B).

After examining whether concepts were (a) merely presented or (b) explained and developed, topics that contained “applications” of the concepts to solving problems were considered to see whether the complexity of the applications increased, stayed the same or decreased over the course of the lesson (Figure 5.4)

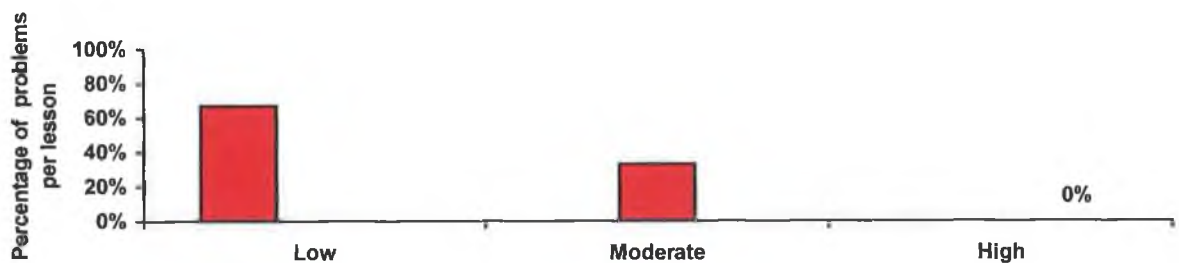
Figure 5.4: Percentage of topics in each lesson that contained procedural applications that increased in complexity, stayed the same or decreased over the course of the lesson, Year 1



One salient characteristic of mathematics encountered in teaching and learning is its complexity and this now examined. The complexity of the mathematics presented in the lessons is difficult to define however, because it can be of many different kinds and what may be difficult to one student may be less complex to his/her classmate. One kind of complexity that can be defined independently of the student is procedural complexity and is helpful to the exploration of the teaching and learning of mathematics. Procedural complexity looks at the number of steps it takes to solve a problem using a common solution method and puts the spotlight on the levels of sophistication involved from lower order to higher order skills. The mathematics problem analysis group for the TIMSS video study of eighth grade mathematics teaching (which is comparable to second year Irish post-primary) (Hiebert et al., 2003, p. 70-71) provided a system of coding the procedural complexity of mathematical content along three levels of complexity: low, moderate and high complexity. Low was defined as a problem that required four or fewer decisions by a student to solve it using a conventional procedure. Moderate difficulty was defined as a problem that required a conventional procedure using more than four decisions by the student to solve it and could contain one sub-problem. High complexity was

defined as a problem that required more than four decisions by a student and at least two sub-problems to solve it. Taking the six recordings from the classrooms together, Figure 5.5 shows the average percentage of problems per lesson that were at each complexity level.

Figure 5.5: Average percentage of mathematics problems per lesson rated as having low, medium and high quality of procedural mathematical content. (Taking all of the recorded lessons into account), Year 1

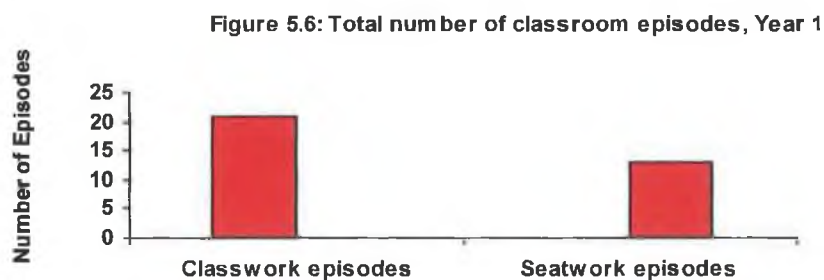


From the videotapes for Year 1 students were not observed being encouraged to develop different methods and/or examine their relative advantages. Students had no choice in determining how to perform tasks. Teachers demonstrated how to solve a problem and then asked students to apply the same method to a similar problem.

Overall it would appear that the goal the teachers set for the lessons was the acquisition of ready-made mechanical skills or procedures for solving a mathematical problem. In all three classrooms a particular mechanical skill or procedure was prescribed for all students to acquire rather than students being encouraged to generate alternative solution methods. The emphasis seemed to be placed on acquiring the mechanics and instruments of the procedure rather than learning why the procedure works (relational knowledge). The procedures to be learned were at a relatively simple/moderate mathematical level. We now turn our attention to examine more fully the methods by which the teachers structured the

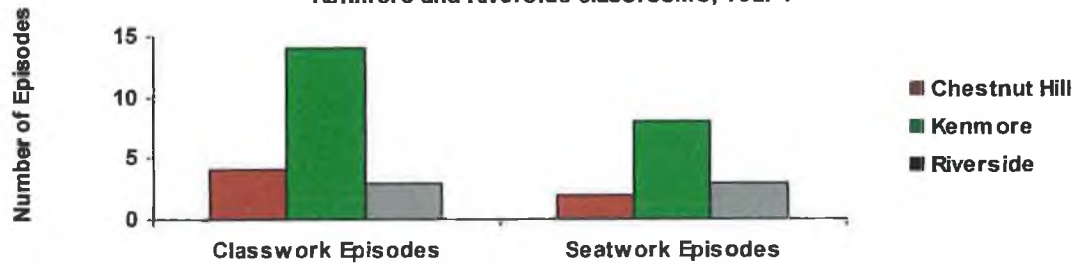
lessons so as to engage the students with the content and discuss the processes of instruction used at Chestnut Hill, Kenmore and Riverside.

An initial viewing of the videotapes suggests that the similarities were more striking than the differences. All three classrooms contained white/blackboards and individual desks for the students arranged in rows. The teachers tended to divide their lessons into periods of classwork and periods of seatwork. Classwork includes those times, or episodes, when teachers are working with all of the class, talk is public and the teachers and students are engaged in learning a new concept, solving a mathematical problem, demonstrating a procedure etc. Seat work means those episodes when students worked independently on assigned tasks individually or in small groups. While students were observed talking to each other during these episodes, it appeared that during these episodes students were engaged in helping each other or checking their answers. In the latter kinds of episodes the students were never directed to do so by their teachers. The total number of each type of episode observed in the 6 recorded lessons is shown below (Figure 5.6).



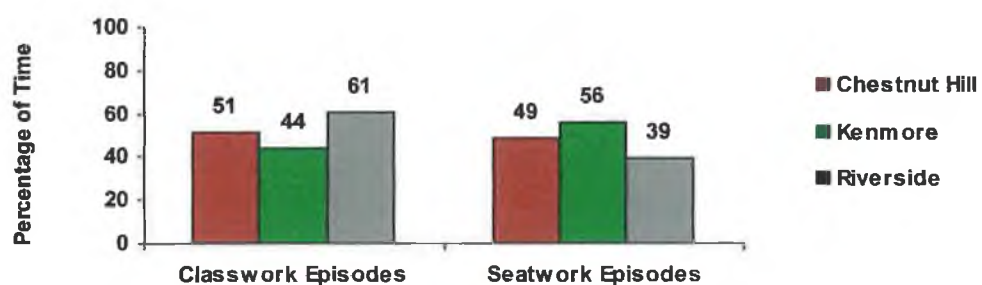
A further analysis of the classes showed a striking difference. Kenmore contained many more classwork and seat work episodes than either Chestnut Hill or Riverside (Figure 5.7).

Figure 5.7: Total number of classwork and seatwork episodes in Chestnut Hill, Kenmore and Riverside classrooms, Year 1



The picture changes when the percentage of time spent in classwork and seatwork episodes was considered (Figure 5.8). All schools now look quite similar.

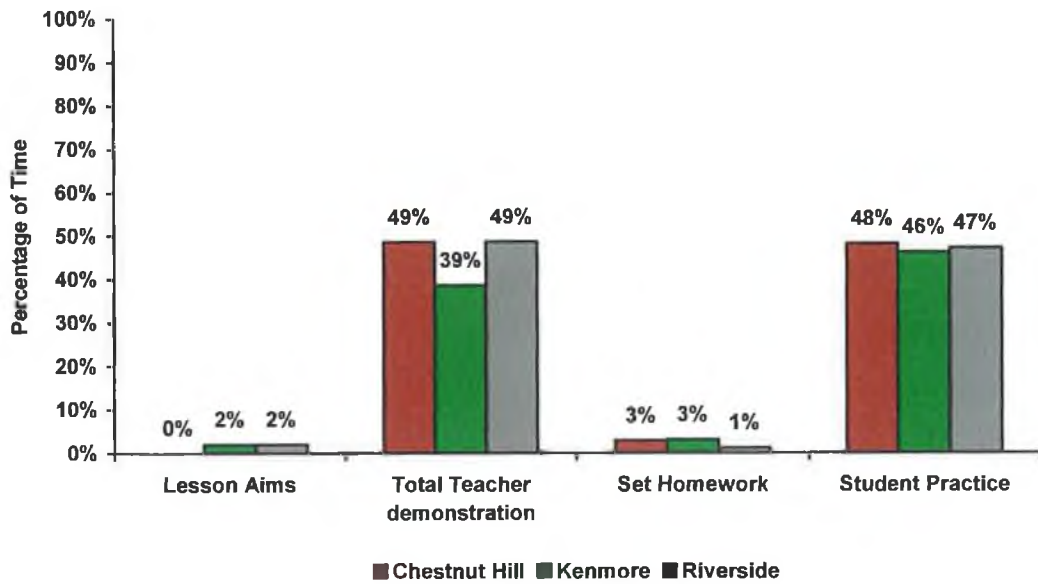
Figure 5.8: Percentage of time during the lesson spent in classwork and seatwork episodes in Chestnut Hill, Kenmore and Riverside classrooms, Year 1



This preliminary analysis of the proportion of time spent in classwork and seatwork episodes represents however only a superficial view of what occurs in a mathematics lesson. The quality of what goes on during these episodes and what goals teachers

are trying to achieve is more important than what a quantitative analysis can disclose. With this in mind the recorded lessons were further divided into activity episodes that served pedagogical purpose such as lesson aims, teacher demonstration/explanation, setting homework and student practice. In each of the six recorded lessons, including the three earlier and three latter sessions, the time spent on stating the aims of the lesson was minimal and not systematic. By stating what material will be covered in the lesson and what relationship this lesson has with topics already covered the teacher is providing an important learning context for students (Brophy, 1999; p. 15). The total teacher demonstration/explanation is where the teacher talks about concepts, ideas, solution strategies and demonstration of solution steps to examples. Here the teachers in each instance were transmitting information to students and the students' role was to listen. Figure 5.9 below indicates that practically half of the class time was spent by the teachers in this public role. An illustrative example of a demonstration at Chestnut Hill is presented in Appendix C. In the extract students were not asked to make any decisions about how to approach the problem – only to follow the exact procedure set out by the teacher. The teaching in this instance was directed and interspersed with questions towards the whole class with no attention being directed to individual students.

Figure 5.9: Percentage of Time Allocated to Pedagogical Practices



Homework tended to be checked in private by the teachers as they walked around the classroom. Students were not observed correcting each other's homework or discussing the homework. When homework was corrected it was done so at a brisk pace with an emphasis on obtaining the right answer. In the homework correction episode captured on the six video-recordings, the teacher calls out the final answer to each problem and corrects a large number of problems in a short period of time. (See Appendix D).

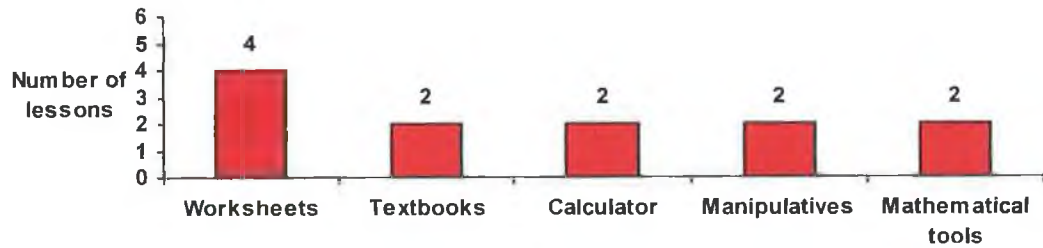
Two lessons were recorded in the case of each of the three teachers in Year 1. All six lessons observed were predominantly dominated by the teacher in various forms of teacher talk – mainly demonstration and explanation. Student practice consisted of repetition of the procedures demonstrated by the teacher (Figure 5.9) and included the doing and correcting of problems assigned to the class by the teacher either

during classtime or for homework. By assigning practice examples, the teachers were able to ascertain whether students had listened to and had an initial understanding of what was demonstrated, but the emphasis was on pressing ahead and getting the course covered. The extract contained in Appendix E is an example of student practice in class that follows on from the same lesson referred to in Appendix B on demonstration.

A variety of instructional materials was observed being used in the six classes recorded. The most commonly used tools were the chalk/whiteboard. The percentage of lessons in which other kinds of materials were used is shown in Figure 5.10. Most are self-explanatory. The term manipulatives refers to concrete materials used to represent quantitative situations, such as cardboard cut-out angles and geostrips used at Riverside. These manipulatives were used by both students and the teacher.

Modern mathematics tools such as Algebra tiles were observed being used by the teacher in Kenmore albeit for a few seconds. Mathematical tools include set squares, protractors and rulers. These were observed being used at Riverside, once again by the teacher and the students. Worksheets were commonly used by all three teachers, but their nature and level of the questions were not unlike the textbook.

Figure 5.10: Use of various instructional materials in the six recorded lessons, Year 1

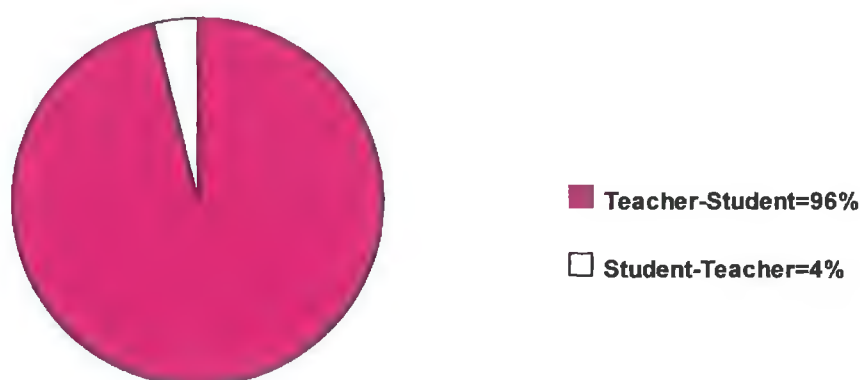


Typically the three teachers organised the lessons with periods of teacher-directed classwork during which they demonstrated step by step procedures followed by seatwork. Earlier in this section we distinguished two ways of including concepts in a lesson. On the one hand concepts might be presented or merely stated by the teacher or students or by contrast concepts might be explained and developed by the teacher, or the teacher and students collaboratively, in order to increase the students' understanding of the concept. Seatwork episodes can play a critical role in the development of mathematical concepts, consistent with giving students themselves more responsibility for the process (Schoenfeld, 1992). In only one of the classes observed (Riverside), the development of a concept occurred during the students' seatwork and the teacher was observed in private exchanges with the students giving individual assistance. The evidence in this study points to the conclusion that students during periods of seatwork were generally not involved in the development of concepts; instead they were involved in the practice of routine procedures of low/moderate procedural complexity. Students were not asked to create or invent solution methods, proofs or procedures on their own. They were not required to think or reason, nor were students asked or expected by their teacher to come up with any different methods. The problems the students worked on were drawn from the textbook or a teacher-made worksheet, and few materials or tools were used other

than pencil and paper. Up to this point our focus has been largely on issues of content. Continuing with this analysis we will now explore the dynamics of interaction: namely the classroom discourse, the patterns of engagement between the teachers and their students and the nature of questioning and processes of instruction.

The researcher counted that there were a total of 440 classroom interactions in the recorded lessons, for Year 1, of which 423 were teacher-initiated (i.e. teacher initiated discourse consisted of 96% of all classroom interactions, Figure 5.11). Relative to their students the teachers talked more. Student-initiated interactions comprised mainly questions that were related to mathematics (as distinct from comments on extra curricular matters) but were of a low cognitive level. (See Appendix F). What was clear from the video analysis was that few students asked questions publicly. Student-initiated public interactions did include one instance where a student offered an alternative method to solving a problem. (See Appendix G).

Figure 5.11: Percentage of all public interactions, Year 1 (n=440)



The kind of utterances between teachers and students were significantly made up of questions. The questions were delivered to the whole class rather than individual students and were questions that intended to elicit an immediate response from students. (Figure 5.12 and 5.13 respectively).

Figure 5.12: Percentage of teacher-student interactions, Year 1

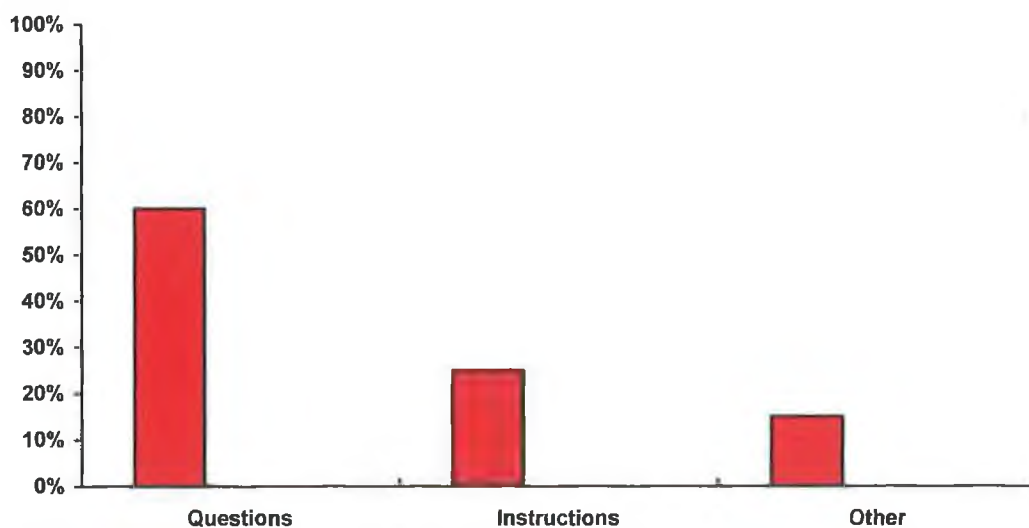


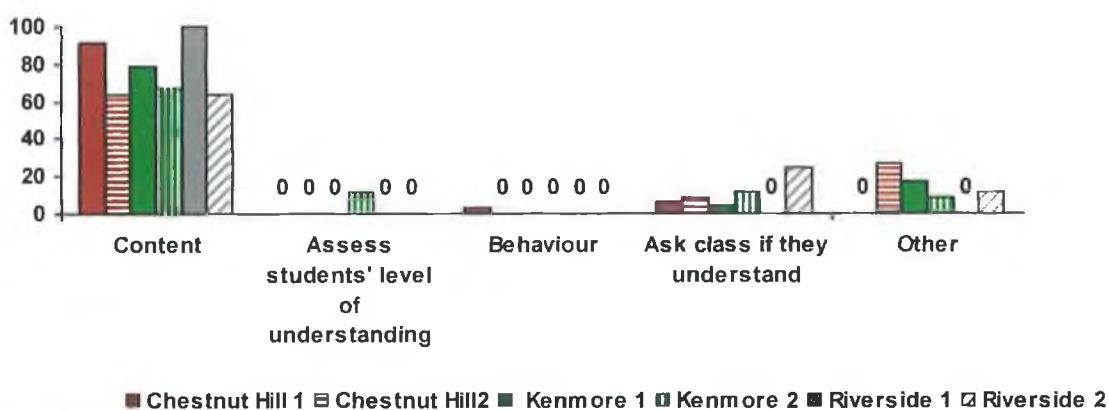
Figure 5.13: Distribution of teacher-student interactions, Year 1



The types of questions overwhelmingly produced by teachers were content elicitations. Students were asked to supply the next step in a procedure to a solution, supply a number, identify a shape, define a term or evaluate an answer. Generally the

three teachers did not ask questions that would assess the students' level of understanding. To assess students' level of understanding all three teachers tended to say periodically: "Do you all understand that", again directed to the whole class. Figure 5.14 below shows the percentage of the type of questions asked by the three teachers in each of the six recorded lessons. Overall in the six classes observed the three teachers managed to ask a total of 256 questions i.e. an average of 42 questions per class. Thus questioning tended to move at a brisk pace. (See Appendix H).

Figure 5.14: Percentage of type of questions asked by teachers, Year 1 (n=256)



As content elicitation generated much of the mathematical content that was discussed in the lesson, a further analysis of these questions was undertaken. The results are shown in Figure 5.15. The results show that the questions that were asked were mainly of a low order. To name or to state requires a relatively short response from students, such as numbers, formulas, a single rule, an answer to some mathematical operation, to read a response from a book. Appendix I to chapter 5 contains vignettes that highlight the low-level questioning in the classes observed and the emphasis on instrumental learning (Skemp, 1978). The teachers' purpose was to assess whether the students knew the answer or were able to produce the correct answer.

It was observed that at times when a student answered incorrectly the teachers moved onto another student until they got the right answer and did not appear interested in pursuing the wrong answer to find out a particular student's thinking. As most questions were directed to the whole class, the response could have been given by any student and at times was supplied by the teacher. Students generally were not observed being asked to report on their individual opinions, ideas or thinking processes. Neither were they observed being asked to evaluate another students' response or answer. Students were not asked to choose among alternatives. This relates to studies by Schoenfeld (1992) and Povey (2004) which highlight that students who do not seek solutions, formulate conjectures, or explore for themselves, are neither empowered or autonomous learners. Neither were students asked to describe or explain a solution method they generated or to give a reason why something was true or not true (Ma, L., 1999). This type of questioning would stimulate students to respond at higher cognitive levels and would necessitate a slower pace than was evident in the videotapes, with less emphasis by teachers on covering material and more emphasis on students' discovery of mathematics. If no specific response was being pursued by the teacher, responses from students would therefore be less likely to be evaluated by the teacher as right or wrong (Ernest, 2004; Thompson, 1992). In the classes observed the teachers tended to look for specific right answers.

Figure 5.15: Percentage of type of content questions asked, Year 1 (n=205)

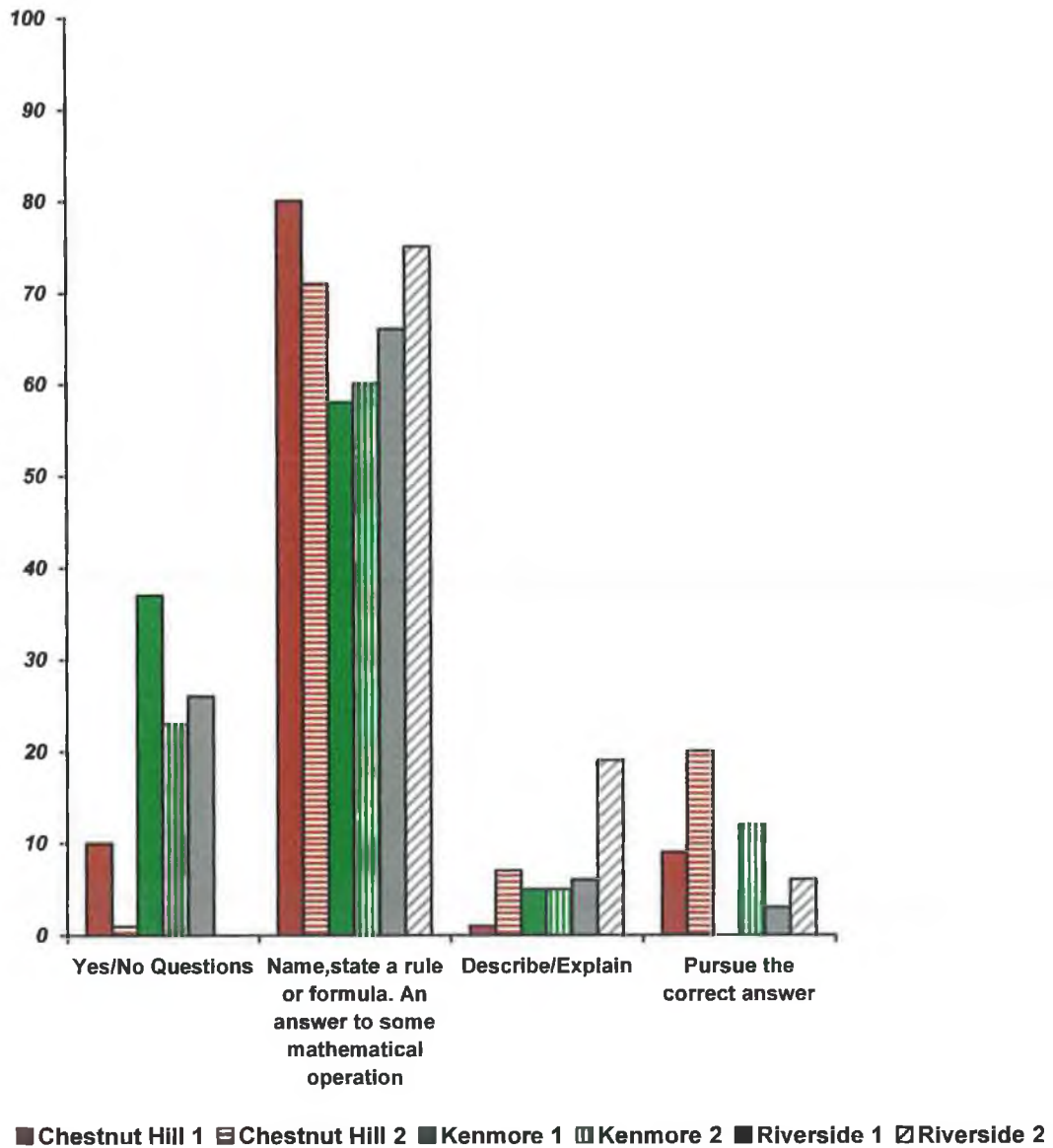
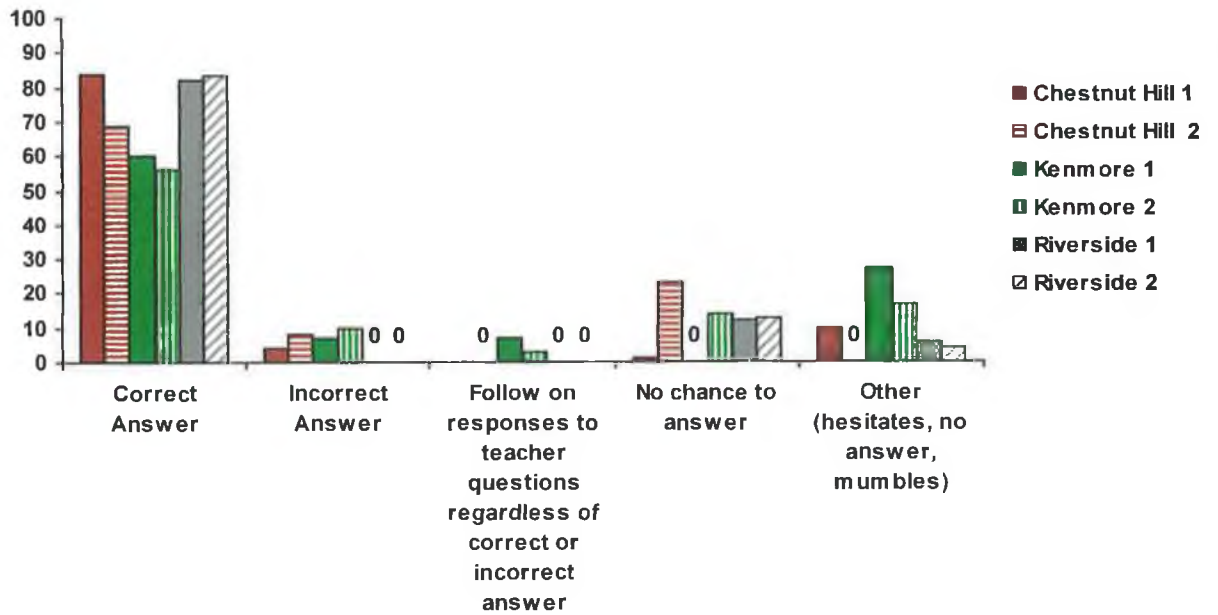


Figure 5.16 shows the responses that students made to questions asked by the teacher taking all three schools together. The students mainly gave correct answers. Incorrect responses accounted for a very small percentage of all student answers. There were significantly few follow-on responses to teacher's questions. In some instances the students were not given a chance to answer at all, as the teacher on these occasions intervened. Other categories included hesitates/no answer/mumbles.

Figure 5.16: Percentage of student responses to teachers' mathematics questions, Year 1 (n=232)



As so many correct answers were given by students, the type of responses the teachers made to these answers was analysed. Figure 5.17 below shows the teachers' reactions to correct responses. This shows that teachers mainly accepted correct answers without praising the students. Repeating the students' correct answer was the next most common type of feedback accounting for 34% of all teacher feedback. Other types of feedback included asking an additional question of the student who provided the correct answer or checking with the whole class to see who got the particular correct answer.

Figure 5.16 showed that there were very few incorrect answers in response to questions from the teacher. The teacher's reaction to incorrect responses is illustrated in figure 5.18 showing that teachers mainly dismissed an incorrect answer or answered the question correctly themselves. Pursuing what the student has in mind

accounts for only 12% of all teacher feedback to incorrect answers given by students. An example of how teachers reacted to incorrect answers is to be found in Appendix J.

Figure 5.17: Percentage of teacher feedback to correct responses, n=170

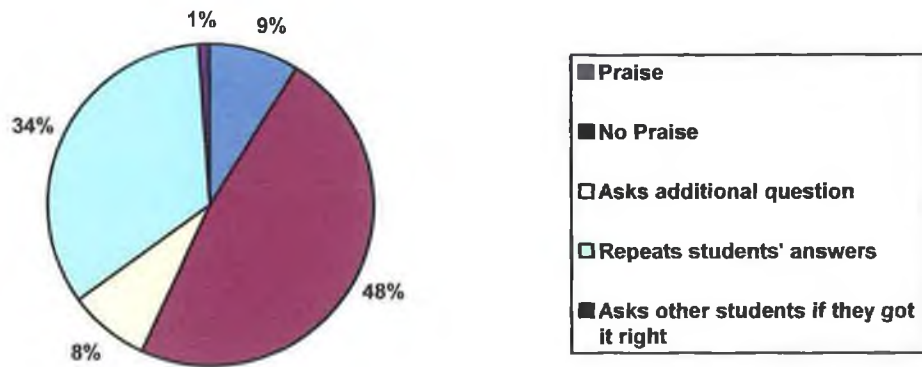
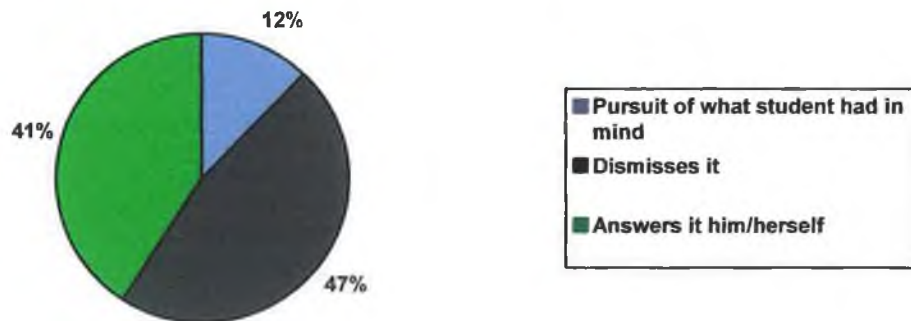


Figure 5.18: Teachers' response to incorrect answers, n=13



Review of Key Issues

Having analysed the main points of comparison and contrast in the recorded lessons from a critical standpoint we will now review the findings from Year 1 of this study.

Overall the six lessons videotaped typically followed similar patterns of acquisition/application script (Hiebert et al., 1996 p.12-21). During the acquisition phase, students were expected to learn how to solve particular types of problems, through demonstration by the teacher. During the application phase the students were expected to practise the routine step-by-step procedures they had learned. A transmission-style approach was used to varying degrees by the teachers, consistent with the absolutist view of mathematics (Burton, 1992; Thomson, 1992; Ernest, 2004). The teachers showed the students “how” to do sums, which results in instrumental understanding (Dossey, 1992; Skemp, 1978). The students were involved in carrying out unrelated tasks and applying learnt procedures and “rules without reasons” (Skemp, 1978). (See Appendix K). For students, such an approach deprives them of the conditions necessary for independent learning (Brousseau et al., 1991). It appeared that the objective, although not stated, was to ensure that students perfected their procedural skills. When students answered incorrectly the teachers showed signs of disapproval (Ernest, 2004; Thompson, 1992) instead of allowing the students locate their errors and help the student make sense of their mathematics (Nickson, 1992; Goos et al., 2004). The emphasis seemed to be on covering material with few interruptions from the teachers’ point of view and from a students’ point of view being able to complete and answer questions and problems quickly, which does not promote relational understanding (Hiebert & Carpenter, 1992; Ma, L., 1999). The students appeared to accept the methods they were shown and carried them out without any discussion (Brousseau et al., 1991; Belenky, 1986).

In terms of current ideas on teaching and learning mathematics (fallibilist view) where the focus is on process rather than product, the lessons observed fell short

(Burton, 1995; Ernest, 2004). Students were generally not actively engaged in questioning, constructing their own ideas or testing their own conjectures; accomplishments that are required to produce mathematical knowledge and relational understanding (Thompson, 1992; Skemp, 1978). Students were not encouraged to approach a task in a variety of ways but only made aware of the strategy demonstrated by the teacher, which leaves students with the belief that they cannot solve any problem as the teacher is the expert and must show them how to do the problem (Davies, 1992). Students taught in this way often stop thinking for themselves and become dependent on the teacher (Povey et al., 2004) (See Appendix L). We will now examine the teachers and students' perspectives on the teaching and learning of mathematics.

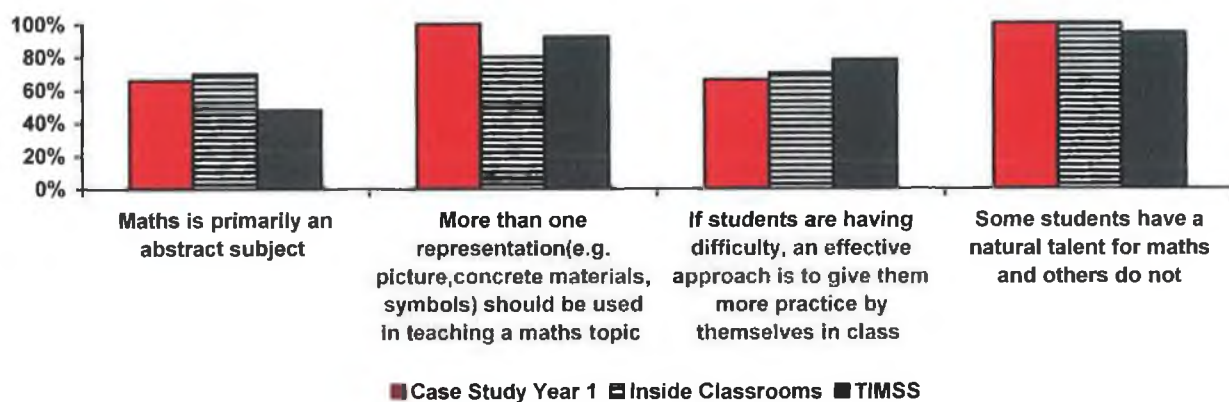
Section 2: Teachers' and Students' perspectives: Findings from First Year

Teachers' Perspectives

National data on Irish teachers' perspectives on mathematics was available from the TIMSS 1994-1995 and from a study of ten Junior Certificate mathematics teachers in the *Inside Classrooms* study of 2003. While the present study presents the perspectives of only three teachers (and does not intend to make generalizations from such a small number), the author found that comparing her findings with this national data served as a fertile starting point. Figures 5.19 and 5.20 present the results of the teachers' responses to the questionnaire items about the nature of mathematics and teaching mathematics. The first column presents the responses of the three teachers in this study to each item, while the second column presents the responses of the ten

teachers in the *Inside Classroom* study and the third column presents the national results of the Irish mathematics teachers who participated in the TIMSS 1994-1995 study (Beaton et al., 1996).

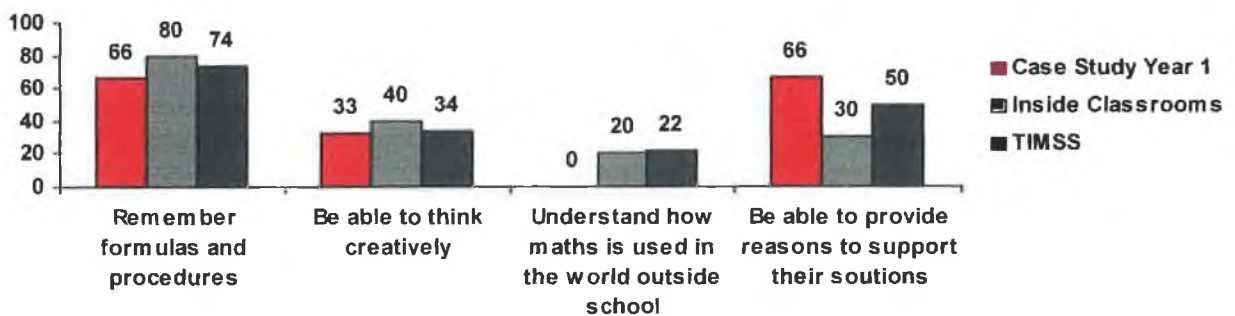
Figure 5.19: Percentage of mathematics teachers who agree or agree strongly with statements about the nature of mathematics and mathematics teaching



In comparison with the 10 teachers' responses to questionnaire items for *Inside Classrooms* and the Irish mathematics teachers who participated in the TIMSS 1994-1995 study (Beaton et al., 1996) the views of the teachers in this study are consistent with their findings. The 2 teachers who agreed or agreed strongly that mathematics is primarily an abstract subject hold a formal view of mathematics. These two teachers also agreed about the benefits of students practising procedures during class, a view that resonates strongly with their formal view of mathematics. All three teachers in the study appear to hold strong views about mathematical abilities (some students have a natural talent for mathematics others do not) and shared the same views on their approach to teaching (more than one representation should be used in teaching a maths topic mathematics).

Figure 5.20 outlines the attitudes of teachers to the cognitive demands of mathematics. It documents the skills the teachers regarded as “very important” or “important” for succeeding in school mathematics. As with Figure 5.19 the views of teachers in the case study are compared with those of teachers nationally and *Inside Classrooms* respectively. There was a high level of congruence between the views of

Figure 5.20: Percentage of teachers who think that particular abilities are important or very important for students' success in mathematics in school



the teachers in this study and the views of the teachers nationally and from *Inside Classrooms* as to the importance placed on being able to remember formulae and procedures, but the teachers do not place a high value on students being able to think creatively. Significant differences were evident between the views of the teachers in this study on the one hand and the views of the teachers nationally and from *Inside Classrooms* on the other. These differences were particularly evident in the importance placed on:

- 1) students' ability to understand how mathematics is used in the real world and
- 2) students' ability to be able to provide reasons to support their solutions.

In relation to the first issue above the current study proved to be a little more disappointing than the national and the international study. The claims made by the teachers in this study as to the importance they placed on the second issue above were not borne out by the video evidence. In the international TIMSS study, mathematics teachers in Ireland (74%) attributed more importance to memorising formulae and procedures than teachers in other countries (40% was the mean in other countries without Ireland). Two out of the three teachers in this study expressed similar views to teachers nationally and *Inside Classrooms*, regarding the priority given to memorisation of formulae and procedures. In the international TIMSS study, the majority of teachers internationally expressed the view that it was very important for students to be able to think creatively, and to understand how mathematics is used in the real world and to be able to provide reasons to support their solutions. The first two of these skills (i.e. to be able to think creatively and to understand how mathematics is used in the real world) were not rated highly in Ireland in the national study. Equally the findings in Figure 5.20 show that the teachers in this study do not attribute a high level of importance to these two skills. Beaton et al. (1996, p.139) expressed surprise at the low ranking attributed by teachers in Ireland to understanding and thinking creatively.

Overall, the teachers in this study generally seem to regard mathematics in a traditional way. It is seen as an abstract subject, where students are either talented or not. While they believe that varied teaching methods and practice improve student learning, learning seems to be equated with learning formulae and procedures rather than being able to think creatively. Two of the three teachers however believe that being able to support solutions is important. This is at variance with what was

observed in the videotapes. None of three teachers believed that understanding how mathematics is used in the real world was either important or very important. These findings show that the three teachers in general have an absolutist rather than a fallibilist view of mathematics. This absolutist view of mathematics has been shown by Dossey (1992) among others to have an impact on a teachers' classroom practice, the focus being on the "how" rather than the "why" of mathematics.

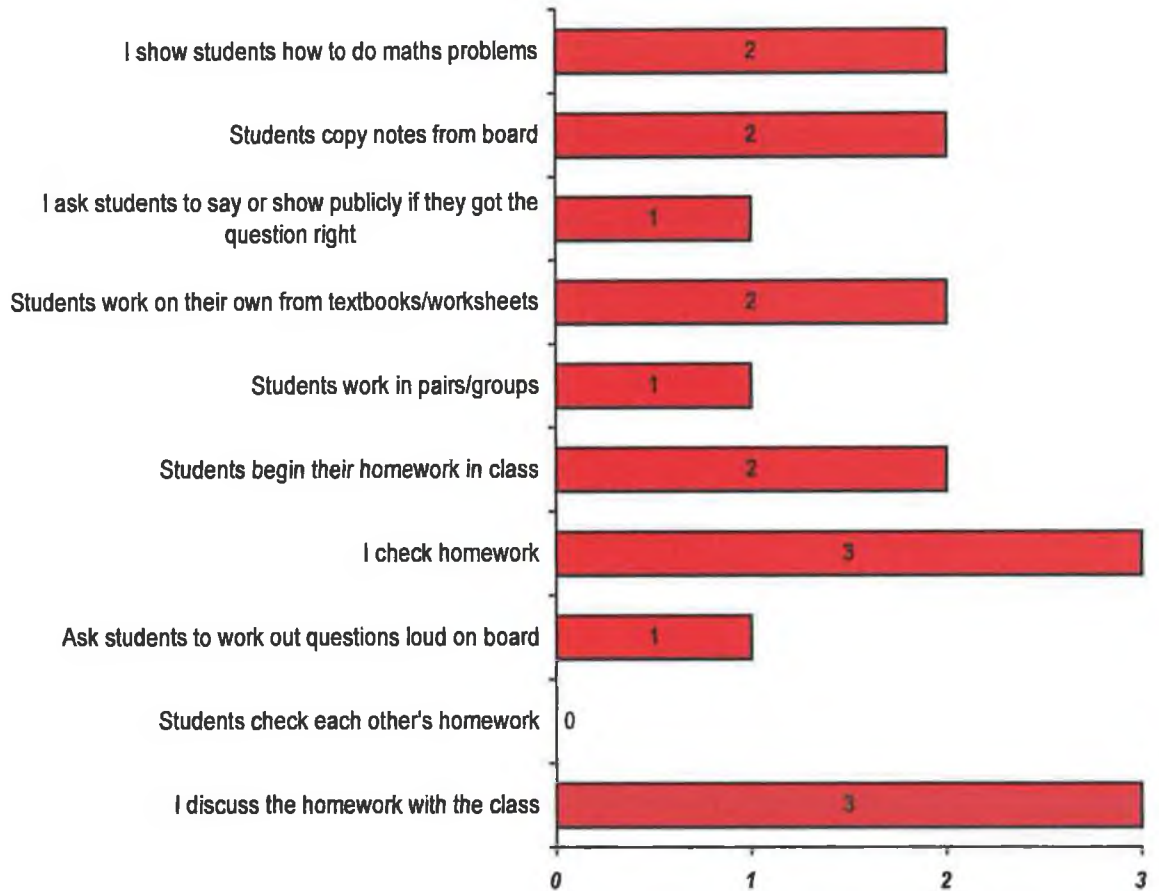
Figure 5.21 presents the teachers' reports from questionnaires on their level of usage of particular methodologies in their Junior Cycle lessons. These findings provide a context for the qualitative accounts that follow.

Figur 5.21: Teachers' reports about whether events happen in "every lesson" or "most lessons"
(Base: 3 teachers)

3= All three teachers in the case study

2=Two out of three teachers in the case study

1=One out of three teachers in the case study



From the findings outlined in Figure 5.21 it can be concluded that the teachers in this study are most concerned with discussing and checking students' homework to ensure that the material is adequately covered and carried out by the students with little emphasis on understanding. Teachers' accounts of their work in this respect confirm the findings from the video analysis. Given the strong evidence of demonstration-led teaching from the video analysis it is not surprising that teachers report that most lessons involve showing students how to do mathematics problems,

students copying notes from the board and students working from textbooks or worksheets. It is clear that the absolutist view of mathematics held by the teachers impacted on their classroom practice, leading to a focus on procedures. The claims of teachers from Figure 5.21 above, regarding the public nature of the work in class, resonate with what was observed in the lessons. While appraisal through questioning in demonstration/student practice phases is conducted publicly, the teachers are less likely to ask students to do questions on their own out loud, on the board, or show publicly if they got the question right. The mathematics problems from the classes observed tended to be broken down into small parts with a range of students being asked to offer solutions at different stages. Finally work by students on their own or in small groups happened infrequently, and was confirmed by the video analysis. Teachers reported that the students did not correct each others' homework and this was also borne out by the video evidence. The data would suggest that there is a high level of congruence between teachers' accounts of their pedagogical practices, which are conservative for the most part, and their actual practice which is also conservative for the most part.

In the semi-structured interviews-cum-discussion with the three teachers they were asked to describe in their own words their approach to teaching mathematics. All three teachers described their approach to teaching mathematics in quite traditional terms. The teachers at Riverside and Kenmore both spoke about the need to change their style of teaching. (See Appendix M).

The teachers' pedagogical approach can be further divided into two main categories, namely personalised and depersonalised accounts. In the personalised accounts, references were made to improving the quality of the learning experience by using humour and encouragement. Teaching is depersonalised in other accounts with the emphasis on students being defined in terms of ability groupings. The teacher from Kenmore described his approach to teaching mathematics in a person-centred way. He referred to ways of making mathematics more enjoyable, trying to meet the needs of individual students and the importance of being friendly with them.

Teacher from Kenmore: I encourage the kids; I try to make it as much fun as I can. Well, I try to facilitate all students. I would deal with each student individually if they're having a problem, I'd deal with it there in the classroom, in my own time, quite often, taking them out outside of the class time; take them at lunch time or whatever. I cajole them around to doing a bit of work. I chat to them about football as much as anything else. It's important to be friendly with them.

The teacher from Riverside also described her approach to teaching in a person-centred way and commented on the need for students to enjoy the subject:

Teacher from Riverside: Encouraging all the time, that it's okay to get them wrong and just have a try. To have confidence in themselves, I'd be very conscious of trying to, boost their confidence. Girls, I find, are very quick to say 'I can't do this'.

The teacher from Chestnut Hill also referred to the need for students to have confidence at mathematics but had a less person-centred approach making references to foundation level students as "those type of kids", and ordinary and higher level classes as "normal classes".

To explore teachers' views of learning they were asked if they could identify any students who had "made progress" over the course of the school year and equally if

they could identify students who “had fallen back”. All three teachers could identify students who had made progress during the year. The teacher from Riverside explained this in terms of the students enjoying the class because they were doing different topics compared to Primary school. The teachers from Chestnut Hill and Kenmore also explained it in terms of the students enjoying the class and that they actively encourage the students to have confidence in their ability to do mathematics. None of the teachers could identify any students who had fallen back, the teachers from Kenmore and Riverside did comment that some students were “making slower progress” or “finding it difficult”.

When asked if they felt students ask for help if they need it, the responses varied. The teachers from Kenmore and Riverside were aware of this in person-centred terms.

Teacher from Kenmore: You try to get around to them, they don't like being singled out. You go round to them when they are on their own and point out how to do something to them help them at the table, that's fine. I think they ask more so now than at the beginning of the year. I think once they know you ,and know you are approachable and can ask you questions, they will. At the beginning of the year they don't ask you anything; they are First Years; they don't do anything; they just sit there with their newly sharpened pencils and pressed shirts and they do nothing. Well that's my experience of them anyway. I suppose they are coming from a country Primary school. We have six or seven feeder schools here from a population of 100 kids to 800 kids which is what we have here. It's new, it's scary, they don't know me. They have a clatter of new teachers they haven't seen before. They have been used to having just one teacher in Primary school. They are basically scared.

Teacher from Riverside: No, there'd be a couple who would never ask. They appear to me, I only have to see them in a classroom situation, but they appear to me just to be shy and quiet. A couple would sort of give you the eye, would look at you, they wouldn't want to draw attention to themselves, they wouldn't want to be seen to be putting up their hand. And they

know that I'll come down. But, because I'm going round everybody it's not as if they're being picked out for special attention. But others that are very weak are quite happy to ask and speak up.

The teacher from Chestnut Hill felt that the students do ask for help if they need it but that some students who need help may not actually realise they need it.

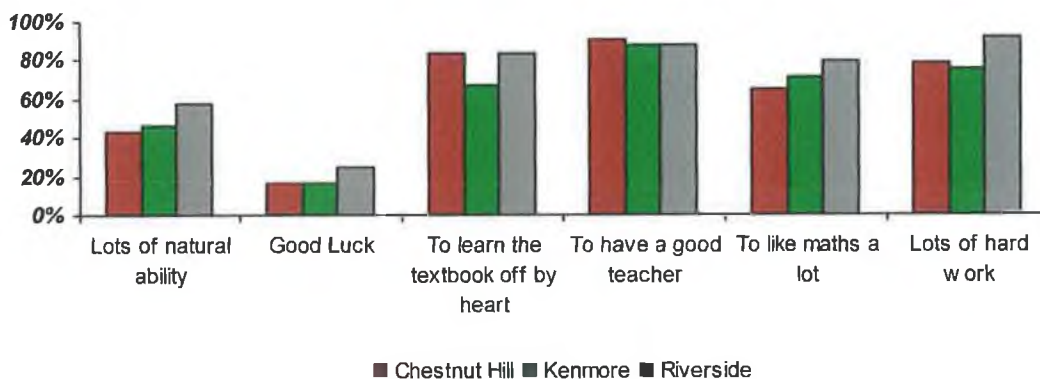
Teacher from Chestnut Hill: The weaker ones mightn't realise that they actually need help, so they mightn't ask. They mightn't be conscious of the fact that they need help, but other than that, the ones who are sort of average to - you know - good at maths would ask.

Students' Perspectives

We will now consider the students' perspectives on learning mathematics, and in particular their experience of learning mathematics in class. International and national evidence shows that students' attitudes towards and performance in mathematics are strongly influenced by their mathematical experiences, and in particular by the way mathematics is taught in school (Dick and Rallis, 1991; Johnston, 1994; Lyons et al., 2003; Ma, X., 1997). Students were asked to indicate their level of agreement or disagreement with six statements about what is required for success in school mathematics. Figure 5.22 shows the level of students' agreement with each of the statements. "Having a good teacher" received the highest level of agreement among students. Most students also agreed "to learn the textbook off by heart", "lots of hard work" and "to like the subject a lot" were important for success in mathematics. Few regarded "good luck" as a requirement for success. While the majority of students from Riverside felt that natural ability was important for success in mathematics, fewer students in the other two classes felt so. Overall,

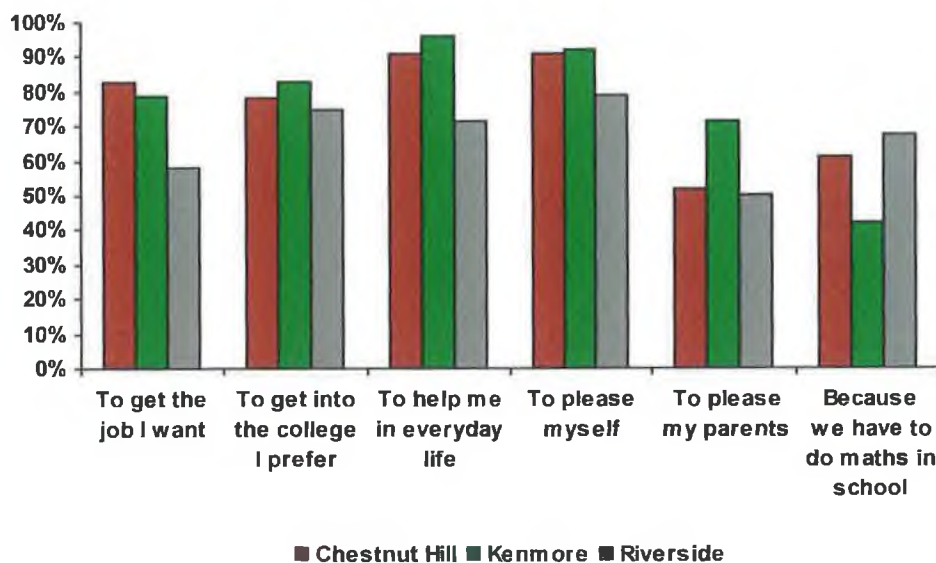
students felt that success in mathematics is the outcome of good teaching, hard work, good memorisation and a liking for the subject.

Figure 5.22: Students' views of requirements for success in school mathematics
(Base: 71 students)



Students were also asked for their views on “why they need to do well in school”. They were asked to indicate the extent of their agreement or disagreement with a number of statements. The results are shown below in Figure 5.23 and would suggest that students are quite aware of the importance of mathematics in their lives. The students’ responses were very positive about the value and importance of mathematics. They realise mathematics is important for employment, although noticeably less so in Riverside. The importance of mathematics for further education is also acknowledged. Its importance in everyday life is acknowledged by students at Chestnut Hill and Kenmore but noticeably less so in Riverside. The short term views on why they need to do well in mathematics in school are more about pleasing themselves than their parents. Whilst only 4 in 10 students from Kenmore feel that they need to do well in mathematics in school because it is compulsory, a majority of students in Riverside (6 out of 10) and a majority of students at Chestnut Hill feel that they need to do well in mathematics at school is because it is compulsory.

Figure 5.23: Students' views of why they need to do well in school mathematics, Year 1 (Base=71 students)



Having observed students in classes, questionnaires (given to all students present in the three First Year classes) were used to explore students' own views on their classroom experience of learning mathematics. (See Appendix N for construction of scales). The following four areas were examined:

1. students' attitudes to mathematics: perceptions of school mathematics, that is how difficult, useful, interesting, enjoyable or boring mathematics is perceived to be and whether it was listed as a first or second favourite subject;
2. their academic self-image in relation to maths, self-assessed ability in school mathematics in the context of their peer group;
3. their perceptions of a positive classroom interaction with their teacher, perceptions of frequency of interaction with their teacher and level of reward for achievement in class;

4. their perceptions of a negative classroom interaction with their teacher, correction/sanctioning for poor work or bad behaviour.

The mean scores for each class on each of the scales are presented in Appendix to chapter 5 O: Table 2. As data was available from the ten case study schools from *Inside Classrooms* (2003), the findings from the three schools in this study are presented in comparison with the ten case study schools, giving a total of thirteen schools. The comparisons of the findings of this study with *Inside Classrooms* can be found in Appendix 0 to chapter 5: Table 3 (i) and (ii). One of the most striking findings of this study was that the classes from Kenmore, Riverside and Chestnut Hill ranked 9th, 10th and 11th respectively out of the thirteen schools in their “attitude towards mathematics” when compared with the schools from Lyons’ study and would be classified by Lyons et al., as having a “negative attitude” towards mathematics. All three classes did have a more positive perspective in terms of mathematics self-image. The students from Kenmore (ranked 5th in comparison with schools from *Inside Classrooms*) had the most positive self-image of the three schools in this study, followed by Chestnut Hill and Riverside (ranked joint 7th in comparison with schools from *Inside Classrooms*) with the same mean. The differences between the three classes were not statistically significant.

Two scales were constructed to examine the students’ experiences with the class teacher; these were positive and negative classroom interaction. Within the three schools in this study the students from Chestnut Hill and Riverside (ranked 5th and 6th in comparison with schools from *Inside Classrooms*), reported higher levels of

positive interaction with their mathematics teacher than students from Kenmore (ranked 8th in comparison with schools from *Inside Classrooms*); the differences again were not significant. What was significant was that students from Kenmore and Riverside (ranked 1st and 3rd in comparison with schools from *Inside Classrooms*) reported receiving low levels of negative attention i.e. sanctioning/correction for work-related and non-work related behaviour. The opposite applied in Chestnut Hill (ranked 11th in comparison with schools from *Inside Classrooms*) where there was a considerable difference. The data indicates that in general all three classes did not have very positive attitudes to mathematics, yet had a fairly high mathematics self-image. Their experiences with their classroom teacher were fairly positive, however the students from Chestnut Hill reported more negative interactions with their teacher.

Of the three focus group discussions with students, the students from Chestnut Hill and Kenmore found it difficult to discuss mathematics as a subject and those from Chestnut Hill in particular had little interest in discussing it. When discussing their experiences of learning mathematics all three schools discussed mathematics in terms of their teachers. Students from Kenmore and Chestnut Hill did not discuss the subject at all but students from Riverside did. Students at Riverside and Kenmore identified algebra as an area where they were having difficulty but could not articulate why they were having problems with it. Both sets of students used similar language to describe their difficulty: “don’t get it”, “can’t pick it up”, “not in my head”, “mind blown by it”, and “rules are confusing”. As there is considerable international and national evidence that students’ attitudes and performance are strongly influenced by their mathematical experience, and in particular how

mathematics is taught (Dick and Rallis, 1991; Johnston, 1994; Ma, X., 1997, 1999; Reynolds and Walberg, 1992; McLeod, 1992, 1994), the next section examines students' experiences of learning mathematics. The insights are from the focus-group interviews-cum-discussions with students.

When asked about their current experience of learning mathematics the students in the focus group from Riverside were positive. Their responses were mainly articulated in terms of their teacher but their experience of learning geometry arose spontaneously and they spoke at length about the difference in their present teacher's style in teaching it compared to primary school, how much they were enjoying the topic as a result, the resources being used and their parents' comments when they saw how they were learning it. One class of geometry was observed in the videos and has already been referred to in the video analysis (See Appendix B) and by the teacher in her interview (See Appendix M). The extract of the interview with the focus group from Riverside can be found in Appendix P (i). In Kenmore both the boys and the girls spoke about mathematics in terms of being harder than in Primary school and moving faster. In Chestnut Hill the students appeared to find it difficult to articulate how they found mathematics in first year. (See Appendix P (ii)).

Students were also asked what a good mathematics teacher is. At Riverside and Kenmore students spoke solely in terms of their teacher. Students from all focus groups spoke about the importance of thorough explanation. In addition the students from Riverside (See Appendix P (iii)) felt that it was important for teachers to give students encouragement and confidence and to be approachable. Students from

Kenmore referred to qualities their teacher had which they felt made him a good mathematics teacher: “a sense of humour”, “not putting on too much pressure” and not “going too quickly”. Students at Chestnut Hill did not mention their teacher in reply to this question; they commented on the importance of a teacher who cares, is patient and will not shout at you if you get a question wrong.

Another question that arose during the focus group discussions with students centred on asking questions in class time. In answering this question the students said that they would ask the teacher for help but preferred not to ask in front of the class and would seek out help instead from a friend, siblings or parents. Asking a question in front of the class seemed to threaten them and invariably the answers given were in terms of their emotions. At Riverside the girls said that, while they would ask the teacher a question, they sometimes felt “afraid”, “scared”, “annoyed”, “frustrated”, “foolish”. At Kenmore the students reported that they would ask the teacher a question but felt “embarrassed” about doing so. At Chestnut Hill the students spoke about feeling anxious and afraid that “people might laugh at you” if you don’t understand. (See Appendix P (iv)). Considerable research (Donady and Tobias, 1977; Tobias and Weissbrod, 1980, Richardson and Suinn, 1972; Hembree, 1990; Wine, 1971) has documented the consequences of feeling anxious about mathematics as an inability to do mathematics, a decline in achievement and a disturbance of recall of mathematics already learned.

Students were also asked did they like mathematics. The answers given were many and varied from the focus group interviews. The responses from the students at Riverside were generally positive and they credited their teacher for this. One student spoke about

the importance of mathematics in getting a job and the world in general. The students at Kenmore were not as positive in their responses to this question; they spoke about liking the class but not the subject and the boys several times used the word “boring” to describe how they felt about mathematics. Similarly the students at Chestnut Hill were not very enthusiastic about their liking for the subject. (See Appendix P (v)).

Conclusion

The analysis of the six videotapes revealed four important findings.

1. The mathematical content of the lessons was low/moderate, both in procedural complexity and application over the course of the six lessons observed. Without rich mathematical content it is unlikely that many students will construct a deep relational understanding of mathematics from a lesson.
2. To engage students with the mathematical content teachers must consciously structure their lessons. The classes observed were organised in terms of periods of teacher talk/demonstration and student practice. Generally the performance expectation of teachers was for students to practise routine procedures on their own with a resultant lack of opportunity for the students to participate or engage in their own learning. Students were not asked to struggle with a problem for which they might not have been taught a solution, then present the solutions they generated to their classmates. Presentation and discussion of alternative solution methods may provide a natural opportunity for engaging the students in mathematical discourse.

3. Mathematical concepts in the videotapes were, for the most part, just presented or stated by the teacher or students but were not explained or developed by the teacher or the teacher and students collaboratively in order to increase students' relational understanding of the concept. Consequently teachers were far more likely to use lower-order questions rather higher-order discussion type questions to teach concepts. The emphasis was on instrumental understanding rather than relational understanding. The researcher is aware however that during the course of the year the teachers may have developed and explained these concepts.

4. Answers were classified as either right or wrong, with clear signals from the teachers that they were interested only in the right answer. The subject then is one where there was a very definite judgement of the students' work. This judgement was sometimes made public. The potential effect of this emphasis on right or wrong answers could lead to anxiety and tension for students in relation to mathematics. Considerable research (Donady and Tobias, 1977; Tobias and Weissbrod, 1980, Richardson and Suinn, 1972; Hembree, 1990; Wine, 1971) has documented the consequences of feeling anxious about mathematics as an inability to do mathematics, a decline in achievement and a disturbance of recall of mathematics already learned.

The data from questionnaires with the three teachers show that they generally regarded mathematics as a formal way of representing the world and as a subject where natural ability plays an important role in determining learning outcomes. While they believed that varied teaching methodologies and practice at the subject improved learning, learning was equated with formulae and procedures rather than

thinking creatively, or understanding how mathematics is used in the real world.

There was a high degree of consistency between teachers' reports of their pedagogical practices as observed in the videotapes, except for the importance they claimed to place on students being able to provide reasons to support solutions.

Teachers reported that much of their time in mathematics lessons involved demonstration, monitoring student progress and checking homework. This was consistent with the findings from the video analysis. Their reports that much of the students' time is spent copying examples from the board and practising procedures was also borne out by the video analysis. Teachers also reported that students are not involved in discussions relating to mathematics. Overall, the data confirmed that the didactic approach to teaching mathematics was the predominant style embraced by the three teachers in this study, both in theory and in practice. Teachers generally attributed student progress to enjoying the class and the encouragement given to them by the teacher. Teachers did not notice any student falling behind over the course of the year but felt that some students were making slower progress than others.

Most students believed that success in mathematics depended on: having a good teacher, learning the textbook off by heart, doing lots of hard work and liking the subject a lot. Students did not attribute as much importance to natural talent as their teachers did. Students were aware of the importance of mathematics both in the short and in the long term. Analysis of data showed little difference between the three classes in the students' attitude toward, and experience of school mathematics. The three classes were broadly definable as being quite negative. There was a considerable difference in their perceptions of classroom interaction with their

teacher. The focus group discussions did highlight the importance of the style of the teacher in determining the students' attitudes towards and experience of learning mathematics.

All students spoke about mathematics in terms of their teacher's particular approaches, as distinct from speaking about mathematics as a subject in itself. Students at Riverside however were extremely articulate in discussing geometry and expressed a liking for it more than any other topic they had done that year. For this topic the teacher was trying out a new approach which involved more active learning on behalf of the students. This exception highlights the point that the traditional procedural approaches typically fail to engage the student and bring them inside the subject as such. This finding concurs with international and national research that students' view of the subject is strongly influenced by their classroom experience of learning it (Dick and Rallis, 1991; Johnston, 1994; Lyons et al., 2003; Ma, X., 1997).

Students spoke about teaching practices in mathematics that make them feel uneasy in class, such as going too fast, being put under pressure and the teacher expressing annoyance when a student gets something wrong. Students described a good teacher as someone who gives encouragement, is patient, explains things clearly and uses humour. Students expressed anxiety about their reluctance to ask the teacher questions in front of the class. This may be due to the fact that when teachers asked questions they tended to ask the whole class questions and there was a culture of the teachers wanting rapid response, correct answers from students. They did not at any time show that it was okay to get an answer wrong or show an interest in what was in

the students' individual mind when they gave an incorrect answer. Research (Donady and Tobias, 1977; Tobias and Weissbrod, 1980; Richardson and Suinn, 1972; Hembree, 1990; Wine, 1971) has documented the consequences of feeling anxious about mathematics as an inability to do mathematics, a decline in achievement and a disturbance of recall of mathematics already learned. Students hid their lack of understanding from both their peers and their teacher. Students also referred to not liking the subject and the reason they gave was that they found it boring. This finding seems to indicate that the students were not too deeply engaged with the mathematics they were being taught by their teachers.

In this chapter we have reviewed the findings and issues from Year 1 of this study. In the next chapter we will continue to review the study's investigations from Year 2 and compare them with that of Year 1. The chapter will focus on an in-depth investigation of the classroom interaction from the video tape recordings. It will also look at the teachers' views on teaching and learning mathematics. Students' views on learning mathematics are described and finally the central themes that emerge in the teaching and learning of mathematics in second year mathematics will be outlined.

Chapter 6

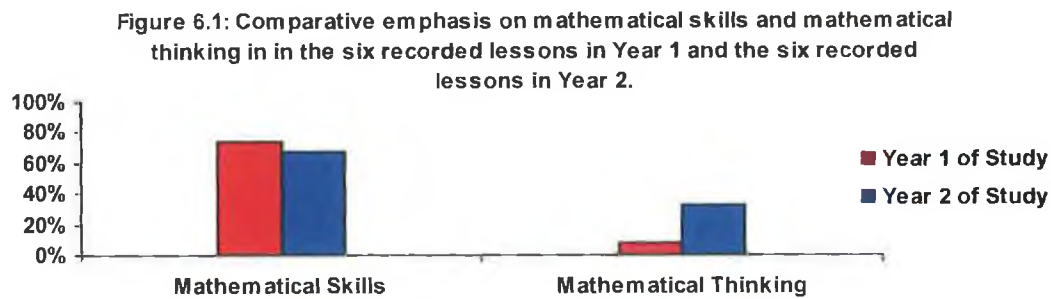
Findings from Second Year

Section 1: Videotape Analysis: Pedagogical Practices and Classroom Interactions.

The importance of the mathematics content presented during the lesson derives in part from the fact that the prescribed content defines the parameters within which students work. If students are presented with a topic they have an opportunity to learn something about the topic. Moving beyond the textbook and the syllabus, an examination of the content of the videotaped lessons reveals the implemented curricula and the focus here is on what the students actually encountered in the classroom. A general description of the mathematical content of each of the mathematics lessons that were videotaped in the second year of the study is contained in Appendix A Table 1 for chapter 6. It also includes a description of how teachers described the content of the videotaped lessons on a continuum from “all new” to “all review”.

A key variable that shapes the nature of the teaching is the set of learning goals toward which the teacher is working (Hiebert et al., 1997). The three teachers in this study were asked to describe the “main thing they wanted students to learn from today’s lesson?” The findings from Year 2 of the study are compared to Year 1 (Figure 6.1). The answers show that the teachers listed skills quite highly in Year 2,

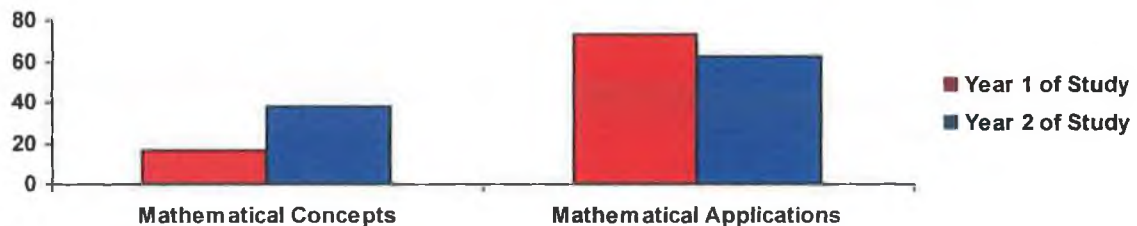
as in Year 1, but also identified mathematical thinking as a goal to a much greater extent in Year 2 than in Year 1.



Having determined what appeared to be the goals for the six recorded lessons (two from each of the three classrooms Chestnut Hill, Kenmore and Riverside), each lesson was analysed by the author of this study to see what type of mathematics was evident in the lessons videotaped. For this purpose the mathematics presented in the six lessons was divided into two categories: mathematical concepts and mathematical applications. These categories were defined to broadly catch those instances in which students might have constructed concepts or learned how to apply them. “Concept” in this instance means when a mathematical property, formula or theorem is explicitly referred to, defined or explained to the class e.g. “Here we are using the Sine ratio” or “The formula for Pythagoras’ theorem is the area of the square on the hypotenuse is equal to sum of the areas of the squares on the other two sides” or “The distance travelled equals the speed multiplied by the time”. “Application” means the explanation of the concept was not included but that the concept itself was used without explicitly referring by name to it to develop skills for specific types of mathematical problems. Just as in using grammar in a foreign language we would have to say for example “Now we will use the future tense”. However, when we are

already well versed in a language we don't have to refer explicitly to the grammar. Application then is something we do when we are fluent without referring to the concept/grammar we are using. But for learners we need to refer explicitly to the concept we use whether it be the concept of Pythagoras' Theorem or the future tense otherwise a presumption is being made in the application that the concept is understood. While the writer is not making a criticism here good practice would be for the teacher to intervene and not presume that all students know which concept is being used or why they are using it. These presumptions occur as a matter of course in customary practice. The average percentage of topics in each lesson that include mathematical "concepts" or "application" is summarised in Figure 6.2.

Figure 6.2 Percentage of topics in each lesson that include mathematical concepts or mathematical applications

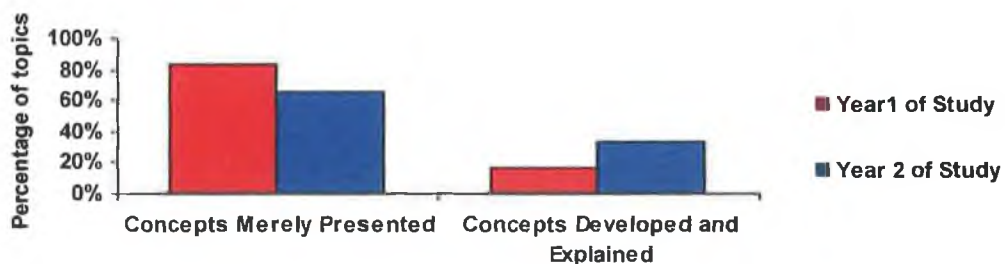


The analysis of the recorded lessons sought to discover if "concepts" that were introduced in the lessons were (a) merely presented by the teacher or students in order to guide the students to the solution of a problem or (b) explained and developed collaboratively by experimentation, by conjectures, verification, demonstration of results and using logically connected sequence of steps. In the case of (a) the focus is on mathematical information and mechanical skills rather than on the process and development of mathematical reasoning. On the other hand in the

case of (b) the focus is on increasing students' relational understanding (Skemp, 1978; Nickson, 1992; Dossey, 1992). In the classes observed 66% of topics contained concepts that were merely presented without further explanation in order to solve problems as opposed to 33% that were developed and explained (Figure 6.3).

Compared to Year 1 of this study there were many more examples from the lessons videotaped where concepts were developed by the teacher and/or the students collaboratively. These examples were predominantly to be found at Riverside. In contrast to Year 1 (where the only example of developing concepts by students took place privately), this collaboration at Riverside tended to be conducted publicly and the students observed appeared very comfortable in doing so. (See Appendix B).

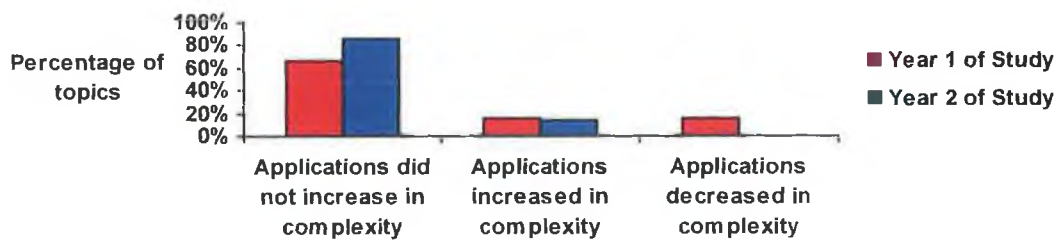
Figure 6.3: Average percentage of topics in maths lessons that contained concepts that were merely presented or developed



After examining whether concepts were merely (a) merely presented or (b) explained and developed, topics that contained “applications” of the concepts to solving problems were considered to see whether the complexity of the applications increased, stayed the same or decreased over the course of the lesson (Figure 6.4).

Generally the mathematical problems were the same, or mostly the same, as a preceding problem in the lesson; that is, they required essentially the same operations to solve although the numerical or algebraic expression might be different. When topics did increase in complexity it included extending or elaborating a previous problem.

Figure 6.4: Percentage of topics in each lesson that contained procedural applications that increased in complexity, stayed the same or decreased over the course of the lesson



One characteristic of mathematics encountered in teaching and learning is its complexity and this is now examined. The complexity of the mathematics presented in the lessons is difficult to define however because the complexity of a problem for a learner depends on a number of factors, including the experience and capability of the student. One kind of complexity that can be defined independently of the student is procedural complexity - the number of steps it takes to solve a problem using a common solution method. The mathematics problem analysis group for the 1999 TIMSS video study of eighth grade mathematics teaching (which is comparable to first/second year Irish post-primary) (Hiebert et al., 2003, p70-71) developed a scheme for coding the procedural complexity of mathematical problems. Problems were sorted into low, moderate or high complexity according to the following definitions:

- Low complexity: Solving the problem, using conventional procedures, requiring four or fewer decisions by the students (decisions could be

considered small steps). The problem contains no sub-problems, or tasks embedded in larger problems that could themselves be coded as problems;

- Moderate complexity: Solving the problem, using conventional procedures, requiring more than four decisions by the students and can contain one sub-problem;
- High complexity: Solving the problem, using conventional procedures, requiring more than four decisions by the students and containing two or more sub-problems.

Taking the six recordings from the classrooms together, Figure 6.5 shows the average percentage of problems per lesson that were at each complexity level. Figure 6.6 shows the number of problems per lesson at Kenmore, at Riverside and at Chestnut Hill that were at each complexity level. Kenmore is a Foundation Level class and by its nature it has predominantly more of low level content than Chestnut Hill and Riverside, which are both Higher level classes.

Figure 6.5: Average percentage of mathematics problems per lesson at each level of procedural complexity. (Taking all of the recorded lessons into account).

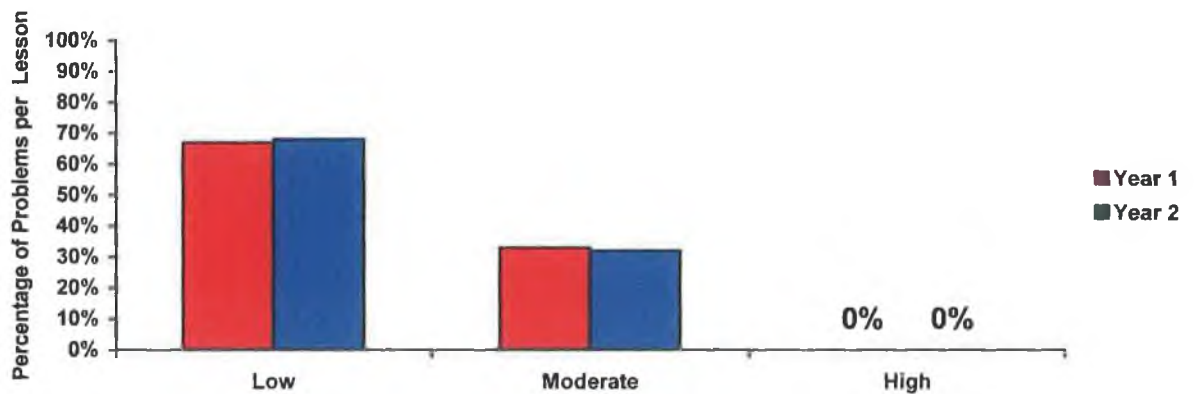
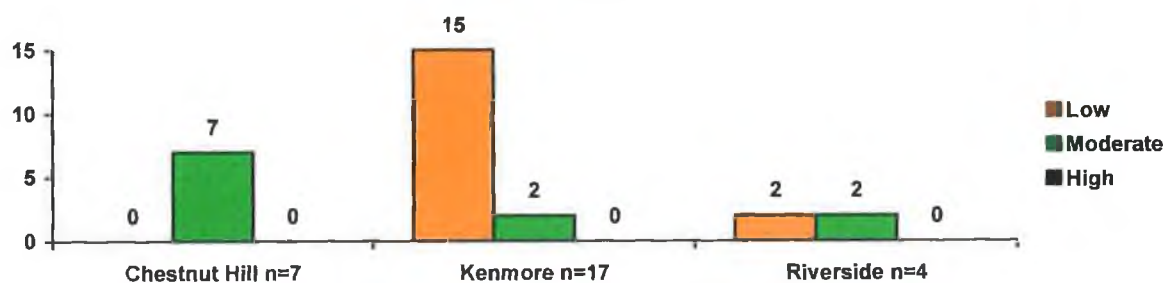
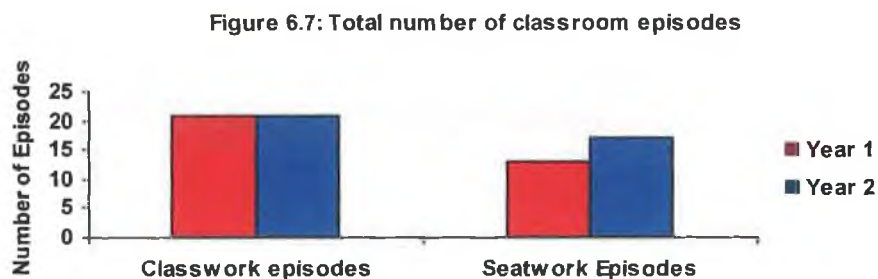


Figure 6.6: Number of Second Year mathematics problems per lesson at each level of procedural complexity, by class.



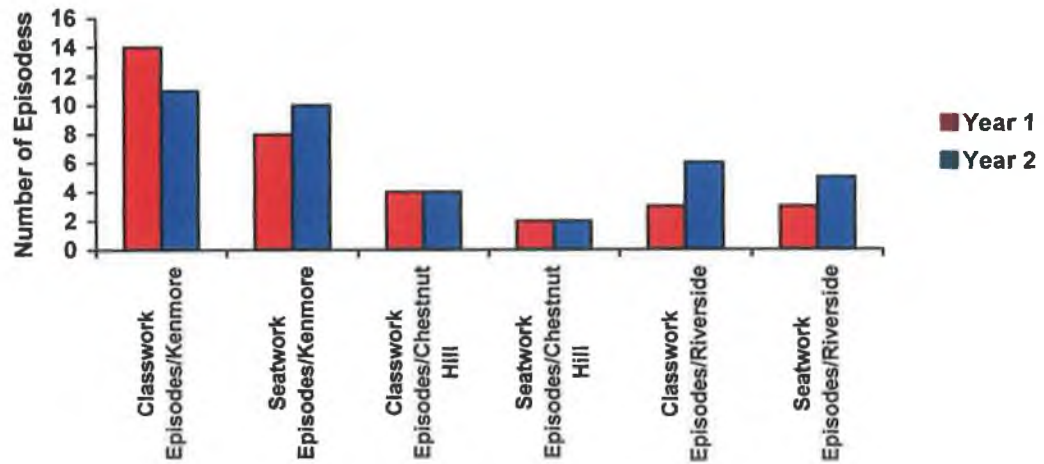
The videotapes for Year 2 Riverside show that lessons were characterised by devoting lesson time to relatively few problems and spending a relatively long time on each one. The emphasis was on introducing new content and lesson time was filled with developing concepts where students were invited to explore the concepts with the teacher. The problems were of both low and moderate complexity and included much public discussion. This profile is in contrast to Kenmore and to Chestnut Hill (which is also a Higher Level class). Chestnut Hill and Kenmore devoted lesson time to a significant number of problems, which consisted mainly of the repetitive practising of content without much development of the concepts. The problems at Chestnut Hill were all of moderate complexity whereas at Kenmore they were mainly of low complexity. The fact that a topic is introduced and many problems completed does not tell us much about the learning opportunities for students - whether they are enabled or constrained, or about how deeply students might learn the topic. To pursue these issues we will now consider how the mathematics lesson environments were organised. This is a key issue, as the organisation of the lesson may constrain both the content that is taught and the way that content is taught.

An element of the classroom organisation that can enable or constrain different kinds of learning experiences for students is the way in which the teacher and students interact (Brophy, 1999; p. 10-12). Many classrooms include both periods of whole-classwork and periods of seatwork which provide activities for students to engage with the subject. Classwork includes those times in which the teacher and students interact publicly with the intent that all students participate (at least by listening) in learning a new concept, solving a mathematical problem or demonstrating a procedure etc. Seat work means those episodes when students complete assignments individually or in small groups, and during which the teacher circulates around the room and assists students who need help. The total number of each type of episode observed in the 6 videotapes is shown below (Figure 6.7)



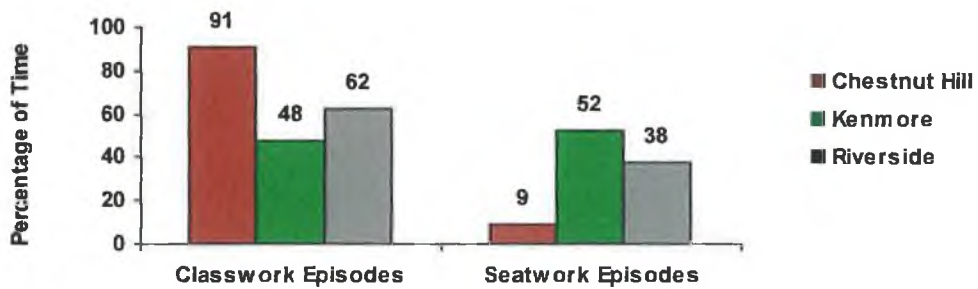
A further analysis of the classes shows a striking difference. Kenmore contains many more classwork and seat work episodes than either Chestnut Hill or Riverside (Figure 6.8). On this account it appears that the students in Chestnut Hill and Riverside seem to be idle much of the time, while students at Kenmore look very occupied. This picture changes when the percentage of time spent in classwork

Figure 6.8: Total number of classwork and seatwork episodes in Chestnut Hill, Kenmore and Riverside classrooms



and seatwork episodes are considered (Figure 6.9). Kenmore and Riverside look quite similar in terms of the classwork and seatwork episodes used to study mathematics. On the other hand Chestnut Hill appears distinctly different. 91% of lesson time involves the students participating in public classwork, mainly by listening to the teacher demonstrate a procedure.

Figure 6.9: Percentage of time during the lesson spent in classwork and seatwork episodes in Chestnut Hill, Kenmore and Riverside classrooms



This preliminary analysis of the proportion of time spent in classwork and seatwork episodes represents however only a superficial view of what occurs in a mathematics lesson. These episodes can be used by teachers to accomplish different purposes.

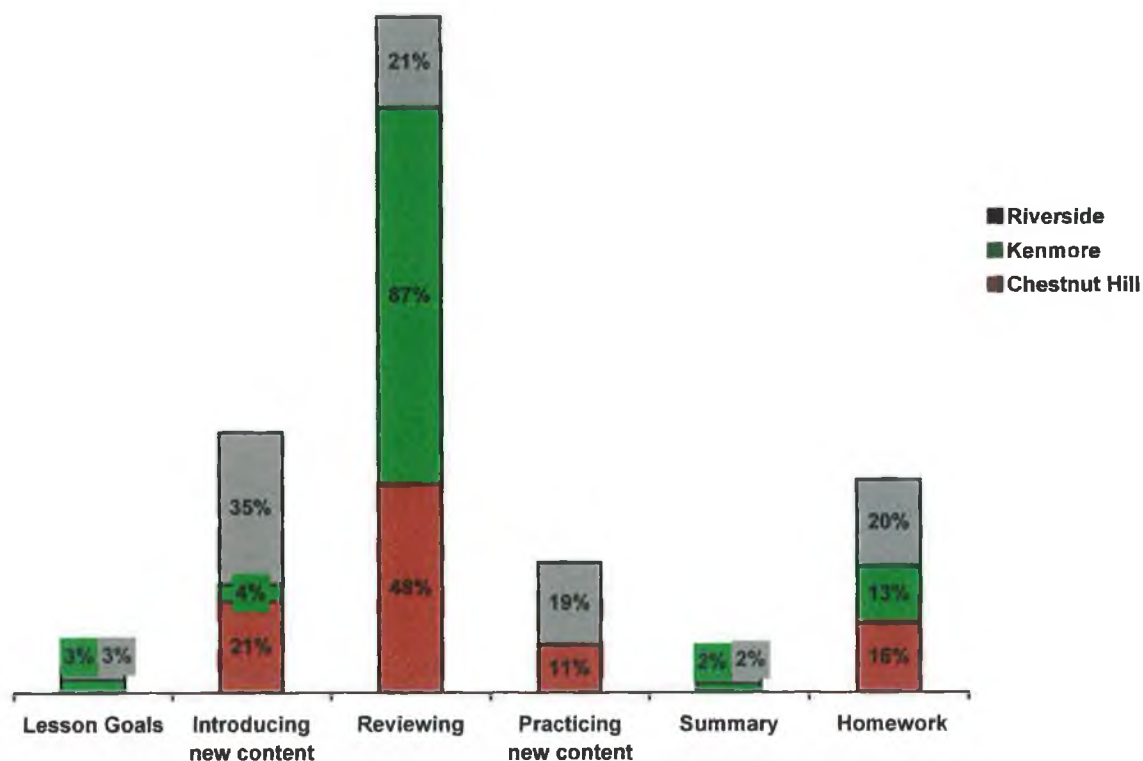
With this in mind the recorded lessons were further divided into activity episodes that served some pedagogical purpose such as:

- goal statements (verbal or written statements by the teacher about the specific mathematical topic(s) that would be covered during the lesson);
- introducing new content (this activity focused on introducing new content that students had not worked on earlier in a previous lesson. Examples of this type of activity included: teacher expositions and demonstrations, teacher and student explorations through solving problems that were different from problems worked on previously and class discussions of new content);
- reviewing (this activity focused on addressing content introduced in previous lessons. These activities typically involved the practice or application of a topic learned in a prior lesson, or the review of an idea or procedure learned previously);
- practising new content (this means students practising or applying new content introduced in the current lesson);
- lesson summary statements (statements made by the teacher describing the key mathematical point(s) of the lesson);
- homework (this activity focused on setting homework or correcting answers for previously completed homework problems).

Figure 6.10 displays the percentage of lesson time devoted to each activity. In each of the six videotapes – i.e. the three earlier and three later sessions - the time spent on stating the goals of the lesson was extremely low: 3% of lesson time was spent on this at Riverside and at Kenmore and none at all at Chestnut Hill. Yet this is one way

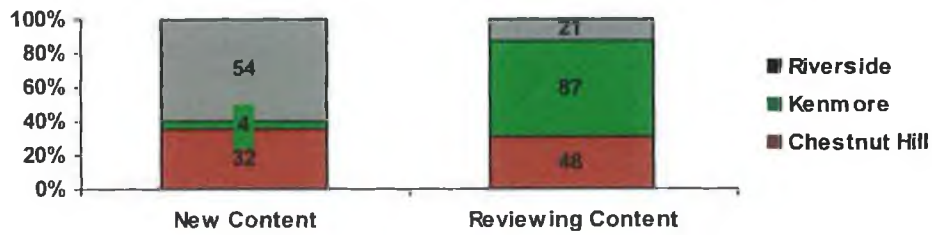
teachers can help students identify the key mathematical points of a lesson (Brophy, 1999; p. 15) and improve the clarity of a lesson. A second kind of aid to help students recognize the key ideas in a lesson is a summary statement which highlights points that have been studied in the lesson. Summary statements were less common than goal statements, with only 2% of lesson time being spent on this again at Riverside and at Kenmore and none at Chestnut Hill. The decision to include the setting of homework within a lesson (as distinct from at the end) and correcting answers for previously completed homework problems can directly affect how that lesson is organized. Figure 6.10 displays the extent to which homework was worked on as part of the lesson time. At Riverside homework was corrected privately but at Chestnut Hill and Kenmore homework was corrected publicly. In no classes were students observed correcting or discussing each others' homework. In homework correction episodes, homework corrected in the public domain (Chestnut Hill and Kenmore) was corrected at a brisk pace, with little explanation when a student gave an incorrect answer to a particular step in the solution. The emphasis by both teachers seemed to be on getting the correct response each time in order to get to the final correct answer. (See Appendix C). Homework was treated as a more central part of the lessons in Chestnut Hill than in the other two classrooms.

Figure 6.10: Percentage of Time Allocated to Activities



Two contrasting examples of how new content was introduced by teachers can be found in Appendix D (i) and (ii) for chapter 6. Appendix D (i) illustrates how the teacher uses a transmission style to introduce new material whereas Appendix D (ii) highlights how students can be actively engaged in the introduction of new content by the teacher. By combining the time spent on the two activities of reviewing and practising new content it is possible to compare the time spent on new content with the time spent reviewing content introduced in a prior lesson. This comparison can be seen in Figure 6.11.

Figure 6.11: Percentage of Second Year mathematics lesson time devoted to various purposes , by class



Riverside (a Higher Level class) spent a greater percentage of mathematics lesson time on introducing and practising new material relative to previously learned material. The reverse occurred in Chestnut Hill (a Higher level class), and in Kenmore (a Foundation level class), where the teachers placed a greater emphasis on reviewing previously learned material. There are two ways to interpret this: On the one hand lessons that spend a lot of time reviewing and practising old content provide students with the opportunity to become more familiar and efficient with content they have already encountered; yet on the other hand they are less likely to have opportunities to learn new material.

Two lessons were recorded in the case of each of the three teachers in Year 2. All six lessons observed were predominantly dominated by the teacher in various forms of teacher talk – mainly demonstration, explanation and questioning. Student practice mainly consisted of applying content introduced in the current or earlier lessons. By assigning practice examples, the teachers were able to ascertain whether students had listened and had an initial understanding of what was demonstrated. The emphasis in Chestnut Hill and Kenmore seemed to be on applying mathematical procedures whereas in Riverside the students appeared to be more actively engaged by the questions the teacher posed in constructing mathematical ideas. (See Appendix D

(ii)). Appendix E is an example of student practice applying procedures from Chestnut Hill that follows on from the same lesson referred to in Appendix D (i) for chapter 6 on introduction of a new topic.

Having presented the elements that structured the learning environments for students it is important to note that through these elements might not influence learning directly, they might set boundaries for students' learning or on the other hand present opportunities. The choices teachers made in structuring their lessons represent stage-setting choices on behalf of the teachers. Up to this point our focus has been on the mathematical content and organisation of the lessons respectively. Continuing with this analysis we will now explore how this content was worked on during the lessons. What were the dynamics of interaction in the classroom? How were students engaged by the teacher in the classroom? What was the nature of questioning and processes of instruction? Answers to these questions are taken up in the next section and provide additional information about Second Year mathematics teaching in each classroom.

The researcher counted that there were a total of 531 classroom interactions in the recorded lessons out of which 484 were teacher-initiated (i.e. teacher initiated discourse consisted of 91% of all classroom interactions). Relative to their students, as was also found in Year 1, the teachers talked much more (Figure 6.12). The average number of teacher words to every one student word per Second Year mathematics lesson, by class is shown in Figure 6.13. Student-initiated interactions were mainly questions of a low cognitive level – e.g. asking if what they are doing is

right. What was clear from the video analysis was that few students asked questions publicly.

Utterances between teachers and students were chiefly made up of questions. However in Chestnut Hill there was a significantly large number of instructions being given to the students by the teacher, compared to Kenmore and Riverside (Figure 6.14 and 6.15 respectively).

Figure 6.12: Percentage of all public interactions, Year 2 (n=531)

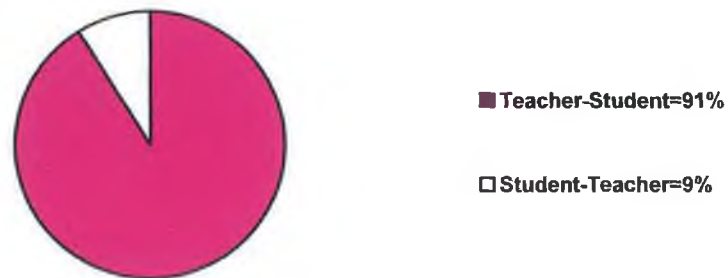


Figure 6.13: Average number of teacher words to every one student word per Second Year mathematics lesson, by class

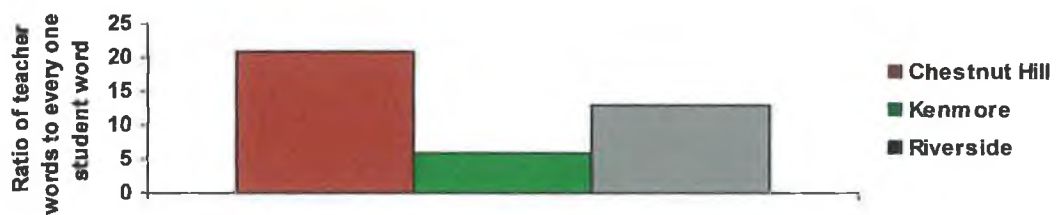


Figure 6.14: Total number of teacher-student interactions.

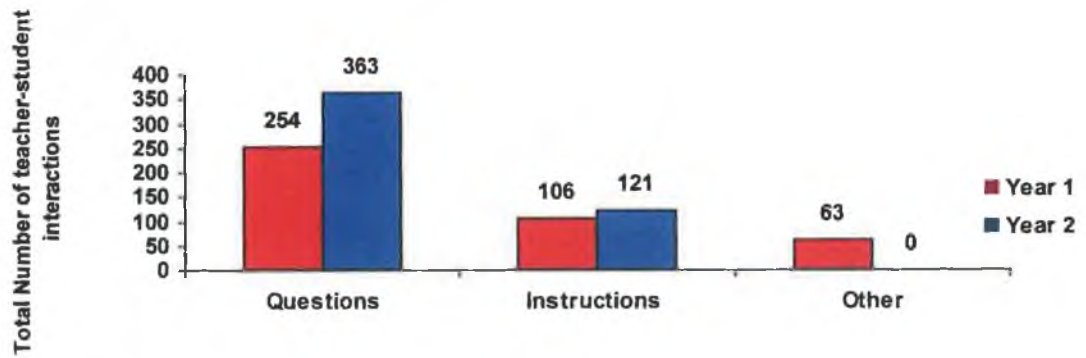
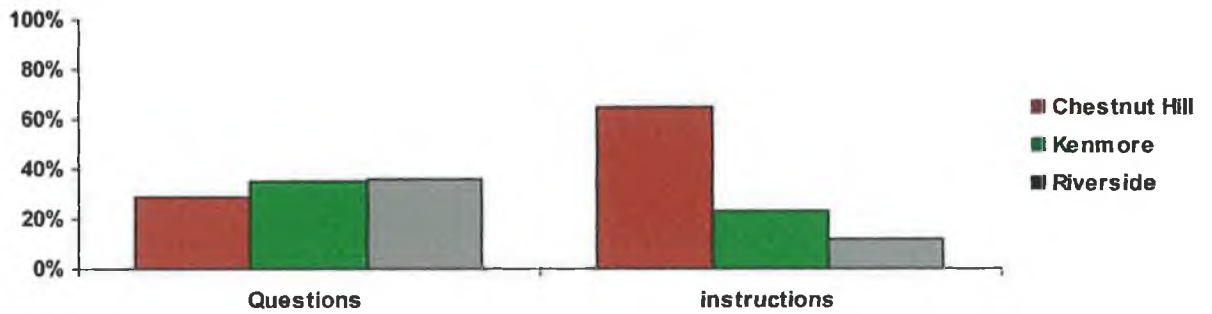
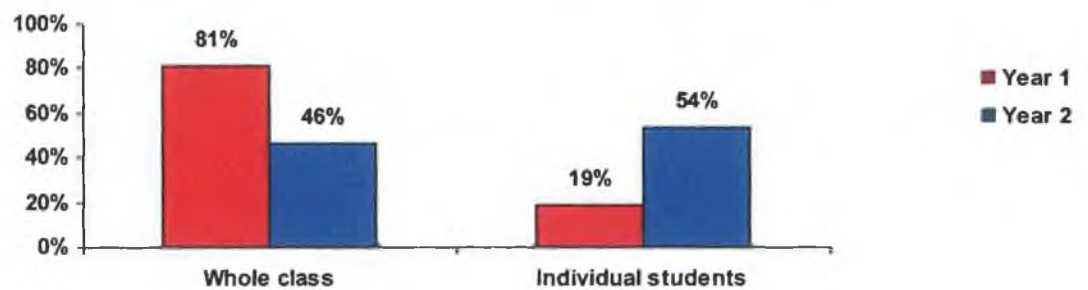


Figure 6.15 Categories of Teacher-student interactions, Year 2 (n=484)



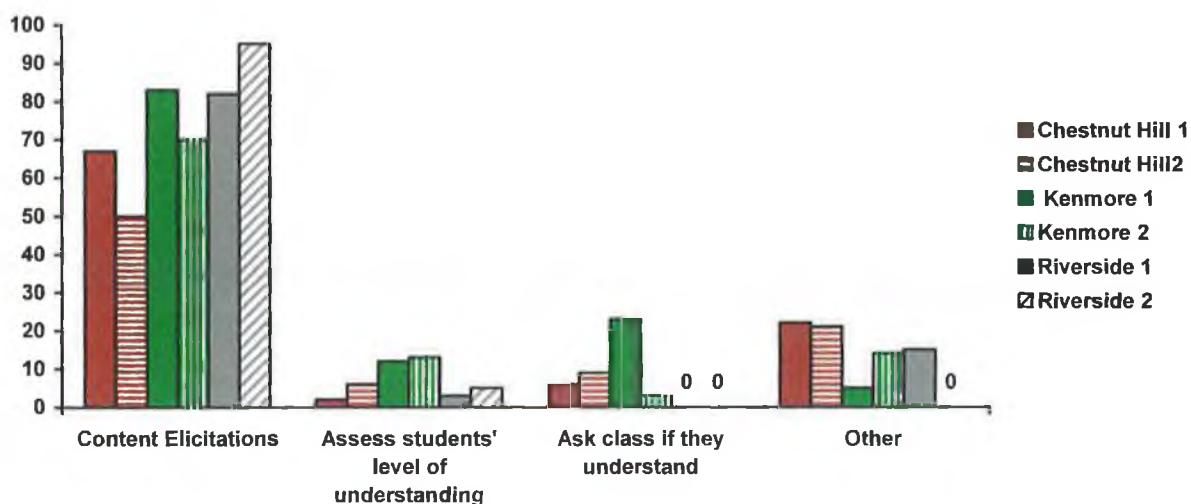
In contrast to Year 1 the questions in Year 2 appear to be delivered quite evenly between individual students and the whole class (Figure 6.16).

Figure 6.16: Distribution of teacher-student interactions (n 484)



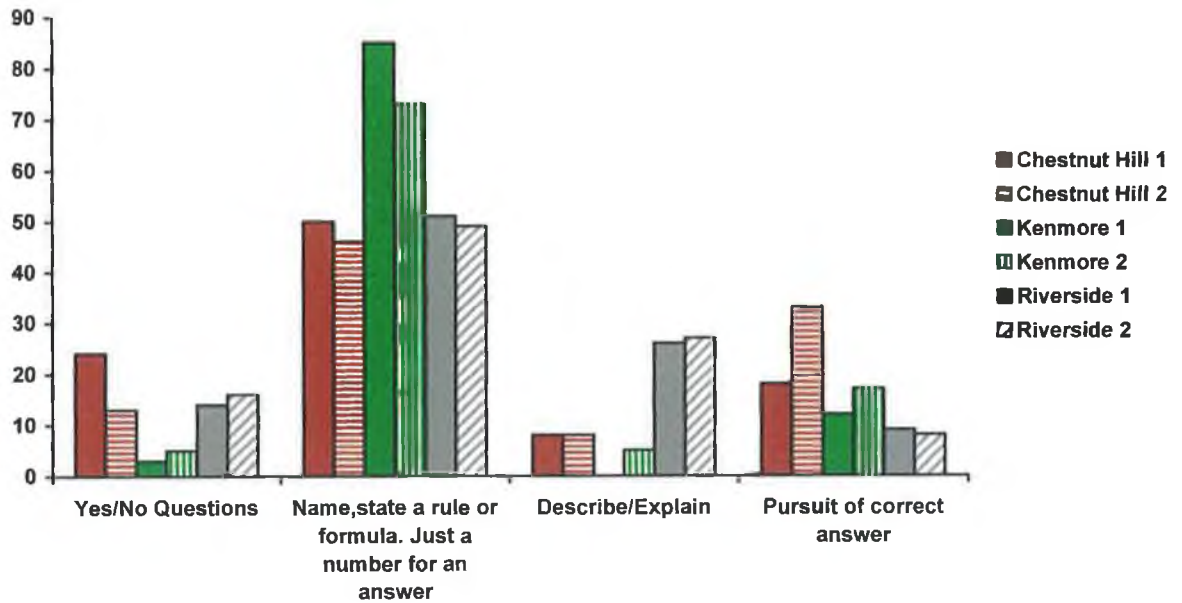
The types of questions overwhelmingly asked by teachers were content elicitation. Students were asked to supply the next step in a procedure to a solution, supply a number, identify a shape, define a term or evaluate an answer. Generally the teachers did not ask questions that would assess the students' level of understanding. The teachers in Chestnut Hill and Kenmore tended to say periodically: "Do you all understand that?" directed to the whole class in order to assess students' level of understanding. Figure 6.17 below shows the percentage of the type of questions asked by the three teachers in each of the six classes observed.

Figure 6.17: Percentage of type of questions asked by teachers Year 2, n=363



As content elicitation generated so much of the mathematical content that was discussed in the lesson, a further analysis of these questions was undertaken. The results are shown in Figure 6.18.

Figure 6.18: Percentage of Type of content questions asked, Year 2 n=270

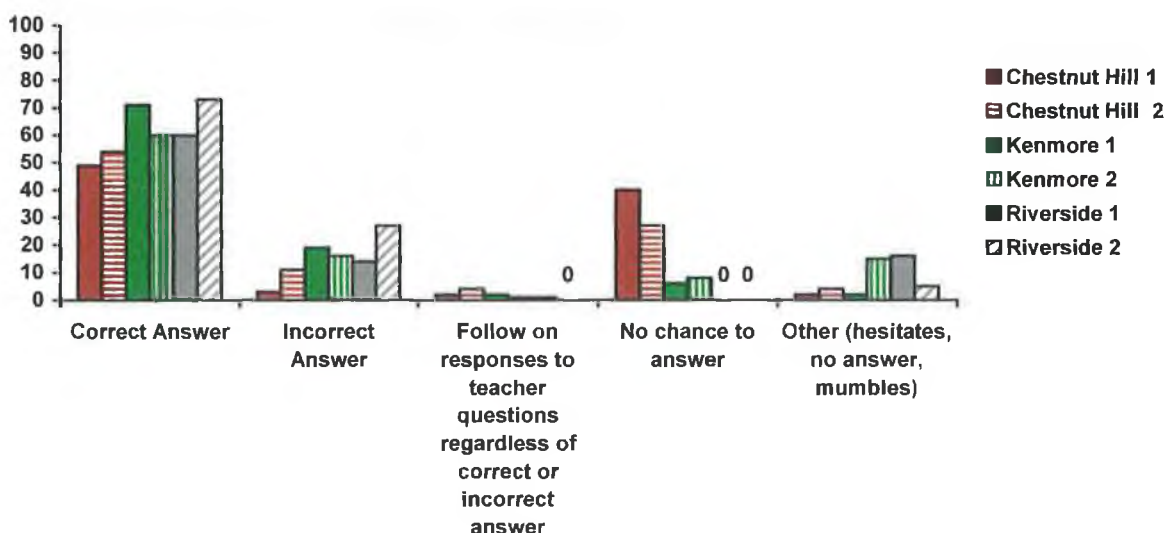


The questions in Kenmore and Chestnut Hill tended to be limited to rapidly-paced recitation type questions that elicited short answers and moved at a brisk pace (Skemp, 1978). At Riverside however questioning tended to feature sustained and thoughtful development of key ideas. Students were observed constructing and communicating content-related understandings. At Riverside it was also noted that the teacher paused when she asked a question to allow students time to process the question and at least to formulate responses, especially when the question required the student to engage in higher-order thinking (Skemp, 1978). An illustrative vignette of the type of questioning that was found in all three classrooms can be found in Appendix F. At Riverside, as no specific response was being pursued by the teacher, responses from students were less likely to be evaluated by the teacher as right or wrong (Ernest, 2004; Thompson, 1992). By contrast, in Chestnut Hill and Kenmore, the teachers tended to look for specific right answers. The purpose of questioning in Chestnut Hill and Kenmore seemed to be to assess whether the students were able to

produce the correct answer. It was observed that at times when a student answered incorrectly the teachers from these classes moved onto another student until they got the right answer, whereas at Riverside the teacher pursued the wrong answer and students were observed abandoning their misconceived answers and adopting correct ideas and answers.

Figure 6.19 shows the responses that students made to questions asked by the teacher. The students mainly gave correct answers. Incorrect responses accounted for a very small percentage of all student answers. There were significantly few follow-on responses to teacher's questions. In some instances the students were not given a chance to answer at all, the teacher on these occasions intervened. Other categories included: hesitates/no answer/mumbles.

Figure 6.19: Percentage of student responses to teachers' maths questions (n=262, Year 2)



As so many correct answers were given by students, the type of responses that the teachers made to these answers was analysed. Figure 6.20 below shows the teachers'

reactions to correct responses. This shows that teachers mainly accepted correct answers without praising the students. Repeating the students' correct answer was the next most common type of feedback, accounting for 26% of all teacher feedback. Other types of feedback included asking an additional question of the student who provided the correct answer or praise which was not specific.

Figure 6.20: Percentage of teacher feedback to correct responses, n=188

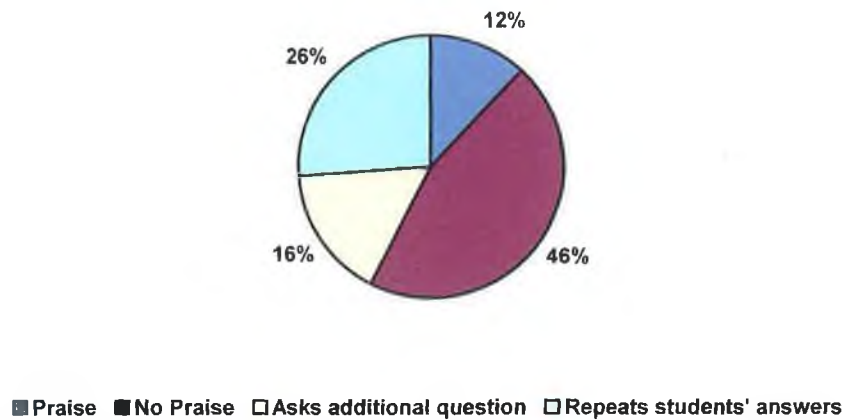
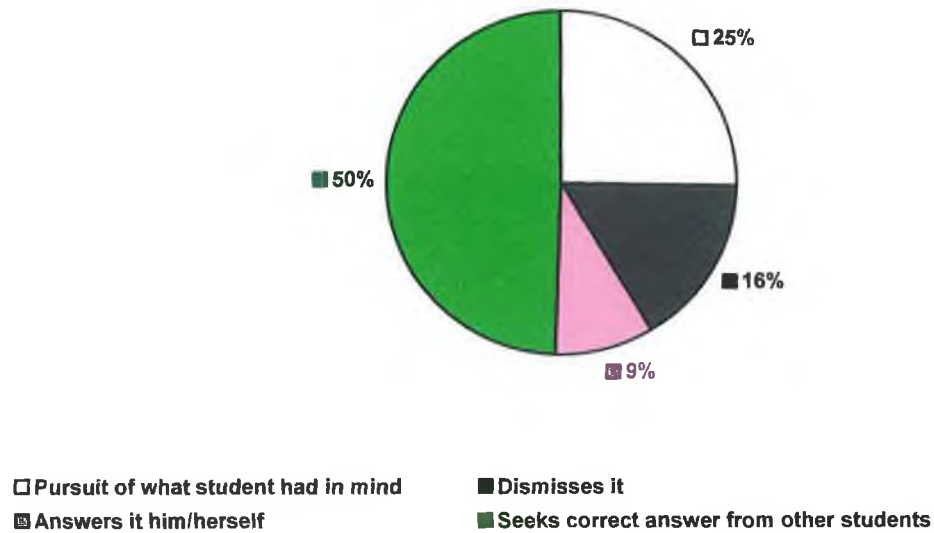


Figure 6.19 shows that there were very few incorrect answers in response to questions from the teacher. The teacher's reaction to incorrect responses is illustrated in figure 6.21. At Chestnut Hill and Kenmore teachers pursued the correct answer by asking other students the same question until they got the correct answer whereas at Riverside although the teacher pursued the correct answer she did so by rephrasing the question to the same student and pursued what the student had in mind (accounting largely for the 25% in figure 6.21) and guiding the student who gave an incorrect answer. Other responses by teachers were to dismiss an incorrect answer or answer the question correctly themselves. (See Appendix G).

Figure 6.21: Teachers' response to incorrect answers (n=55, Year 2)

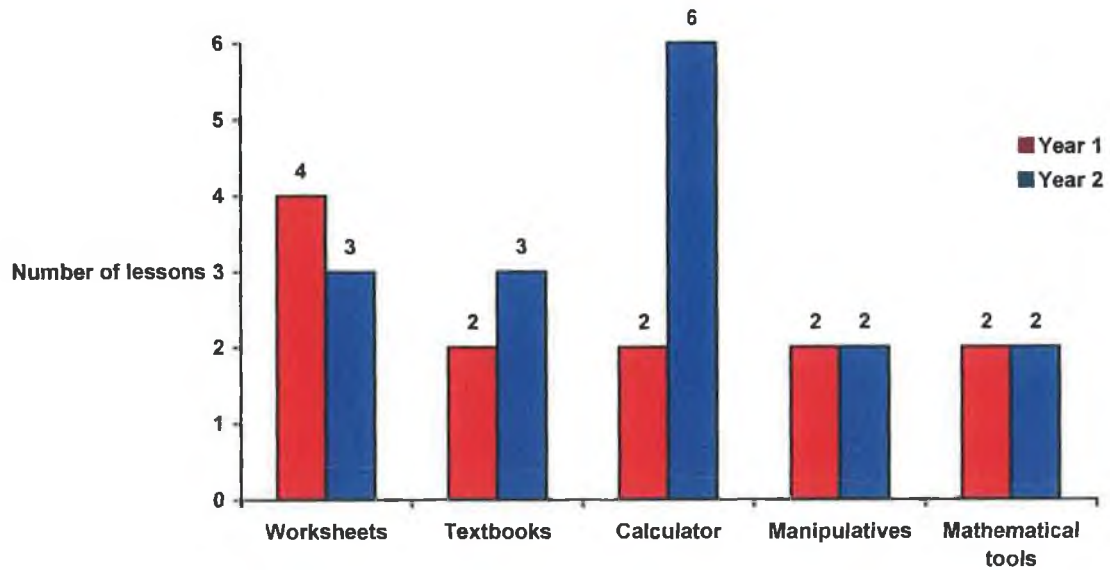


A variety of instructional materials was observed being used in the six classes recorded. The most commonly used tools were the chalk/whiteboard. The number of lessons in which other kinds of materials were used is shown in Figure 6.23. Most are self-explanatory. A mathematics tool such as the clinometer (Figure 6.22) was observed being used by the teacher in Chestnut Hill and was demonstrated by the teacher and used by several students. In a questionnaire the teacher in Chestnut Hill reported that he had never used this in class before but had been introduced to it at a TL21 workshop. Mathematical tools include protractors and rulers. These were observed being used at Chestnut Hill mainly by the students. Worksheets were commonly used by two teachers, but their nature and the level of the questions were not unlike the textbook. The greatest difference between findings from Year 1 and Year 2 is the greater reliance and use of calculators by students in Year 2.



Figure 6.22: Clinometer

Figure 6.23: Use of various instructional materials in the six recorded lessons



From a wide-angle view, the analyses of the six recorded lessons reveal similar findings to Year 1 in the Second Year mathematics lessons across the three classrooms. Mathematics teachers in all the classrooms organized the lesson to contain some public whole-class work and some private individual work. Teachers in all the classrooms talked more than their students, at a ratio of 21:1 teacher to student words in Chestnut Hill to 6:1 at Kenmore. All the lessons used a textbook or worksheet. Teachers in all the classrooms taught mathematics through doing exercises. And lessons in all the classrooms included some review of previous content as well as some attention to new content. A second close-up lens brings a different picture and different findings into focus. In terms of learning goals all three teachers shared considerable similarity in their learning goals; namely wanting their students to know how to execute routine mathematical procedures. However the teacher from Riverside also identified mathematical reasoning as an important thing for her students to learn. Tracing the interactions from when the teachers presented a

problem to the class to when their solutions were arrived at showed some significant differences across the three schools. At Riverside mathematical problems were approached in a way that suggested making mathematical connections, or explaining mathematical relationships. In contrast problems at Kenmore and Chestnut Hill were tackled in a way that emphasised the learning of procedures. Consequently students from Riverside were engaged in higher forms of mathematical reasoning than those at Chestnut Hill and Kenmore. However, no direct comparison is being made here between Riverside and Kenmore, which is a Foundation Level class. All exercises worked on at the three schools tended to be repetitions.

There were detectable differences among the classrooms in the relative emphasis the teachers placed on introducing new content, reviewing content, practising new content and homework. Riverside lessons focused on presenting new content through solving a few exercises, mostly as a whole-class with each problem requiring a considerable length of time. In Chestnut Hill and Kenmore doing exercises played a more central role, with students spending a larger percentage of time working on exercises, either reviewing old homework or starting new homework or practising repetitions. The three teachers emphasised different purposes in their Second Year lessons. Kenmore emphasised reviewing whereas Riverside, and Chestnut Hill to a lesser extent, emphasised introducing new content.

The ways in which teachers and students interact about mathematics are direct indicators of the nature of teaching and learning taking place, and of the nature of the learning opportunities for students. In all three classrooms the teachers did most of

the talking during the lessons. In broad terms the lessons in all cases provided many brief opportunities for students to talk during periods of public interaction, and fewer opportunities for more extensive discussions. Second Year students in all three classes were not observed presenting and examining alternative solution methods for mathematics problems, despite the emphasis placed on this in the research literature. By piecing these findings together what can be learned about the nature of teaching in these three classrooms? And what are the implications of the teaching practices in these classes for students' learning of mathematics? These questions are taken up directly below.

An absolutist view of mathematics seemed to dominate the classrooms at Chestnut Hill and to a lesser extent at Kenmore, with an emphasis on things like the following: mastery of procedures (Henningesen and Stein, 1997), and a transmission style of teaching and a focus on right versus wrong answers (Burton; 1992, Thompson;1992, Ernest; 2004). A fallibilist view of mathematics, to a point, was evident at Riverside, where students were to a certain extent actively engaged in constructing mathematical ideas (Nickson; 1992, Dossey; 1992). Teachers at Chestnut Hill and Kenmore showed students how to do sums (Brousseau et al., 1991). Students taught in this way tend to view mathematics as learning off by heart a vast number of fixed plans and rules without reasons (Skemp, 1978), which does not promote relational understanding (Brousseau et al., 1991; Hatano,1988; Hiebert and Lefevre; 1986). This style of teaching does not develop effective learning or promote problem-solving skills in students. It also limits their understanding and involvement in mathematics to lower-order activities.

The drill and practise style of teaching and learning that was evident at Chestnut Hill and Kenmore did not invite the kind of interaction or communication in the classroom that is conducive to a higher order engagement with mathematics where mathematical thinking is concerned. Consequently students can become “mindless and voiceless” (Belenky; 1986, Povey et al., 2004). Removal in this way from productive learning experience leads to anxiety on behalf of the students (Buerk; 1985, Buxton; 1981, McLeod; 1992, Isaacson; 1990). According to Povey et al. (2004) amongst others, students who are shown “how” to do sums in a mechanical way can become dependent on their teacher and come to focus on whether their answer is right or wrong (Nickson, 1992). A view of mathematics that emphasises drill or mastery of set procedures is not conducive to fruitful classroom interaction and, for students taught this way, much of mathematics remains inaccessible (Nickson, 1992). Schoenfeld (1992) has pointed to another consequence of this type of learning for students; namely that when they are faced with any problem that deviates from well practised procedures they will give up quickly. Many of the above consequences were highlighted recently in the Chief Examiner’s Report on Leaving Certificate Examinations (2005, www.examinations.ie).

The findings show that at Riverside instruction included “what” to do and “why” to do it, which develops relational understanding (Brousseau 1991, Ma, L. 1999). The consequences for students’ learning are manifold, but primarily students are challenged to think (Thompson, 1992) and this moves them away from “fearful silence” (Povey et al., 2004). The students in Riverside were encouraged by their teacher to construct mathematical ideas. The focus was not on right versus wrong answers but on analysis of answers to increase understanding. Errors at Riverside

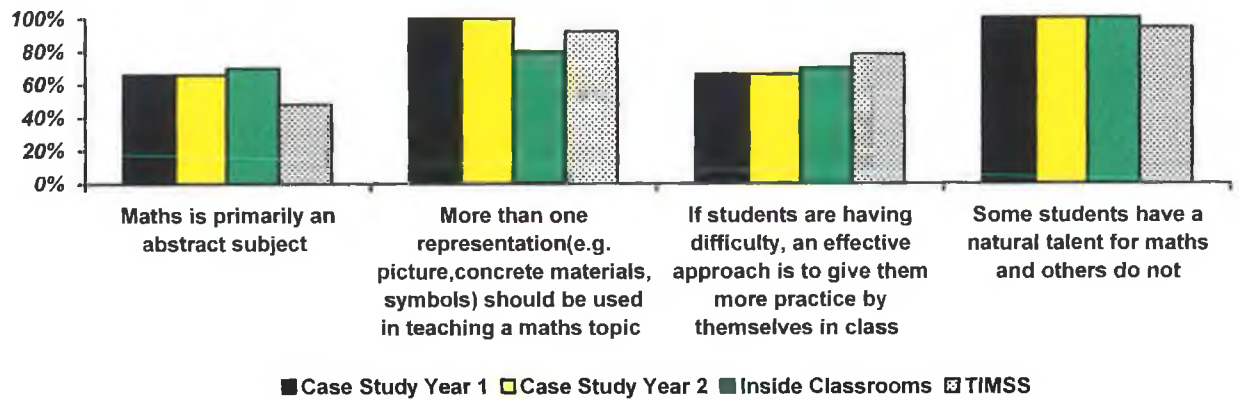
were seen as good for learning and when this happens students come to feel more comfortable with mathematics (Goos et al., 2004).

Section 2: Teachers' and Students' perspectives: Findings from Second Year

Teachers' Perspectives

National data on Irish teachers' perspectives on mathematics was available from the TIMSS 1994-1995 and from a study of ten Junior Certificate mathematics teachers in the *Inside Classrooms* study of 2003. While the present study presents the perspectives of only three teachers (and is alert to the dangers of making generalizations from such a small number), the author found that comparing her findings with this national data served as a starting point. Figures 6.24 and 6.25 present the results of the teachers' responses to the questionnaire items about the nature of mathematics and teaching mathematics. The first and second columns present the responses of the three teachers in Year 1 and Year 2 of this study to each item, while the third column presents the responses of the ten teachers in the *Inside Classroom* study and the fourth column presents the national results of the Irish mathematics teachers who participated in the TIMSS 1994-1995 study (Beaton et al., 1996).

Figure 6.24: Percentage of mathematics teachers who agree or agree strongly with the four statements about the nature of mathematics and mathematics teaching

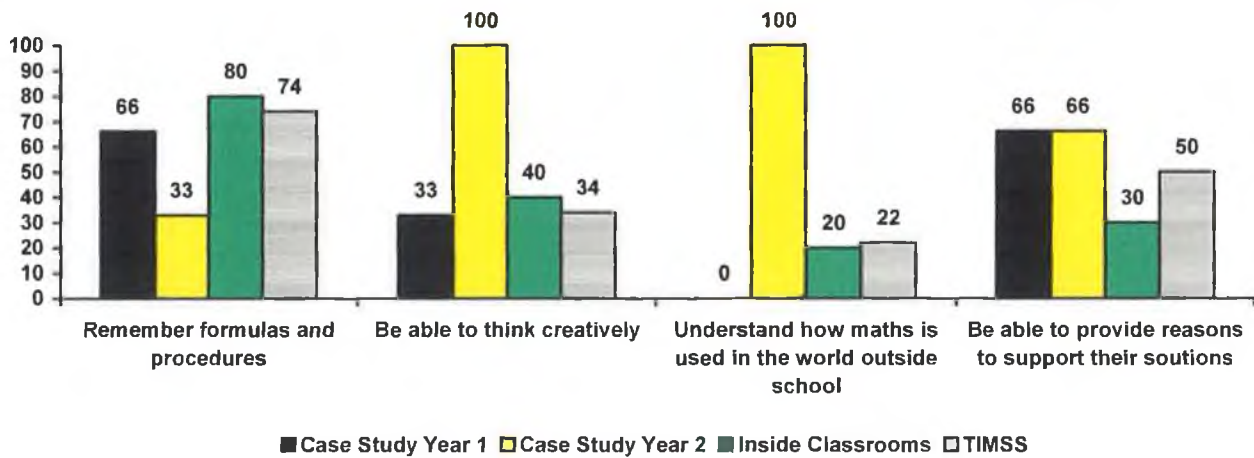


The bar chart above shows that the views of the three teachers in the current study are largely consistent with those of the Irish teachers who participated in the TIMSS 1994-1995 study (Beaton et al., 1996). Two of the three teachers in the current study agreed or agreed strongly that mathematics is primarily an abstract subject. These two teachers also agreed about the benefits of students practising procedures during class, a view that resonates strongly with their formal view of mathematics. All three teachers in the study held strong views about mathematical abilities (some students have a natural talent for mathematics others do not) and shared the same views on their approach to teaching mathematics (more than one representation should be used in teaching a maths topic).

Figure 6.25 presents the attitudes of teachers to the cognitive demands of mathematics. It documents the skills the teachers regarded as “very important” or “somewhat important” for succeeding in school mathematics. As with Figure 6.24 the views of teachers for Year 2 are compared with Year 1 and with those of teachers nationally and *Inside Classrooms* respectively. Significant differences are evident

between the views of the teachers from Year 1 to Year 2 of this study. There are also significant differences between the views of the three teachers in Year 2 of this study and the views of the Irish teachers who participated in the national/international study. Only one teacher in Year 2 of this study felt that being able to remember formulae and procedures was very important. All three teachers in this study felt that it was “somewhat important” to be able to think creatively and to understand how mathematics is used in the real world. Two of the three teachers felt that it was “somewhat important” for students to be able to provide reasons to support their solutions. In relation to these issues above, the findings from the current study (Year 2) proved to be a little more encouraging than Year 1 and than the national/international study. However the claims made by the teachers in this study as to the importance they placed on being able to think creatively and being able to

Figure 6.25: Percentage of teachers who think that particular abilities are important or very important for students' success in mathematics in school



provide reasons to support solutions were in general not borne out by the video evidence. Overall, the teachers in this study, both for Year 1 and Year 2, seem to regard mathematics in a traditional way. It is seen as an abstract subject, and the students are regarded as either talented or not in mathematics. While they may

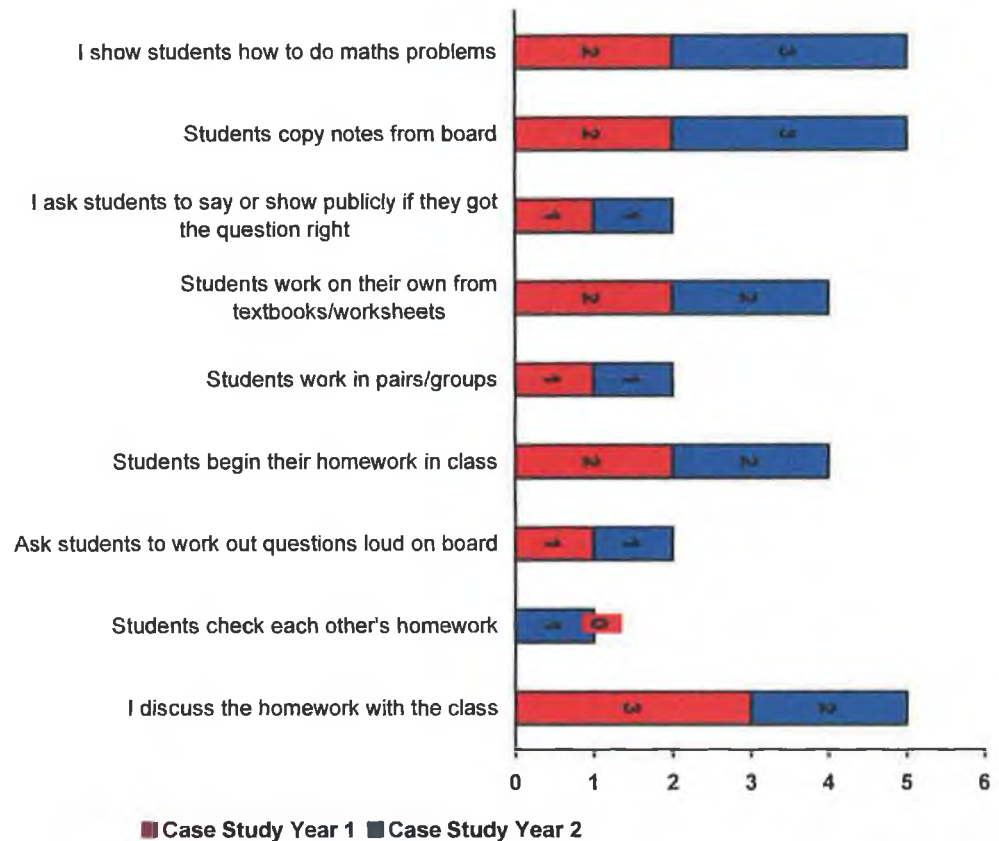
believe that varied teaching methods and practise improve student learning, from the evidence in the videotapes learning mathematics seems to be equated with reproducing learnt procedures rather than being able to think creatively. Whilst their views on what is important for students' success in mathematics leans more towards a fallibilist viewpoint than an absolutist view, with the exception of the teacher at Riverside the other two teachers' philosophy appears to be at variance with their practice. The absolutist view of mathematics has been shown by Dossey (1992) among others to have an impact on a teachers' classroom practice, the focus being on the "how" rather than the "why" of mathematics and this seems to be the case in Year 2 of the current study.

Figure 6.26 presents the teachers' reports from questionnaires on their level of usage of particular methodologies in their Junior Cycle lessons. These findings provide a context for the qualitative accounts that follow. From the findings presented in Figure 6.25 it can be concluded that the teachers in this study are most concerned with showing students how to do mathematics exercises, getting students to copy notes from the board, and checking homework. Teachers' accounts of their work in this respect confirm the findings from the video analysis, where there was strong evidence of demonstration-led teaching. It is clear that the absolutist view of mathematics as (i.e." a static, structured system of facts, procedures and concepts", Henningsen and Stein, 1997, p. 524), held by the teachers impacted on their classroom practice, leading largely to a focus on mechanical procedures. The claims of teachers from Figure 6.26 regarding the nature of the public work in class resonates with what was observed in the lessons. While appraisal through questioning during demonstration/ practice phases is conducted publicly, the teachers

are less likely to ask students to do questions on their own out loud, on the board, or show publicly if they got the question right. The mathematics problems in the classes observed tended to be broken down into small parts with a range of students being asked to offer solutions at different stages. Finally work by students on their own happened frequently, but work in small groups happened infrequently, as was confirmed by the video analysis. One teacher reported that the students corrected each others' homework but this was not the case in the lessons recorded. The data would in general suggest that there is a high level of congruence between teachers' accounts of what happens in their classroom and their actual practice, which is traditional for the most part.

Figure 6. 26: Teachers' reports about whether events happen in "every lesson" or "most lessons"

3=All three teachers in the Case study
 2= Two out of three teachers in the Case Study
 1=One out of Three teachers in the case Study



In the semi-structured interviews with the three teachers they were asked to describe:

- their approach to teaching mathematics
- how their approach may have changed over the years
- how their approach may be different in a Higher level class to a Foundation level class
- the main problems in relation to the teaching of Mathematics at Junior Certificate level

All three teachers described their approach to teaching mathematics in quite traditional terms. The teachers at Riverside and Kenmore both spoke about the need to change their style of teaching as they were aware that it was causing problems for

the students, such as student dependency on the teacher and lack of perseverance with problems. The teacher at Chestnut Hill also identified these as problems associated with mathematics at Junior Certificate, but attributed them to the students rather than as a consequence of his teaching. The three teachers also detailed how their approach has changed over the years. The teacher at Chestnut Hill was not specific in how his approach had changed and explained that what had influenced him was reflection on classroom experience. The teacher at Kenmore said that he had become more reflective and now questioned certain beliefs he had held about mathematics. The teacher at Riverside said that she is no longer in a hurry to cover the course and waits more for students to respond to her questions. The teachers from Kenmore and Riverside explained that what had influenced these changes was experience and the TL21 project. They all pointed out however that the way in which they teach Foundation, Ordinary and Higher Level Junior Certificate would vary. (See Appendix H (i)).

To explore teachers' views of learning they were asked if they could identify any students who had "made progress" over the course of the school year and equally if they could identify students who "had fallen back". The teachers from Chestnut Hill and Kenmore could identify students who had made progress during the year. The teacher from Kenmore explained this in terms of the students had "matured and settled down". The teacher from Chestnut Hill explained it in terms of his own confidence in the classroom as he has now taught Higher Level Junior Certificate mathematics for a few years now. The teacher from Chestnut Hill was the only teacher to identify any students who had fallen back, and felt that this was due to

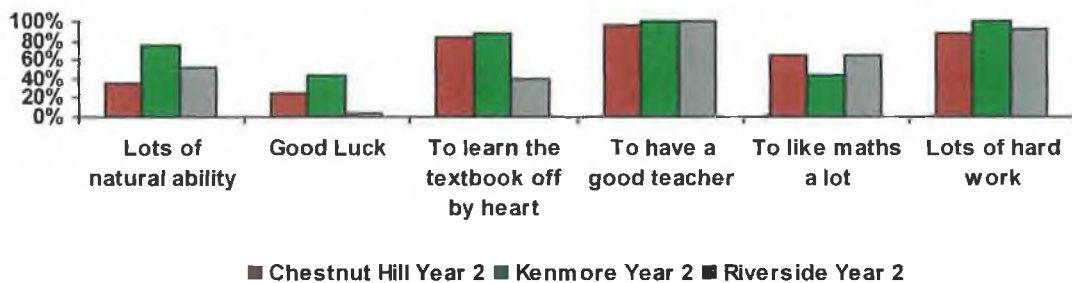
students “stepping up to another level” i.e. promoted to a higher ability class. (See Appendix H (ii)).

When asked if they felt students ask for help if they need it, the responses varied. The teachers from Kenmore and Riverside were aware of this in person-centred terms. The teacher from Chestnut Hill felt that the students do ask for help but from each other first before asking him for help. (See Appendix H (iii)).

Students’ Perspectives

We will now consider the students’ perspectives on learning mathematics and in particular their experience of learning mathematics in class. In questionnaires issued to all students in the three classes, students were asked to indicate their level of agreement or disagreement with six statements about what is required for success in school mathematics. Figure 6.27 shows the level of students’ agreement with each of the statements.

Figure 6.27: Students’ views of requirements for success in school mathematics
(Base: 54 students Year 2)



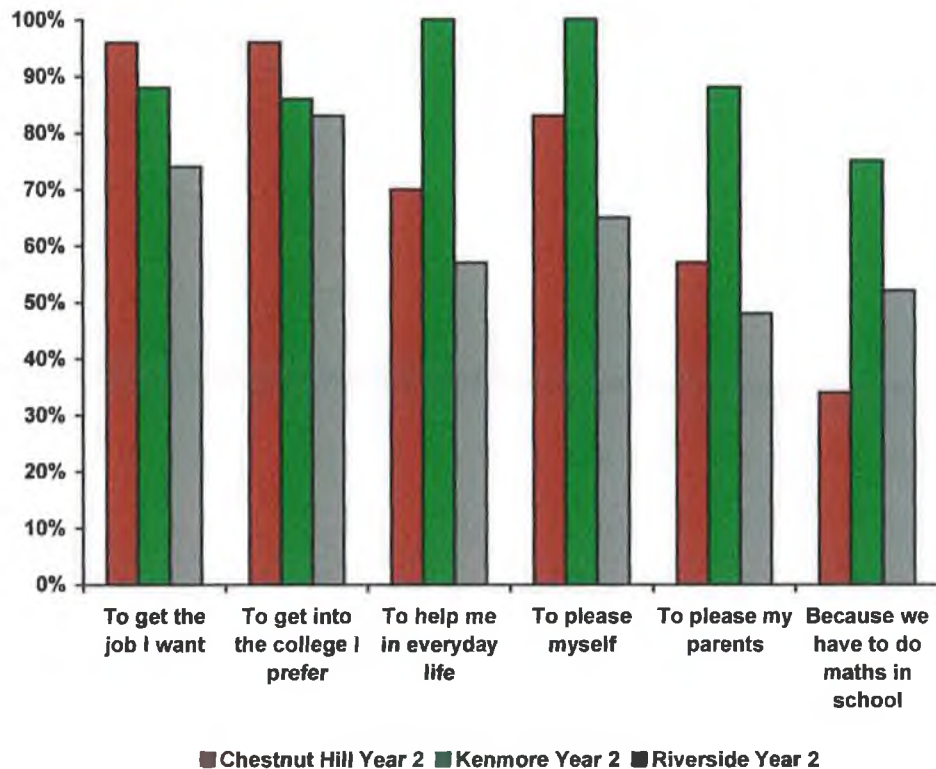
“Having a good teacher” received the highest level of agreement among students in all three classes. Most students also agreed “lots of hard work” was important for

success in mathematics. Few regarded “good luck” as a requirement for success. The similarities in the findings ended here. To like “the subject a lot” was important for the majority of students at Chestnut Hill and Riverside but not perceived as being important by the students at Kenmore. A significant finding was that the majority of students at Chestnut Hill and Kenmore, but not Riverside felt that “to learn the textbook off by heart” was important for success in mathematics. This may attributed to the fact that the teacher at Riverside (from the video evidence) emphasised in her instruction “what” to do and “why” to do it, whereas at Chestnut Hill and Kenmore the emphasis was on the mastery of procedures. Another significant finding was that in the class at Kenmore, who are a Foundation Level class, the majority of students disagreed with the students from Chestnut Hill and Riverside and felt that “lots of natural ability” was important for success in mathematics. Overall, the majority of students felt that success in mathematics is the outcome of good teaching and hard work.

In questionnaires issued to all students in the three classes students were also asked for their views on “why they need to do well in school”. They were asked to indicate the extent of their agreement or disagreement with a number of statements. The results are shown below in Figure 6.28 and would suggest that students showed the same awareness in Second Year as they did in First Year of the importance of mathematics in their lives. The students’ responses were very positive about the value and importance of mathematics. They realise mathematics is important for employment, although noticeably less so in the case of Riverside students. The importance of mathematics for further education is also acknowledged. Its importance in everyday life is acknowledged by students at Chestnut Hill and

especially Kenmore but noticeably less so in Riverside. The short term views on why they need to do well in mathematics in school is more about pleasing themselves than pleasing their parents. Whilst only one third of the students from Chestnut Hill feel that they need to do well in mathematics in school because it is compulsory, fifty per cent of students in Riverside and a majority of students at Kenmore (three quarters) feel that they need to do well in mathematics at school is because it is compulsory.

Figure 6.28: Students' view of why they need to do well in school mathematics (Base 54 students, Year 2)



In the second year of the study the researcher administered a short mathematics test (40 items) to have a measure of students' attainments. The forty item test was based on released items from the TIMSS 1994-1995 study (See in Appendix 1 to chapter 4). The test was administered in the three classes in the second year only of the

study. It provided useful background data on students' skills in mathematics and enabled the researcher to determine what level of competence students had in the different classes in the second year the study was undertaken. The purpose of having a measure of the students' skills at mathematics was to determine the correlation if any between students' attitudes to mathematics and their ability at mathematics. As data was available from the ten case study schools from *Inside Classrooms* (2003), the findings from the three schools in this study are presented in comparison with the ten *Inside Classrooms* schools, giving a total of thirteen schools. A profile of the grades achieved by students in each of the three classes in this study and the ten *Inside Classrooms* schools in the TIMSS related test is presented Figure 6.29 below. (See also in Appendix I Table 2).

It is evident from this that there was a considerable disparity in the attainment level of the students in this test in mathematics at the time the study was undertaken. In this study the highest score (25.6) was in Chestnut Hill (a mixed community school). The students in Riverside (a girls' secondary school) had the next highest score (19.5). The students in Chestnut Hill and Riverside were expected to take the Higher level mathematics paper in the Junior Certificate 2007. The lowest score (11.4) was in Kenmore (a mixed secondary school). These students were expected to take the Foundation level paper in their Junior Certificate 2007.

Figure 6.29: Case-study schools (Year 2 of Study) and 10 Schools from *Inside Classrooms*: Performance in TIMSS-related test*

3 Case Study Schools from above Compared with Case Study Schools from Inside Classrooms Ranking in Brackets	Mean Score on TIMSS-related test	A	B %	C %	D %	E or lower %	Group
Case Study School Inside Classrooms (1)	29.8	16.0	56.0	24.0	4.0	0.0	Top Set
Case Study School Inside Classrooms (2)	28.5	16.0	44.0	28.0	8.0	4.0	Top Band
Case Study School Inside Classrooms (3)	27.2	12.9	38.7	29.0	19.4	0.0	Top Stream
Chestnut Hill (4)	25.6	0.0	43%	38%	14%	5%	Higher Level Group
Case Study School Inside Classrooms (4)	25.6	11.5	26.9	23.1	38.5	0.0	Mixed Group
Case Study School Inside Classrooms (5)	24.4	9.1	27.3	22.7	36.4	4.5	Top Stream
Case Study School Inside Classrooms (6)	21.9	4.2	20.8	29.2	25.0	20.8	Mixed Group
Riverside (7)	19.5	0.0	4.5	23	68	4.5	Higher Level Group
Case Study School Inside Classrooms (8)	18.1	0.0	0.0	33.3	29.7	37.0	Lower Level Group
Case Study School Inside Classrooms (9)	16.5	0.0	0.0	10.0	70.0	20.0	Bottom Band
Case Study School Inside Classrooms (10)	13.3	0.0	0.0	0.0	33.3	66.7	Bottom band
Case Study School Inside Classrooms (11)	13.2	0.0	0.0	0.0	10.0	90.0	Bottom Band
Kenmore (12)	11.4	0.0	0.0	0.0	11	89	Foundation Level

* The national mean for second year students in second-level schools was 25 out of a possible forty. A: 85%-100%, B: 70%-84%, C: 55%-69%, D: 40%-54%, E: 39% or lower.

Questionnaires were used to explore students' own views on their classroom experience of learning mathematics and were given to all students present in the

three Second Year classes. (See Appendix J (i)-(iv) to chapter 6 for construction of scales, the four areas examined and Appendix K: Table 3 to chapter 6 for the findings). As data was available from the ten case study schools from *Inside Classrooms* (2003), the findings from the three schools in the current study are presented in comparison with the schools in *Inside Classrooms*, giving a total of thirteen schools and the comparisons of the findings of this study with *Inside Classrooms*. (See Appendix L: Table 4 (i) and (ii)).

One of the most striking findings from the questionnaires in Year 2 of the current study was that Chestnut Hill ranked fourth, with a mean score of 25.6, Riverside ranked seventh with a mean score of 19.5 and Kenmore ranked last among the thirteen schools on the TIMSS related test with a mean score of 11.4 (out of a possible forty). Yet the attitudes of both Chestnut Hill and Kenmore towards maths were not significantly different. Their “attitude towards learning mathematics”, was negative when compared with the schools from Lyons’ study, would be classified by Lyons et al. as being a “negative” one. Riverside did not fall into this category and would be categorised as largely “positive” about learning mathematics. Chestnut Hill students and Riverside students had both a moderate self image in relation to mathematics, even though Chestnut Hill students achieved much higher scores in the TIMSS test than Riverside. Kenmore students had the lowest self-image. The differences between Chestnut Hill and Riverside students on the one hand, and Kenmore students on the other, were statistically significant.

Two scales were constructed to examine the students' experiences with the class teacher; these were for positive and negative classroom interactions. Within the three schools in this study the students from Kenmore and Riverside (ranked 5th and 6th in comparison with schools from *Inside Classrooms*), reported higher levels of positive interaction with their mathematics teacher than students from Chestnut Hill (ranked 8th in comparison with schools from *Inside Classrooms*). The differences were not significant. Given the differences in the reporting of positive interaction, it is not surprising to find that the students in Kenmore and Riverside also reported receiving less negative attention than the students in Chestnut Hill.

The data indicates that in general all three classes did not have very positive attitudes to mathematics, despite the high score achieved by the students from Chestnut Hill in the TIMSS test. Chestnut Hill students had a slightly higher mathematical self-image compared with Riverside students, although the difference was not significant. The students at Kenmore had the lowest self-image in relation to maths. The experiences of the students at Kenmore with their classroom teacher were fairly positive; however the students from Chestnut Hill reported more negative interactions with their teacher. Given the variability in the character of the classes in the current study the students' response to positive and negative interaction suggests that individual teachers play an important role in creating the classroom climate and students' attitudes towards mathematics.

When discussing their experiences of learning mathematics, all three focus groups found it difficult to discuss mathematics as a subject in itself and could only identify

with the subject as presented by their teachers. Students identified algebra and geometry as areas where they were having difficulty. Students said they found mathematics “harder” than other subjects and even though one might work hard at the subject one can still do badly in examinations. This was felt by some students to diminish their confidence. The students spoke about their difficulties which arose along three lines: (i) the way questions are phrased, (ii) their inability to remember how to do sums (iii) so many rules and formulas to learn. Students in all classes used similar language to describe their difficulty: “don’t get it” and “rules are confusing”. (See Appendix M (i) to (iv)).

When asked about their current experience of learning mathematics the students in the focus groups in Chestnut Hill and Kenmore generally felt that they had made some progress at Mathematics in Second Year and attributed this to their teacher. Their responses were mainly positive about their experiences. In Riverside the students referred to the fact that their teacher never rushes and that they felt comfortable saying that they don’t understand. Some students in Riverside and Chestnut Hill said that they felt more nervous and more pressure about mathematics in Second Year and attributed this to doing Higher Level. (See Appendix M (i)).

Students were also asked what is a good mathematics teacher. Students from all three focus groups spoke about the importance of thorough explanation. Students from all groups also spoke about the teacher “breaking it down”, giving individual help and going over it until “you can do it”. The students from Riverside said it was important for the teacher to explain why you are doing something, not just how to do the sums.

Students at Riverside also felt it was important to have a good relationship with your teacher and that the teacher is organised, fair and doesn't shout. Students from Riverside and Kenmore regarded as a positive attribute of their teacher that he/she did not go too fast, whereas students at Chestnut Hill mentioned as a negative quality that their teacher went very fast. (See Appendix M (ii)).

Another question that arose during the focus group discussions with students centred around asking questions in class. In answering this question the students in Chestnut Hill and Riverside said that they would ask the teacher for help but preferred to ask the student sitting beside them. Asking a question in front of the class seemed to threaten them and invariably the answers given in the interviews called attention to their own emotions. At Riverside the girls said they would ask the teacher a question, but that they sometimes felt "stupid", "worried", "afraid" and "scared" that their answer might be wrong. At Chestnut Hill the students also reported that they would ask the teacher a question but felt "afraid that they might be wrong", "nervous that they might not be right" and "anxious about what others might think they were stupid". As noted previously in Chapter 5, considerable research (Donady and Tobias, 1977; Tobias, S. and Weissbrod, C. 1980, Richardson and Suinn, 1972; Hembree, 1990; Wine, 1971) has documented the consequences of feeling anxious about an inability to do mathematics, a decline in achievement and a disturbance of recall of mathematics already learned. At Kenmore the students said they would ask the teacher rather than a student beside them. One student commented that the reason he wouldn't ask a question was that he couldn't be "bothered" and another student said that they "would sit there and hope for the best" rather than ask a question. (See Appendix M (iii)). Students were also asked to describe any time (not just in Second Year) when they remembered feeling "happy or anxious in

mathematics class". All the students focused on feeling anxious rather than happy in mathematics class and described tests and examinations as the chief cause of this anxiety, which is perhaps understandable. However the language used by students at Chestnut Hill and Kenmore was extremely emotive. (See Appendix M (iv)).

Conclusion

An analysis of the second year of the study reveals some issues in a number of areas:

1. the way the lessons are structured and delivered, and how this can constrain and/or enable particular kinds of learning opportunities;
2. the kind of mathematics taught, the kind of thinking students engage in during the lessons and how deeply students learn the topic;
3. the role of the teacher in the classroom;
4. the nature and effect of students' anxiety and attitude towards mathematics.

In this concluding section, these issues will be considered in turn.

1. The way the lessons are structured and delivered and how that this can constrain and/or enable particular kinds of learning opportunities.

This study has found that the three teachers focused on skills (students being able to do something, perform a procedure, solve a particular type of problem) more than on thinking and understanding (students being able to understand mathematical concepts or ideas). However, compared to Year 1 of the study there was an increased emphasis on thinking and understanding. This was more in evidence in Riverside than in the other two classrooms. The teacher from Riverside constructed the

mathematics lesson in a different way to the teachers at Chestnut Hill and Kenmore. Lessons at Chestnut Hill and Kenmore tended to have two types of phases. In the first phase the teacher demonstrated and/or explained “how” to solve a problem. The explanations were almost purely procedural. The goal in both classrooms was to teach students a method for solving specific examples. In the second phase, or application phase, students practised solving similar examples on their own while the teacher helped individual students who were having difficulties. Students were constrained in the kind of learning opportunity (instrumental understanding) provided for them. Lessons at Riverside followed a different structure. Problem solving came first, where the teacher in collaboration with the students explored /reflected on how to solve a particular problem by considering “what and why”, in an attempt to increase the students’ understanding of the problem. Students at Riverside were enabled to make sense of the problem they were trying to solve and this increased their relational understanding.

The videotaped lessons revealed a distinction between the structure of the lessons and the role of problem solving at Riverside versus Chestnut Hill and Kenmore. These different patterns seemed to follow from the different emphasis (understanding versus skills) set by the teachers, which in turn seemed to define: (i) the kind of mathematics taught (i.e. the kind of thinking students engaged in during the lessons and how deeply students learned the topic); (ii) the role of the teacher in the classroom; (iii) the kind of anxieties felt by the students and the effects of these anxieties on their attitudes to learning mathematics.

2. The kind of mathematics taught, the kind of thinking students engage in during the lessons and how deeply students learn the topic

An analysis of the content of the videotaped lessons revealed that most lessons included some mixture of concepts and application of those concepts to solving problems. Students from Chestnut Hill and Kenmore were exposed to mere statements of the relationships such as $\text{Sin}=\text{O}/\text{H}$ or $\text{D}=\text{S} \times \text{T}$ and lacked opportunities to engage in developing concepts, whereas at Riverside the concept “For what values of x is the graph positive” was derived over the course of the lesson, and students at Riverside were sometimes engaged in concept development. At Chestnut Hill and Kenmore the emphasis was on review and was consistent with a high percentage of repetition type problems as the most common activity for teaching and learning mathematics. By contrast, at Riverside there was a greater emphasis on introducing new material and a low percentage of repetition type exercises (mainly assigned as homework). In all cases the nature of the content defined the parameters of the learning opportunities within which students worked and the kind of thinking students engaged in during the lessons.

3. The role of the teacher in the classroom

This study also found that the teachers adopted different roles of responsibility in the classroom and this had decisive consequences for how they controlled the learning environment. Based on an analysis of the videotapes and data from questionnaires and interviews, the teachers at Chestnut Hill and Kenmore, and to a lesser extent at Riverside, took responsibility for shaping the learning tasks into pieces that were manageable for most students. In the lessons observed it was clear that the teachers felt that mathematics is best learned by mastering material incrementally. This was

especially evident in Chestnut Hill and Kenmore. The students at Chestnut Hill for example first mastered drawing a right-angled triangle and correctly identified the three sides. They were then shown what sine, cosine and tan are equal to; then shown how and when to use these three identities and warned about common errors before practising the more difficult examples/exercises, such as the identities combined with Pythagoras' Theorem. Similarly, at Kenmore students first mastered the triangle with the letters D, S and T to help learn what Distance, Speed and Time equals. Then they were shown how to fill in the various formulae $D=S \times T$, $S=D/T$ and $T=D/S$. At Riverside the teacher discussed with the students where the graph was positive and negative, then discussed the range of values of x along the X-Axis before proceeding to the more difficult problem of finding the range of values of x for which the graph is positive. The difference at Riverside was that while it appears the teacher is teaching this incrementally, mathematical relationships made with other concepts along the way were significant and the teacher with the students cultivated an understanding of what it means for the graph to be positive. If this topic had been taught like at Chestnut Hill or Kenmore, the teacher would have simply stated at the start of the class: "The graph is positive above the X-Axis".

The teachers from Chestnut Hill and Kenmore, whose goal was mainly for students to learn a set of procedures, adopted a role of helping students become proficient in this, and in executing the procedures. They provided sufficient information and repeatedly demonstrated how to complete an example just like those assigned for homework/practise. When they noticed confusion, they quickly assisted students by providing whatever information it took to get the students back on track. The emphasis seemed to be on speed and if a student could not complete a procedure

quickly this was taken as a sign that students did not understand it properly. Teachers from Chestnut Hill and Kenmore assigned students seatwork problems and circulated around the room, helping and monitoring students' progress. When both of these teachers spotted a problem they were inclined to speak to the whole class and tried very hard to reduce confusion by presenting full information about how to solve the problems, for example: "Right would everyone make sure to get that right, that you've got your calculator in the right mode. You've got to make sure that (lifting a student's calculator and looking at it) your calculator (referring to calculator in hand) is in radians (from Chestnut Hill). Whilst this practice is not blameworthy, more could have been done by these teachers to examine students' confusion/mistakes critically in a constructive way.

The teacher from Riverside apparently took responsibility for a different aspect of classroom activity. She chose to start the lesson with a problem and to lead the students through questioning to understand the problem and its solution before they worked on the solution to other similar problems. During the questioning stage the teacher encouraged the students to struggle in the face of difficulty and offered them hints to support their progress. Rarely did the teacher move to take over the solving of the problem.

Another issue is if students are watching the teacher demonstrate a procedure, they need to attend to each step. If their attention wanders, they may well be lost when they try to execute the procedures on their own. This explains more deeply why 60% and 25% of the teacher interactions with students at Chestnut Hill and Kenmore respectively were based on instructions e.g. telling the students what they should be

drawing, where they should be drawing it and how they should be drawing it. To hold students' attention these teachers used a variety of other techniques such as: an increased pace of the activities, non-specific praising of students, humour etc.

In contrast to the lessons at Chestnut Hill and Kenmore, the pace was slow at Riverside and support was given here in a different way to Chestnut Hill and Kenmore. The lessons at Riverside seemed to be generated by a somewhat different belief about the subject. The teacher acted as if mathematics is not just a set of procedures but also a set of relationships between concepts, facts and procedures. These relationships were revealed by developing, studying and talking about the relationships, working increasingly towards solving the problem.

4. The nature and effect of students' anxiety and attitude towards mathematics

The structure and delivery of the lessons, the kind of thinking students engaged in during the lessons, and the role of the teacher in the classroom influenced the students' anxiety and attitudes towards learning mathematics in a number of areas:

i. One might expect students who were high achievers to be the most positive about the subject, but this study did not find this. Instead, negative views of mathematics were evident in Chestnut Hill and Kenmore, although both of these classes are different in academic track. The most negative attitudes were expressed by some Chestnut Hill students, an Honours Level class that scored highly on the TIMSS related test (fourth in rank order of thirteen schools). The other class, Kenmore, is a Foundation Level class that scored poorly on the TIMSS related test (last in rank order of thirteen schools). However these students at Chestnut Hill and Kenmore

described their teacher as “good” in terms of explaining material, as did students at Riverside. However Riverside students were mainly classifiable as positive in their experience of mathematics.

ii. The majority of students across the classes felt that success in mathematics is the outcome of good teaching and hard work. While most students in Chestnut Hill and Kenmore believed that success in mathematics depended on learning mathematics off by heart, analysis of data revealed a considerable difference, and only a minority of students from Riverside expressed this view.

iii. One of the most important issues to emerge from the focus group discussions with students regarded students’ anxiety over tests. A synthesis of the comments made by the students in this study highlights that this anxiety is subject specific to mathematics. Students from Chestnut Hill and Kenmore frankly expressed their anxiety about mathematics tests. Some expressed fear about getting stuck on a problem in the Junior Certificate. Students from Chestnut Hill reported the frustration of studying hard for a test (i.e. learning the methods off by heart) and then doing poorly and the loss of self-confidence that accompanies this. Generally students at Chestnut Hill reported that they did not feel they had made progress in Second Year whereas students at Riverside felt they had made progress. Anxiety was higher among students from Chestnut Hill and Kenmore rather than Riverside. Their mathematics anxiety was not purely restricted to testing, as the students from these two classes also seemed to have a general fear of contact with mathematics, including classes, homework and tests.

In this chapter we have reviewed the findings and issues from Year 2 of this study. In so doing we compared them with that of Year 1. In the next chapter we will continue to review the study's investigations from Year 3 and compare them with Year 1 and Year 2.

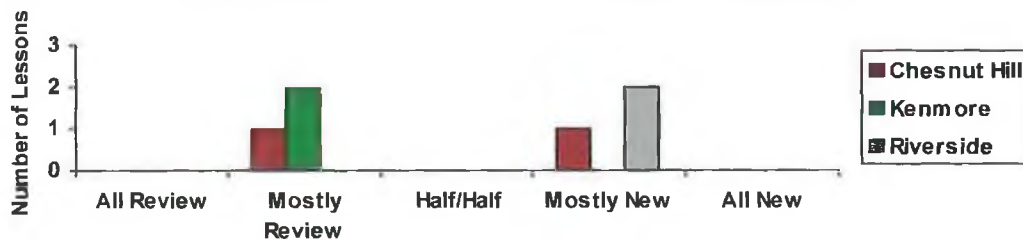
Chapter 7

Findings from Third Year

Section 1: Videotape Analysis: Pedagogical Practices and Classroom Interactions.

Students' opportunity to learn mathematics is shaped in part by the content of the mathematics presented (American National Research Council; 2001). Moving beyond the intended curriculum contained in textbooks and the syllabus, the six videotaped lessons reveal the implemented curricula and the focus here is on what the students actually encountered in the classroom. A general description of the mathematical content of each of the mathematics lessons that were videotaped in the

Figure 7.1: Teachers' description of the content of the videotaped lessons on a continuum from "all review" to "all new".

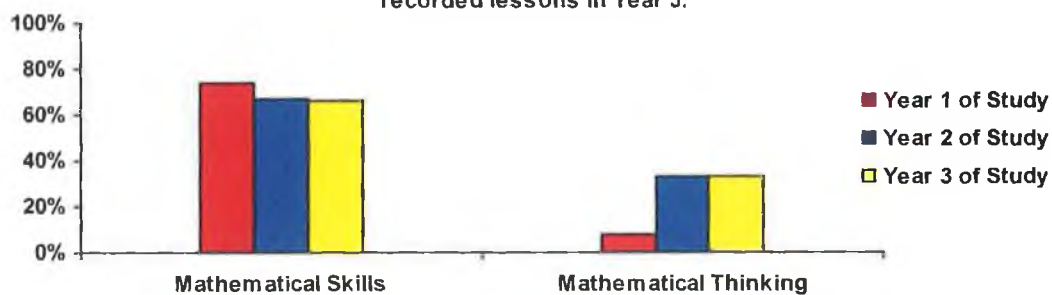


Third Year of the study is contained in Appendix A Table 1. The three teachers were subsequently asked whether the content of the lesson was "all review", "all new", or somewhere in between. Responses to this question are shown in Figure 7.1.

A key variable that shapes the nature of the teaching is the set of learning goals toward which the teacher is working (Hiebert et al., 1997). The three teachers in this study were asked to describe the "main thing" they wanted students to learn in the

lessons that were videotaped. Figure 7.2 presents the comparative emphasis identified by the three teachers as their goal for the six videotaped lessons. The three teachers listed skills (how to solve specific kinds of problems) quite highly in Year 3, but also identified mathematical thinking (emphasis on students' exploration, development and comprehension of mathematical concepts or the discovery of alternative solutions to a problem) as a goal to a much greater extent in Year 3 and Year 2 than in Year 1.

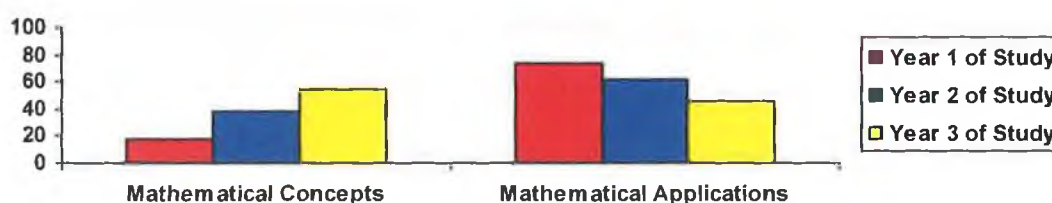
Figure 7.2: Comparative emphasis on mathematical skills and mathematical thinking in in the six recorded lessons in Year 1, Year 2 and the six recorded lessons in Year 3.



Having determined what appeared to be the goals for the six recorded lessons (two from each of the three classrooms hestnut Hill, Kenmore and Riverside), each lesson was analysed by the author to see what type of mathematics was evident in the lessons videotaped. For this purpose the mathematics presented in the six lessons was divided into two categories: mathematical “concepts” and mathematical “applications”. “Concept” in this instance means when a mathematical property, formula or theorem is explicitly referred to e.g. “Today we are using the Theorem of Pythagoras” or “Here we need to use the Sine Rule”. “Application” means the use of a mathematical “concept” without explicitly referring by name to it. Just as in using grammar in a foreign language we would have to say for example “Now we will use the future tense”. However, when we are already well versed in a language we do not

have to refer explicitly to the grammar. Application then is something we do when we are fluent without referring to the concept/grammar we are using. But for learners we need to refer explicitly to the concept we use whether it be the concept of Pythagoras or the future tense, otherwise a presumption is being made in the application that the concept is understood. While the writer is not making a criticism here good practice would be for the teacher to intervene and not presume that all students know which concept is being used or why they are using it. These presumptions occur as a matter of course in customary practice. The average percentage of topics in each lesson that include mathematical “concepts” or “applications” is summarised in Figure 7.3. (The findings from Year 1, Year 2 and Year 3 are compared in Figure 7.3).

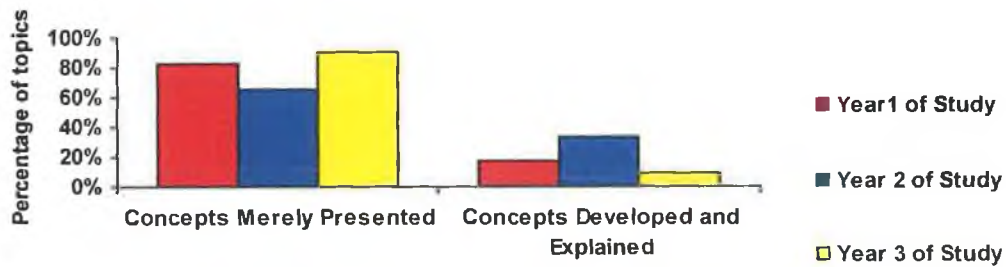
Figure 7.3: Percentage of topics in each lesson that include mathematical concepts or mathematical applications



The analysis of the recorded lessons sought to discover if “concepts” that were introduced in the lessons were (a) merely provided by the teacher or students in order to guide the students to the solution of a problem or (b) explained and developed collaboratively by experimentation, by conjectures, verification, demonstration of results or using logically connected sequence of steps. In the case of (a) the focus is on mathematical information and mechanical skills rather than on the process and development of mathematical reasoning. On the other hand in the case of (b) the focus is on increasing student’s relational understanding (Skemp, 1978; Nickson,

1992; Dossey, 1992). In the six classes observed 91% of topics contained concepts that were merely presented with the presumption that the students were fluent in them, understood them and used them to solve problems as opposed to 9% that were developed and explained (Figure 7.4).

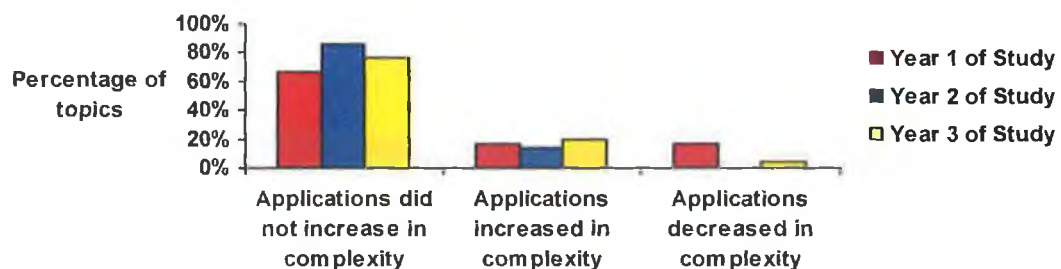
Figure 7.4: Average percentage of topics in maths lessons that contained concepts that were merely presented or developed



Three lessons were videotaped, one each from Chestnut Hill, Kenmore and Riverside in early December 2006 and the three later lessons were videotaped in late January 2007. The teacher from Kenmore was observed concentrating on examination questions in December 2006. The teachers from Chestnut Hill and Kenmore were both observed practising examination questions in January 2007 in preparation for the Pre Junior Certificate examinations (the Junior Certification examination takes place in June of each year). This may explain why 91% of topics contained concepts that were merely presented without further explanation, as their aim was to practise questions similar to what would appear on examination papers. (See Appendix B (i) and (ii) for examples of student practice of applying procedures in preparation for the Pre/ Junior Certificate examination questions). Compared to Year 2 of this study there was only one example from the lessons videotaped where a concept was developed by the teacher and/or the students collaboratively. (See Appendix C).

After examining whether concepts were (a) merely presented or (b) explained and developed, topics that contained “applications” of the concepts to solving problems were considered to see whether the complexity of the applications increased, stayed the same or decreased over the course of the lesson (Figure 7.5). Generally the mathematical problems were the same, or mostly the same, as a preceding problem in the lesson; that is they required repeating procedures that had been demonstrated earlier in the lesson or learned in previous lessons. When topics did increase in complexity this included doing “something other” than repeating learned procedures. “Something other”, might have been developing solution procedures that were new for students or modifying solution procedures they already had learned.

Figure 7.5: Percentage of topics in each lesson that contained procedural applications that increased in complexity, stayed the same or decreased over the course of the lesson



One salient characteristic of mathematics encountered in teaching and learning is its complexity and this is now examined. The complexity of the mathematics presented in the lessons is difficult to define however, because the complexity of a problem for a learner depends on a number of factors, including the experience and capability of the student. One kind of complexity that can be defined independently of the student is procedural complexity - the number of steps it takes to solve a problem using a common solution method. The mathematics problem analysis group for the 1999 TIMSS video study of eighth grade mathematics teaching (which is comparable to

second year Irish post-primary) (Hiebert et al., 2003, p.70-71) developed a scheme for coding the procedural complexity of mathematical problems. Problems were sorted into low, moderate or high complexity, according to the following definitions:

- Low complexity: Solving the problem, using conventional procedures, requiring four or fewer decisions by the students (decisions could be considered small steps). The problem contains no sub-problems, or tasks embedded in larger problems that could themselves be coded as problems;
- Moderate complexity: Solving the problem, using conventional procedures, requiring more than four decisions by the students and can contain one sub-problem;
- High complexity: Solving the problem, using conventional procedures, requiring more than four decisions by the students and containing two or more sub-problems.

Taking the six recordings from the classrooms together, Figure 7.6 shows the average percentage of problems per lesson that were at each complexity level. Figure 7.7 shows the number of problems per lesson at Kenmore, at Riverside and at Chestnut Hill that were at each complexity level for Year 2 and Year 3. Kenmore is a Foundation Level class and by its nature it has predominantly more of low level content than Chestnut Hill and Riverside, which are both Honours level classes.

Figure 7.6: Average percentage of mathematics problems per lesson at each level of procedural complexity. (Taking all of the recorded lessons into account).

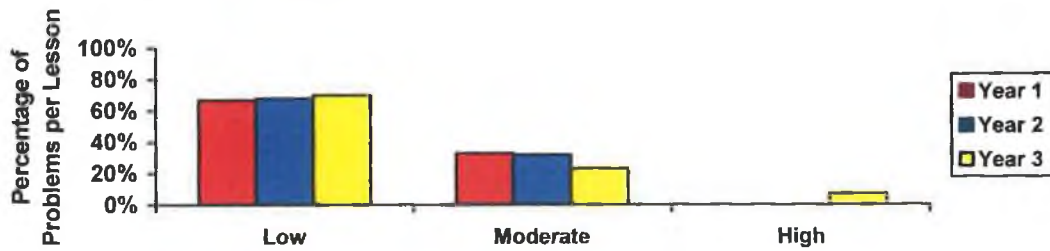
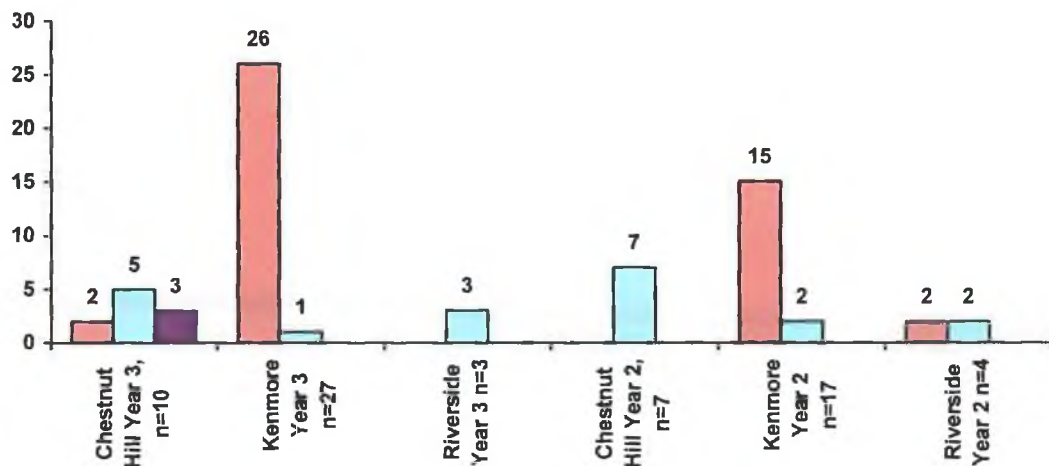


Figure 7.7: Number of mathematics problems per lesson at each level of procedural complexity, by class Year 2 and Year 3.

■ Low Procedural Complexity □ Moderate Procedural Complexity ■ High Procedural Complexity



The six videotapes for Year 3 suggest that the teachers placed a higher emphasis on students knowing “how” to solve specific mathematical problems than on students’ exploration, development and comprehension of mathematical concepts. Students were not encouraged in any of these instances to discuss or present their own different (i.e. different from what the teacher presented) solution methods to problems. This may give students the impression that there is one and only one way to solve a problem and it has to be handed by the teacher to the students.

Mathematical concepts introduced in the recorded lessons were, for the most part, merely presented in order to solve specific mathematical problems. Here once again the emphasis is on the acquisition of isolated mathematical skills rather than on the development of mathematical reasoning. Students were mainly engaged in repeating procedures that had been demonstrated by the teacher earlier in the lesson.

The videotapes from Year 3 Riverside show that lessons were characterised by devoting lesson time to relatively few problems of moderate complexity and spending a relatively long time on each one. The problems related to a single topic and students were invited to explore the concepts with the teacher. This profile is in contrast to Kenmore and to Chestnut Hill (which is also a Higher Level class). However it would be a mistake to assume that students in both of these classes had a similar experience. Chestnut Hill devoted lesson time to a significant number of problems of low, moderate and high procedural complexity. The problems in Chestnut Hill related to a single topic and were found to focus on the procedures needed to solve problems without examining the underlying concepts or mathematical reasoning involved while solving the problem; however the author realises that this reasoning may have occurred in previous lessons other than those that were recorded. Kenmore was the only class in this study in which students were observed beginning the lesson working on sets of problems. The delivery of content was accomplished primarily by working through problems of low procedural complexity related to various topics.

The kind of mathematical problems worked on in the six recorded lessons shapes the learning environment for students but this does not tell us much about the learning opportunities for students. This brings our focus to the second of our three questions: How is the learning environment organised? This section fills in additional information on the mathematical content which we have just focused on, by exploring areas of classroom organisation. The first feature of classroom organisation that we will look at is the way in which teachers and students interact. These interactions are then examined to see what pedagogical purposes they served: In what kind of context were the problems embedded? What were students expected to do when they were working on their own? Answers to these kinds of questions add key elements to how the organisation of the lesson may constrain both the content that is taught and the way that content is taught.

An element of the classroom organisation that can enable or constrain different kinds of learning experiences for students is the way in which the teacher and students interact (Brophy, 1999; p.10-12). The vast majority of class time in the six recorded lessons was spent either in periods of whole-classwork, in which the teacher and students interact publicly, or periods of seatwork which provide activities for students to engage with the subject. Classwork includes those times in which the teacher and students interact publicly and it is intended for all students to participate (at least by listening) in learning a new concept, solving a mathematical problem, demonstrating a procedure etc. Seat work means those episodes when students complete assignments individually or in small groups, and during which the teacher circulates around the room and assists students who need help. The total number of each type of episode observed in the 6 videotapes is shown below (Figure 7.8).

Figure 7.8: Total number of classroom episodes

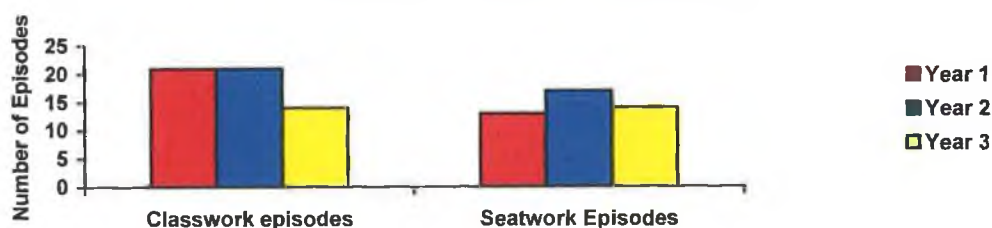
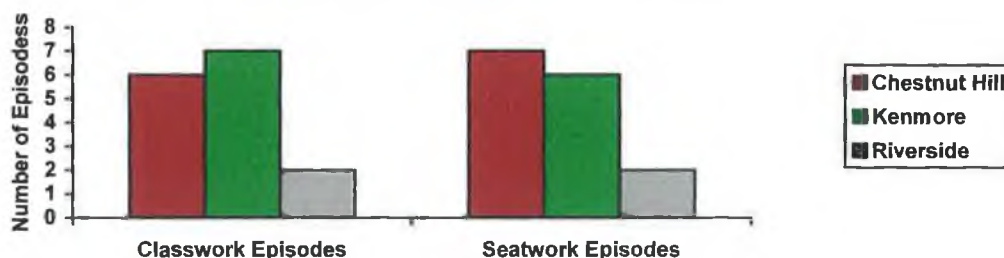


Figure 7.9 displays the total number of classwork and seatwork episodes in Chestnut Hill, Kenmore and Riverside. Third Year mathematics teachers in Chestnut Hill and Kenmore made more shifts in interaction types than the teacher in Riverside. On average they changed interaction types between 6 and seven times each lesson. Each interaction episode lasted, on average, between 5 and 6 minutes. Riverside divided its time between classwork and seatwork episodes somewhat differently than at Chestnut Hill and Kenmore. By shifting between interaction types, the teacher can modify the environment and ask students to work on mathematics in different ways (Hiebert et al.; 2003, p. 56).

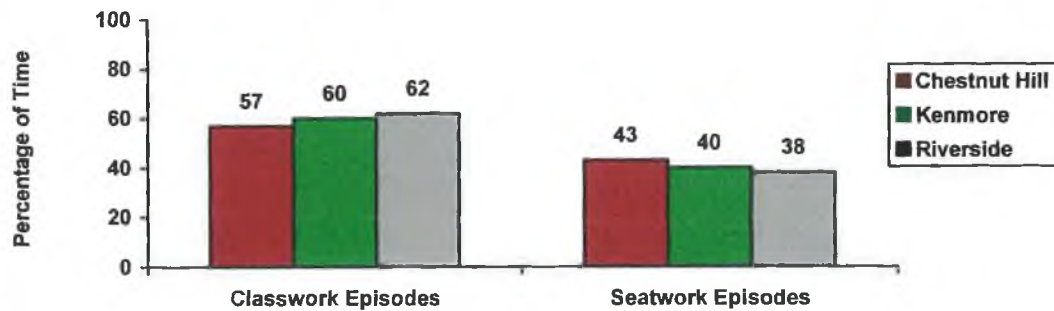
Figure 7.9: Total number of classwork and seatwork episodes in Chestnut Hill, Kenmore and Riverside classrooms, Year 3



However when the percentage of time spent during the lessons is considered Riverside looks quite similar, which is displayed in Figure 7.10 below. As noted earlier, seatwork was defined as the time when students worked individually, or in

small groups. Across the six videotaped lessons only one lesson, at Chestnut Hill involved students working in groups.

Figure 7.10: Percentage of time during the lesson spent in classwork and seatwork episodes in Chestnut Hill, Kenmore and Riverside classrooms



This preliminary analysis of the proportion of time spent in classwork and seatwork episodes represents however only a superficial view of what occurs in a mathematics lesson. These episodes can be used by teachers to accomplish different purposes. With this in mind the recorded lessons were further divided into activity episodes that served pedagogical purpose such as:

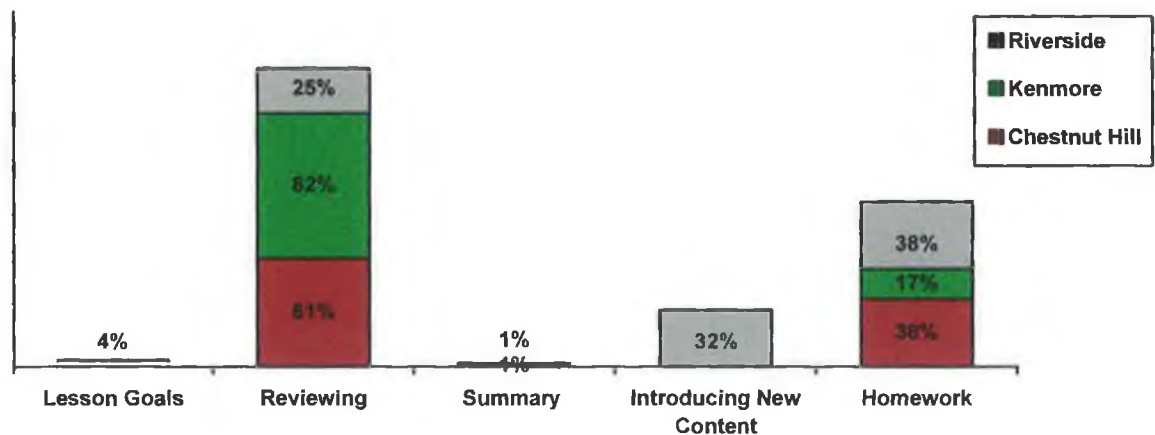
- goal statements (verbal or written statements by the teacher about the specific mathematical topic(s) that would be covered during the lesson);
- introducing new content (this activity focuses on introducing new content that students had not worked on earlier in a previous lesson. Examples of this type of activity include: teacher expositions and demonstrations, teacher and student explorations through solving problems that are different from problems worked on previously and class discussions of new content);

- reviewing (this activity focuses on addressing content introduced in previous lessons. These activities typically involve the practice or application of a topic learned in a prior lesson, or the review of an idea or procedure learned previously);
- practising new content (this means students practising or applying new content introduced in the current lesson);
- lesson summary statements (statements made by the teacher describing the key mathematical points(s) of the lesson);
- homework (this activity focused on setting homework, allowing students to begin homework in class or correcting answers for previously completed homework problems).

Figure 7.11 displays the percentage of lesson time devoted to each activity. In each of the six videotapes – i.e. the three earlier and three latter sessions, the time spent on stating the goals of the lesson was less than one and a half minutes in each lesson at Riverside and no learning intention was set at all at Chestnut Hill and Kenmore. Yet this is one way teachers can help students identify the key mathematical points of a lesson (Brophy, 1999; p.15) and improve the clarity of a lesson. A second kind of aid to help students recognize the key ideas in a lesson is a summary statement which highlights points that have been studied in the lesson. Summary statements were not made in any of the six recorded lessons at Chestnut Hill, Kenmore or Riverside. The decision to correct answers for previously completed homework problems and to include homework within a lesson (allowing students to begin their homework in class as distinct from setting homework at the end) can directly impact on how that

lesson is organized. Figure 7.11 displays the extent to which homework was worked on as part of the lesson time. At Riverside homework was corrected privately but at Chestnut Hill and Kenmore homework was corrected publicly. At Chestnut Hill students were observed correcting and discussing each others' homework albeit for two minutes in one of the recorded lessons. In homework correction episodes, homework corrected in the public domain (Chestnut Hill and Kenmore) was corrected at a brisk pace, with little explanation when a student gave an incorrect answer to a particular step in the solution. The emphasis by both teachers seemed to be on getting the correct response each time in order to get to the final correct answer. (See Appendix D (i) and (ii)). Homework was treated as a central part of the lesson in Chestnut Hill and Riverside, taking up 38% of class time with students observed being allowed to begin working on their homework in class.

Figure 7.11: Percentage of Time Allocated to Activities



By combining the time spent on the two activities of reviewing and practising new content it is possible to compare the time spent on new content with the time spent

reviewing content introduced in a prior lesson. This comparison can be seen in Figure 7.12 (i) and (ii).

Figure 7.12 (i): Percentage of total Third Year mathematics lesson time devoted to various purposes, by class

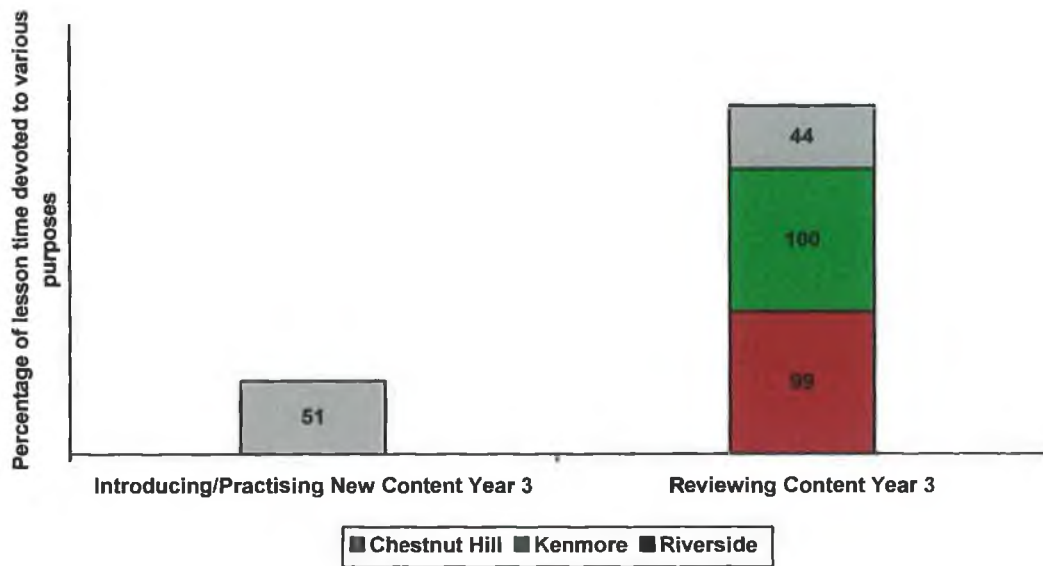
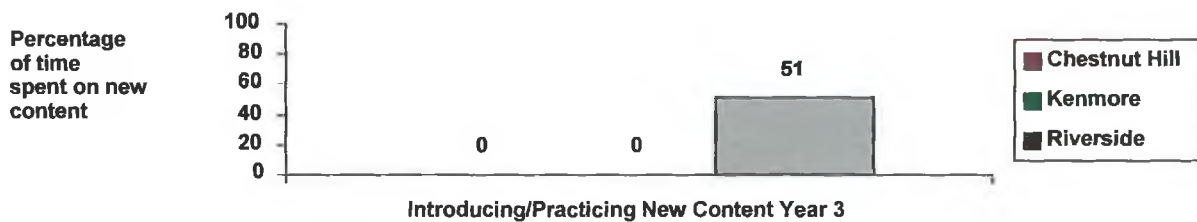


Figure 7.12 (ii): Percentage of Third Year recorded mathematics lesson time devoted to introducing/practising new content



Riverside (a Higher Level class) was the only school in the Third Year of the study observed introducing and practising new content from the syllabus. Although the introduction of new material (mainly dominated by the teacher) seemed to be a primary focus in both lessons recorded at Riverside, starting at the midpoint of each of these, the lessons switched to the more routine procedure of students practising a set of homework problems.

Two lessons were recorded in the case of each of the three teachers in Year 3. All six lessons observed were dominated by whole-classwork where most of the work was done by the teacher. Seatwork episodes could play a critical role in the development of mathematical concepts and give students themselves more responsibility for the process. However the kind of learning experiences provided for the students in all lessons during seatwork episodes was the expectation to practise routine procedure methods. There was no expectation to invent new solutions or create a procedure on their own. The expectation for both teachers and students was not for students to think or reason, but to solve problems by explicitly using one of the solution methods that had previously been presented publicly by the teacher in classwork episodes. In Chestnut Hill (a Higher Level class) and in Kenmore (a Foundation Level class) the teachers placed a greater emphasis on reviewing previously learned material (dominated by the teacher). Throughout the recorded lessons the aim of both teachers (i.e. Chestnut Hill and Kenmore) seemed to be to increase students' confidence and ensure that the students could apply skills in future examinations, such as the Pre and Junior Certificate. There are two ways to interpret this: On the one hand, the lessons at Chestnut Hill and Kenmore, where a lot of time was spent reviewing and practising old content for the Junior Certificate examinations, provide students with

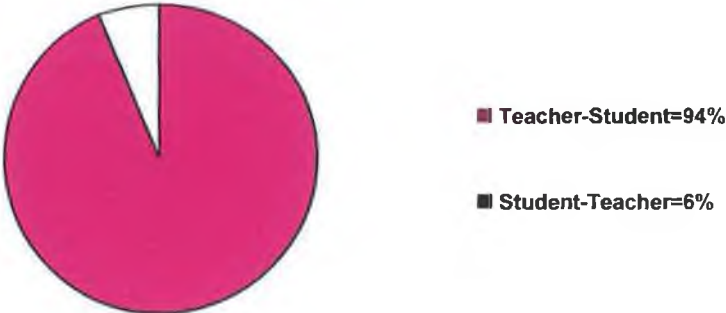
the opportunity to consolidate previously learned content. Yet, on the other hand, the question could be asked did these students at Chestnut Hill and Kenmore have the same opportunity to learn the material as deeply as the students from Riverside, given the larger number of problems Chestnut Hill and Kenmore covered in each class, and the number of shifts in classroom and seatwork episodes in their cases.

Both the content and organisation of lessons are generally planned in advance; they represent conscious decisions on the teacher's part. But not all that happens in classrooms is planned. Some processes become evident as instruction unfolds and sometimes only through detailed analysis which the writer of this thesis carried out on the six recorded lessons. This brings us to the third of the three questions which focuses on the nature of instructional practices taking place in these three classrooms to engage students in working on mathematics. What were the dynamics of interaction in the classroom? How were students engaged by the teacher in the classroom? What was the nature of questioning and processes of instruction? Answers to these questions are taken up in this section and provide additional information about Third Year mathematics teaching in each classroom.

The researcher counted that there were a total of 520 classroom interactions in the recorded lessons, out of which 491 were teacher-initiated (i.e. teacher initiated discourse accounted for 94% of all classroom interactions, Figure 7.13). Relative to their students, as was also found in Year 1 and 2 of this study, the teachers talked much more. There were a total of twenty nine student-initiated interactions. No student-initiated interactions occurred in either of the recorded classes in Chestnut

Hill. Twenty student-initiated interactions occurred in Kenmore and were mainly questions of a low cognitive level – e.g. asking if what they are doing is right. Nine student-initiated interactions occurred at Riverside and were questions in relation to the Junior Certificate. What was clear from the video analysis was that few students asked questions publicly.

Figure 7.13: Percentage of all public interactions, Year 3 (n=520)



Utterances between teachers and students were chiefly made up of questions. However in Chestnut Hill there was a large number of instructions being given to the students by the teacher, compared to Kenmore and Riverside (Figure 7.14 and 7.15 respectively).

Figure 7.14: Total number of teacher-student interactions Year 3, n=491.

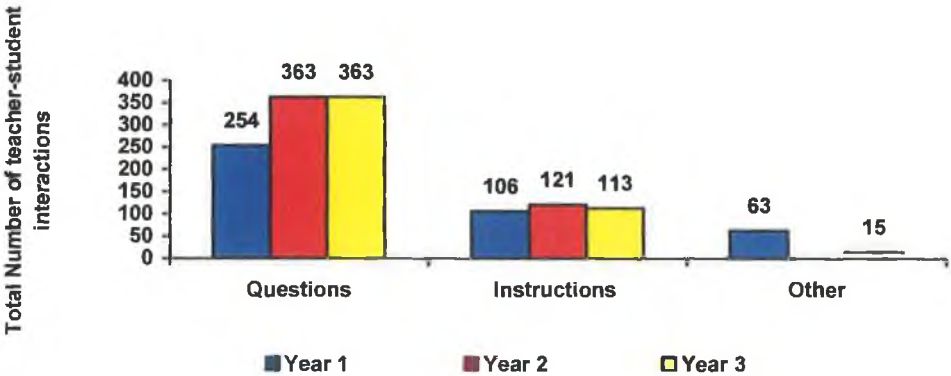
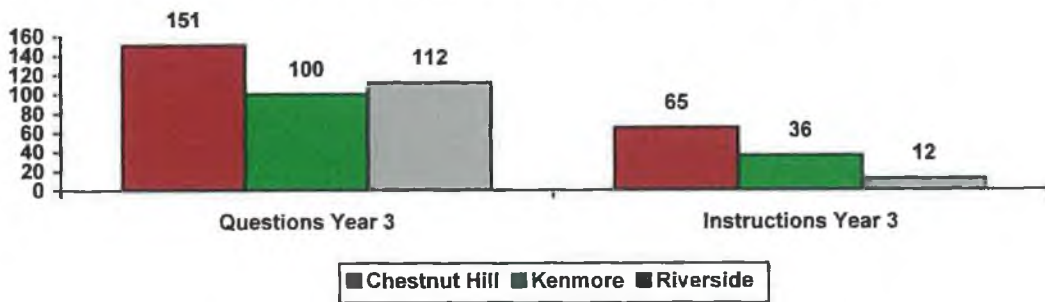
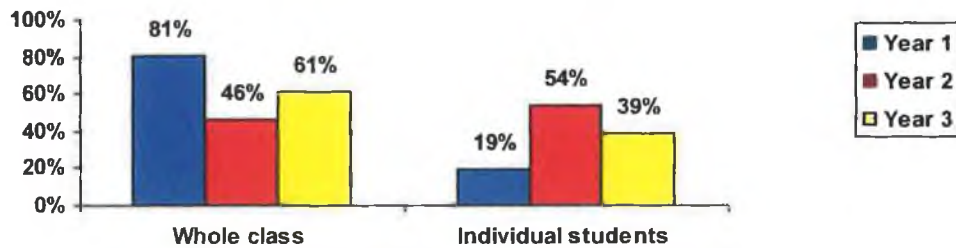


Figure 7.15 Number of Teacher-student interactions, Year 3 (n=491).



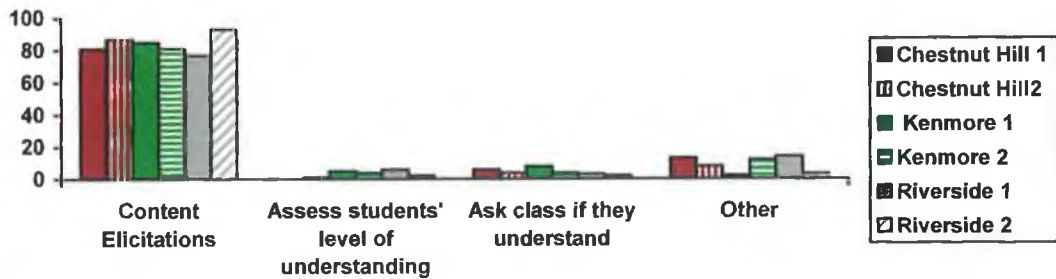
The questions in Year 3 appear to be distributed quite evenly between individual students and the whole class though not as evenly as in Year 2 (Figure 7.16).

Figure 7.16: Distribution of teacher- student interactions



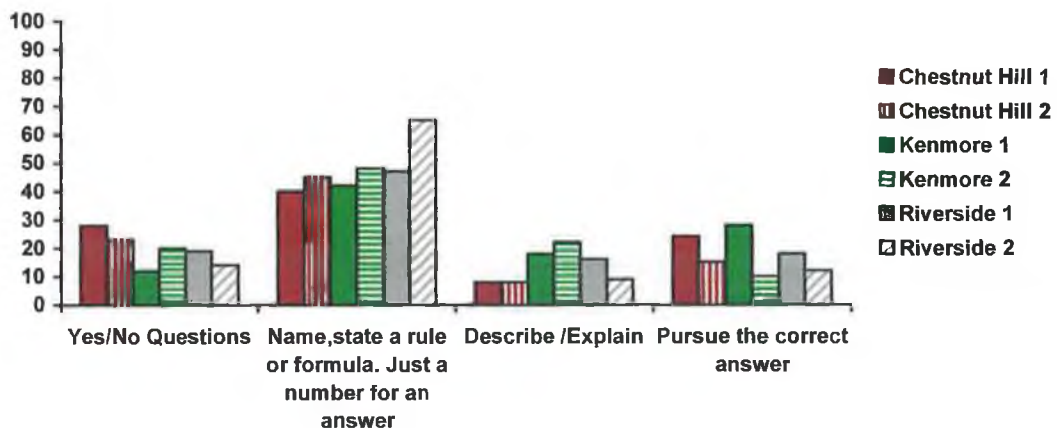
The types of questions overwhelmingly asked by teachers were content elicitation. Such elicitation in this instance may request students to supply the next step in a procedure to a solution, supply a number, identify a shape, define a term or evaluate an answer. Generally the teachers did not ask questions that determined the students' current level of understanding or the students' progress. The three teachers tended to say periodically: "Do you all understand that", directed to the whole class in order to assess students' level of understanding. Figure 7.17 below shows the percentage of the type of questions asked by the three teachers in each of the six classes observed.

Figure 7.17: Percentage of type of questions asked by teachers Year 3, n = 363



As content elicitation generated so much of the mathematical content that was discussed in the lesson, a further analysis of these questions was undertaken. The results are shown in Figure 7.18 below.

Figure 7.18: Percentage of Type of content questions asked n= 297, Year 3



The questions in Chestnut Hill, Kenmore and Riverside tended to be limited to rapidly-paced recitation type questions that elicited relatively short responses. (Skemp, 1978). Questions that required a simple “Yes /No” response from students were also common. In general questioning did not tend to feature sustained and thoughtful development of key ideas. Students were rarely observed being asked to explain a solution method or give a reason why something is true or not true, or indeed given the opportunity to produce long answers, because of the type of

questions asked by the teachers. At Riverside it was also noted that the teacher paused when she asked a question to allow students time to process the question and at least to formulate responses, especially when the question required the student to engage in higher-order thinking; but not to the same extent as in the Second Year of this study (Skemp1978). At Riverside, responses by students were less likely to be evaluated by the teacher as right or wrong. This contrasted with Chestnut Hill and Kenmore; where no specific response was pursued by the teacher. Also in Chestnut Hill and Kenmore the teachers tended to look for specific right answers. The purpose of questioning in Chestnut Hill and Kenmore seemed to be to assess whether the students were able to produce the correct answer. It was observed that at times when a student answered incorrectly the teachers from these classes moved onto another student until they got the right answer, whereas at Riverside the teacher tried to pursue the wrong answer and students were observed abandoning their misconceived answers and beginning to adopt correct ideas and answers. (See Appendix E (i), and (ii)).

Figure 7.19 shows the responses that students made to questions asked by the teacher, taking all three schools together. The students mainly gave correct answers. Incorrect responses accounted for a very small percentage of all student answers. There were significantly few follow-on responses to the teacher's questions. In some instances the students were not given a chance to answer at all, as the teacher on these occasions intervened. Other categories included hesitates/no answer/mumbles.

As so many correct answers were given by students in all recorded lessons, the type of responses that the teachers made to these answers was analysed. Figure 7.20 below shows the teachers' reactions to correct responses. This shows that teachers mainly accepted correct answers without praising the students, repeated the students' correct answer or asked an additional question of the student who provided the correct answer. When praise was given it tended to be not specific.

Figure 7.19: Percentage of student responses to teachers' maths questions (n = 213, Year 3)

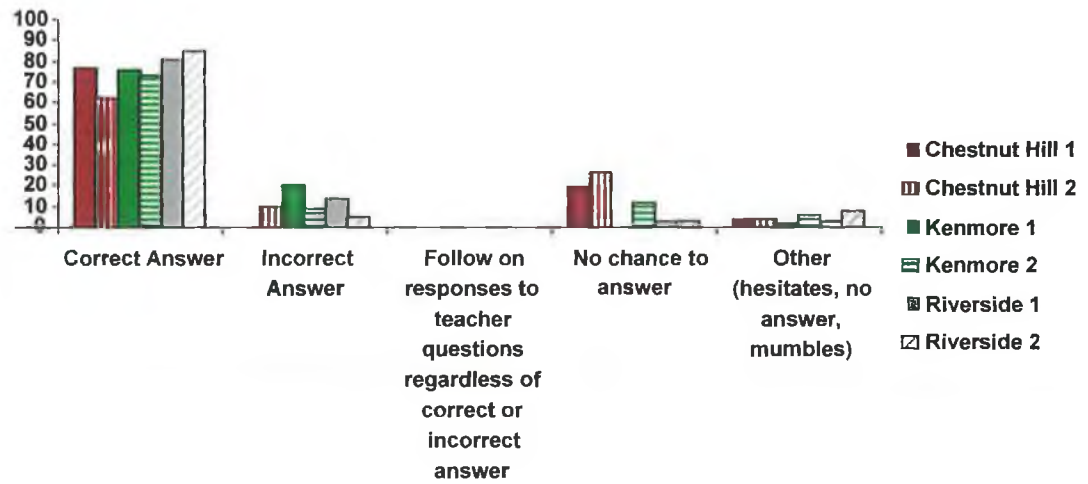
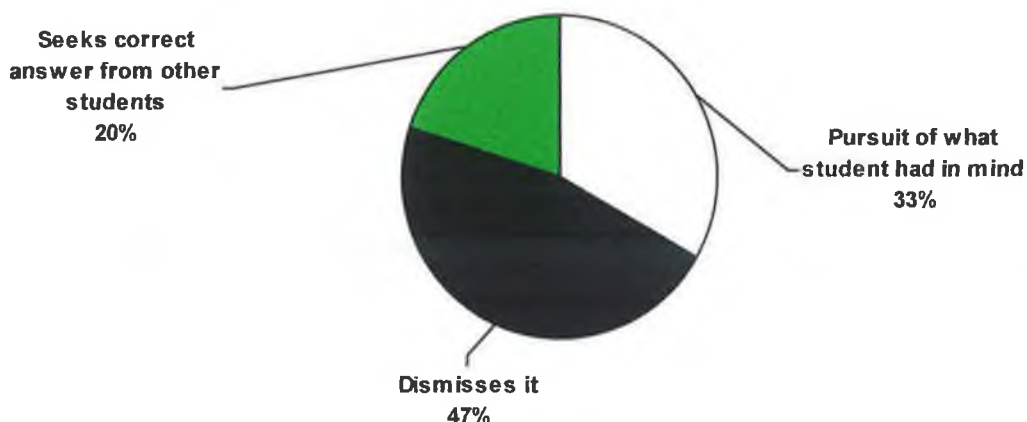


Figure 7.20: Percentage of teacher feedback to correct responses



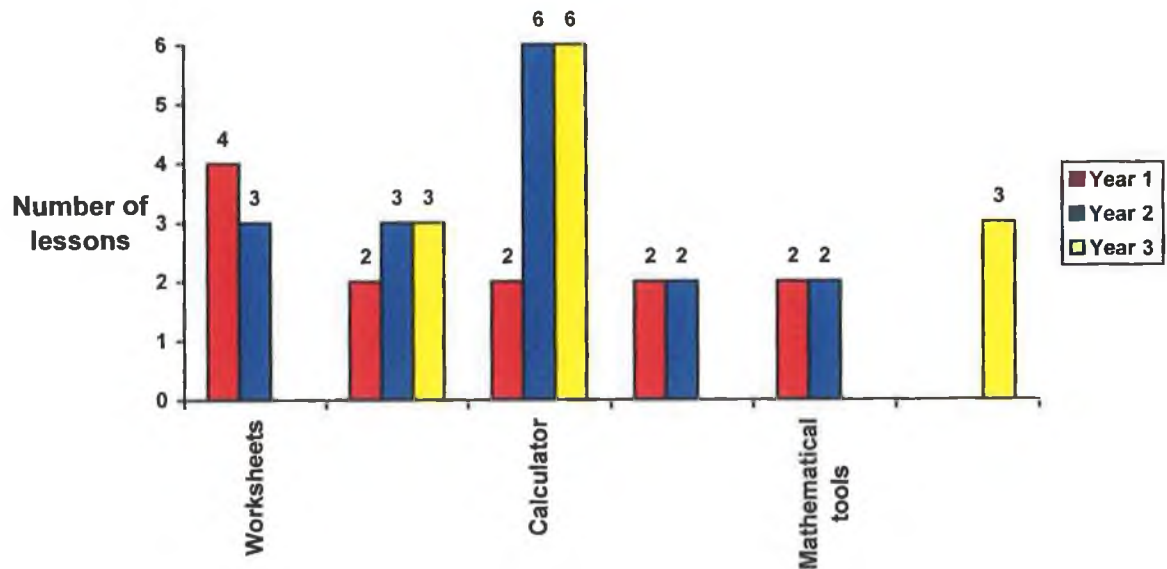
Figure 7.19 shows that there were very few incorrect answers in response to questions from the teacher. The teacher's reaction to incorrect responses is illustrated in figure 7.21. In year 3 at Chestnut Hill and Kenmore, when students gave an incorrect answer teachers pursued the correct answer by asking other students the same question until they got the correct answer. At Riverside, although the teacher pursued the correct answer, she did so by rephrasing the question to the same student and pursued what the student had in mind (accounting largely for the 33% in figure 7.20), by guiding the student who gave an incorrect answer. The other response made by teachers was to dismiss an incorrect answer. (See Appendix F).

Figure 7.21: Teachers' response to incorrect answers (n=20, Year 3)



A variety of instructional materials was observed being used in the six classes recorded. The most commonly used tools were the chalk/whiteboard. The number of lessons in which other kinds of materials were used is shown in Figure 7.22. Most are self-explanatory. Books of Junior Certificate Examination papers were observed being used by two of the teachers. The greatest difference between findings for Year 1 and Year 2 is the greater reliance on and use of calculators by students in Year 2 and 3 and the use of books of examination papers in Year 3.

Figure 7.22: Use of various instructional materials in the six recorded lessons



Review of Key Issues

Having analysed the main points of comparison and contrast in the recorded lessons we will now review from a critical standpoint the three questions posed at the beginning of this chapter: (1) what kind of mathematics content is presented? (2) how is the classroom learning environment was organised? and (3) what kind of instructional practices take place to engage students in working on mathematics?.

(1) What kind of mathematics is presented?

The quality of the content differed across the three classes. Information gathered from the recorded lessons in the Third Year of this study indicates important differences/similarities in what teachers intended students to learn from lessons. The recorded lessons show that the three teachers by and large intended students to memorise mathematical skills rather than to develop mathematical thinking. Doing

routine exercises and learning procedures appears to be the end goal for the teachers, particularly at Chestnut Hill and Kenmore. How well and how quickly the students can do the exercises seemed to be the yardstick by which their success is judged. In Riverside, doing exercises plays a slightly different role. The teacher here identified understanding mathematics and reasoning as an important goal. The teacher used the mathematical exercises to try and develop the students' understanding. Consequently students at Riverside were engaged in higher forms of mathematical reasoning. However no direct comparison is being made between Chestnut Hill and Riverside, which are both Higher Level mathematics classes, and Kenmore, which is a Foundation Level class.

(2) How are lessons structured and delivered?

Looking beyond the content of the lessons, the analyses of the six recorded lessons appear to reveal similar findings in terms of how the lessons were structured.

Mathematics teachers in all the classrooms organized the lesson to contain some public whole-class work and some private individual seat-work. In the public whole-classwork phase the teacher demonstrated and/or explained how to solve an example problem. How the content was delivered and explained was often purely procedural in Chestnut Hill and Kenmore. At Riverside the teacher tried to include students in developing concepts when explaining how to solve a problem. However, the main goal was still the same in the three classrooms: to teach students a method for solving the example problem as distinct from cultivating their mathematical thinking. In the seat-work phase the students practised solving examples on their own while the teacher helped students who were experiencing difficulty. The students in all three classrooms had to follow the teacher as he/she led them through the solution of

example problems during the classwork episodes. Students were never expected to invent their own solution methods or to reflect on those solutions. At Kenmore and Riverside the students were not observed to share their solution methods (if any) that they generated themselves or to work jointly to develop explicit understandings of the underlying concepts in an attempt to increase their understanding. However in Chestnut Hill students were recorded in one lesson involved in group work where they did generate and share the steps needed to rearrange an equation.

There were detectable differences among the classrooms in the relative emphasis the teachers placed on introducing new content, reviewing content, practising new content and homework. Riverside lessons focused on presenting new content through solving a few problems, mostly as a whole-class, with each problem requiring a considerable length of time. In Chestnut Hill and Kenmore solving problems played a more central role, with students spending a larger percentage of time working on problems, either reviewing old homework or starting new homework or practising repetitions. The three teachers emphasised different purposes in their Third Year lessons. Kenmore emphasised reviewing, whereas Riverside, and Chestnut Hill to a lesser extent, emphasised introducing new content.

(3) What kind of mathematical thinking are the students engaged in?

When we examine the kind of work students engaged in during the lesson we find a strong resemblance between the three classrooms. Students in all three classrooms spent their time practising routine procedures. Students' time in class was not spent

inventing new solutions which would require conceptual thinking about mathematics.

The ways in which teachers and students interact about the mathematics are direct indicators of the nature of teaching and learning taking place and of the nature of learning opportunities for students. In all three classrooms the teachers did most of the talking during the lessons. In broad terms the lessons in all cases provided many brief opportunities for students to talk during periods of public interaction and fewer opportunities for more extensive discussions. Third Year students in all three classes were not observed presenting and examining alternative solution methods for mathematics problems, despite the emphasis placed on this in the research literature. By piecing these findings together what can be learned about the nature of teaching in these three classrooms? And what are the implications of the teaching practices in these classes for students' learning of mathematics? These questions are taken up directly below.

An absolutist view of mathematics seemed to dominate the classrooms at Chestnut Hill and Kenmore, and to a lesser extent at Riverside, with an emphasis on things like the following: procedures for students to master (Henningesen and Stein, 1997); the teacher to demonstrate how a manipulation is to be carried out; the teacher to explain how a concept is defined with a strong focus on right versus wrong answers (Burton; 1992, Thompson; 1992, Ernest; 2004). A fallibilist view of mathematics, which emphasises things like finding out why given techniques work, inventing new techniques with a focus on justifying answers, was evident at times in the group work

at Chestnut Hill and during the teacher demonstrations at Riverside where the teacher used questioning to engage in constructing mathematical ideas (Nickson; 1992, Dossey; 1992). A traditional lesson with various combinations of correcting the previous day's work, followed by the teacher presenting one or two problem examples, followed by students working similar problems in their copies or perhaps a few students working at the board, was evident in all three classrooms. The final episode in all three classes involved students working on an assignment for the next day. Teachers at Chestnut Hill and Kenmore showed students how to do sums to a greater extent than the teacher at Riverside and to that extent took some of the learning initiative away from the students. This practice of taking the initiative for the students, for all that may be said in its favour in terms of supporting the students and making them feel comfortable for the examinations, deprives the students of the kind of stimulus and capacity necessary to become better at true understanding and leaves them with a view of mathematics as a subject which requires learning off by heart facts and procedures until they are mastered. This does not promote relational understanding (Brousseau, 1991; Hatano, 1988; Hiebert and Lefevre; 1986). This style of teaching isolates students from effective learning and does not promote problem-solving skills. It also limits their understanding and involvement in mathematics to lower-order activities with no room for ingenuity, no chance for discovery but the deployment of a set of procedures.

The drill and practice style of teaching and learning that was evident at Chestnut Hill and Kenmore did not invite the kind of interaction or communication in the classroom that is conducive to higher order engagement for the students with mathematics, where mathematical thinking and discovery is concerned.

Consequently students can become “mindless and voiceless” (Belenky; 1986, Povey, 2004). Isolated in this way from productive learning can leave students with a collection of techniques that are useful for a student to pass the current Junior Certificate Examination but, as Isaacson (1990) among others points out, this leads to anxiety in mathematics tests on behalf of the students. As noted previously in chapter 6 students who are shown “how” to do sums in a mechanical way can become dependent on their teacher and come to focus on whether their answer is right or wrong (Nickson, 1992, Povey, 1992). Students’ view of mathematics becomes a mastery of techniques defined as knowledge and this becomes an end in itself as students try to absorb what the teacher has transmitted to them. This is not conducive to fruitful classroom interaction and for students taught this way, much of mathematics remains inaccessible (Nickson, 1992). As highlighted in chapter 5 Schoenfeld (1992) has pointed to another consequence of this type of learning for students. It does not enable students to solve a problem that deviates from well-practised procedures and they will tend to give up quickly. Many of the above consequences were highlighted recently in the Chief Examiner’s Report on Leaving Certificate Mathematics Examinations (2005, www.examinations.ie).

The findings show that in the case of Riverside, some of the instruction included not just “what” to do, but also “why” to do it, which develops relational understanding (Brousseau 1991, Ma 1999). The consequences for students’ learning are manifold, but primarily students are challenged to think, develop and apply strategies (Thompson, 1992) and this moves students away from “fearful silence” (Povey 2004). The students in Riverside and Chestnut Hill were encouraged at times by their teacher to understand how mathematical ideas were constructed. However the focus

at Riverside, more so than at Chestnut Hill, was less on right versus wrong answers and more on analysis of answers to increase understanding. To an extent problems were explored in Riverside by the teacher and students together to verify and interpret results. According to Fennema (1999) among others, if students are allowed to do this on their own with the teachers' guidance i.e. through guided discovery (the fallibilist view of mathematics and *Primary School Mathematics Curriculum* (1999 p. 5) which places an emphasis on the child-centred curriculum) then students can become: ..confident in their ability to address real-world problems. If they reason through problem situations, students will develop the habit of making and evaluating conjectures and of constructing, following, and judging valid arguments. (Fennema et al.; 1999).

We will now examine the views of the three teachers and the perspectives of the Third Year students involved in this study.

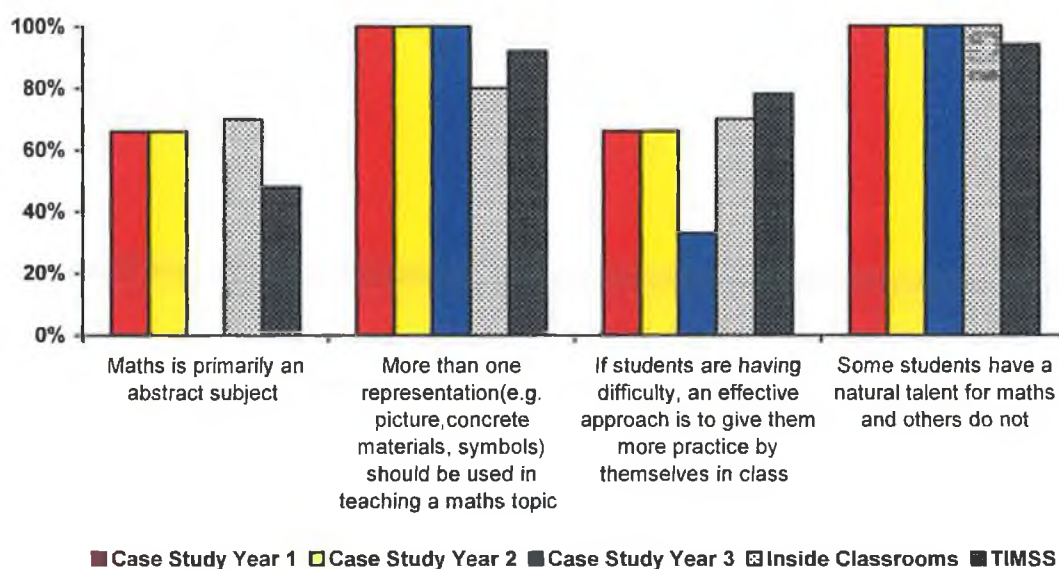
Section 2: Teachers' and Students' perspectives: Findings from Third Year

Teachers' Perspectives

National data on Irish teachers' perspectives on mathematics was available from the TIMSS 1994-1995 and from a study of ten Junior Certificate mathematics teachers in the *Inside Classrooms* study of 2003. While the present study presents the perspectives of only three teachers (and is alert to the dangers of making generalizations from such a small number), the author found that comparing her findings with this national data served as a starting point. Figures 7.23 and 7.24

present the results of the teachers' responses to the questionnaire items about the nature of mathematics and teaching mathematics. The first, second and third columns present the responses of the three teachers in Year 1, Year 2 and Year 3 of this study to each item, while the fourth column presents the responses of the ten teachers in the *Inside Classroom* study and the fifth column presents the national results of the Irish mathematics teachers who participated in the TIMSS 1994-1995 study (Beaton et al., 1996).

Figure 7.23: Percentage of mathematics teachers in different surveys who agree or agree strongly with the four statements about the nature of mathematics and mathematics teaching



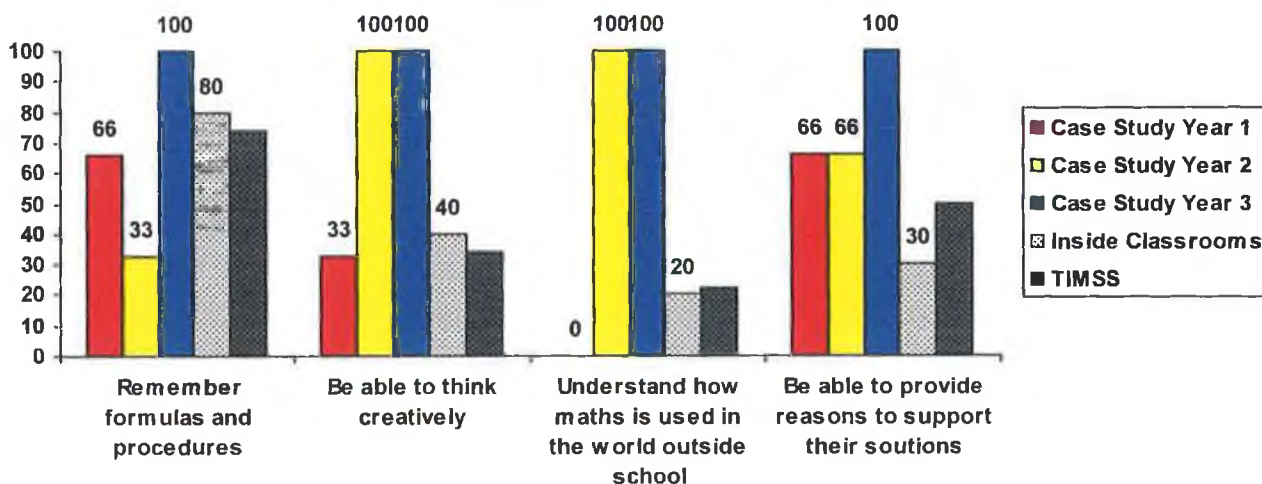
The bar chart above shows that the views of the three teachers in the current study are largely consistent with those of the Irish teachers who participated in the TIMSS 1994-1995 study (Beaton et al., 1996) in the case of two of the statements above (i.e. “Some students have a natural talent for maths and others do not” and “More than one representation e.g. picture, concrete materials, symbols should be used in teaching a maths topic). All three teachers in the study held strong views about the

inherent nature of mathematical abilities i.e. some students have a natural ability for mathematics and some students do not. Some of these views resonate strongly with an absolutist view of mathematics. What is a significant finding in Year 3 of this study is that none of the teachers in the current study agreed that mathematics is primarily an abstract subject. This fact betokens something significant. It suggests a major shift in their view of the nature of mathematics and mathematics teaching. This change in attitude bodes well for their own continuing professional development as mathematics teachers into the future.

Figure 7.24 presents the attitudes of teachers to the cognitive demands of mathematics. It documents the skills the teachers regarded as “very important” or “somewhat important” for succeeding in school mathematics. As with Figure 7.23 the views of teachers for Year 3 are compared with Year 1, Year 2 and with those of teachers nationally (TIMSS 1994-1995) and *Inside Classrooms* respectively. Significant differences are evident between the views of the teachers from Year 2 to Year 3 of this study. There are also significant differences between the views of the three teachers in Year 3 of this study and the views of the Irish teachers who participated in the national/international study. In Year 3 the teachers were giving a new importance to remembering formulae and procedures but gave less importance to this in Year 2. This might be explained by the fact that these students are now preparing for their Junior Certificate and their teachers are aware that students need to remember formulas and procedures for success in the State Examinations. All three teachers in this study felt that it was “very/somewhat important” to be able to think creatively, to understand how mathematics is used in the real world and for students to be able to provide reasons to support their solutions. In relation to these 3

issues, findings from the current study (Year 3) proved to be more encouraging than Year 1, Year 2 and than the national/international study. However the claims made by the teachers in this study as to the importance they placed on being able to think creatively and being able to provide reasons to support solutions were in general not borne out by the video evidence in Year 3.

Figure 7.24: Percentage of teachers who think that particular abilities are important or very important for students' success in mathematics in school



As we have seen in previous chapters the teachers in this study seem to regard mathematics in a traditional way. However there appears to be a change in Year 3 of this study. It is no longer seen by them as an abstract subject (See Figure 7.23). Nevertheless while they may now believe that varied teaching methods improve student learning, from the evidence in the videotapes learning mathematics still seems in Year 3 to be equated more with reproducing learnt procedures than being able to think creatively (impending examinations may have influenced this). In Year 3 whilst their views on what is important for students' success in mathematics leans more towards a fallibilist viewpoint than an absolutist view, their beliefs about how to teach mathematics still appear to be at variance with their practice. The three

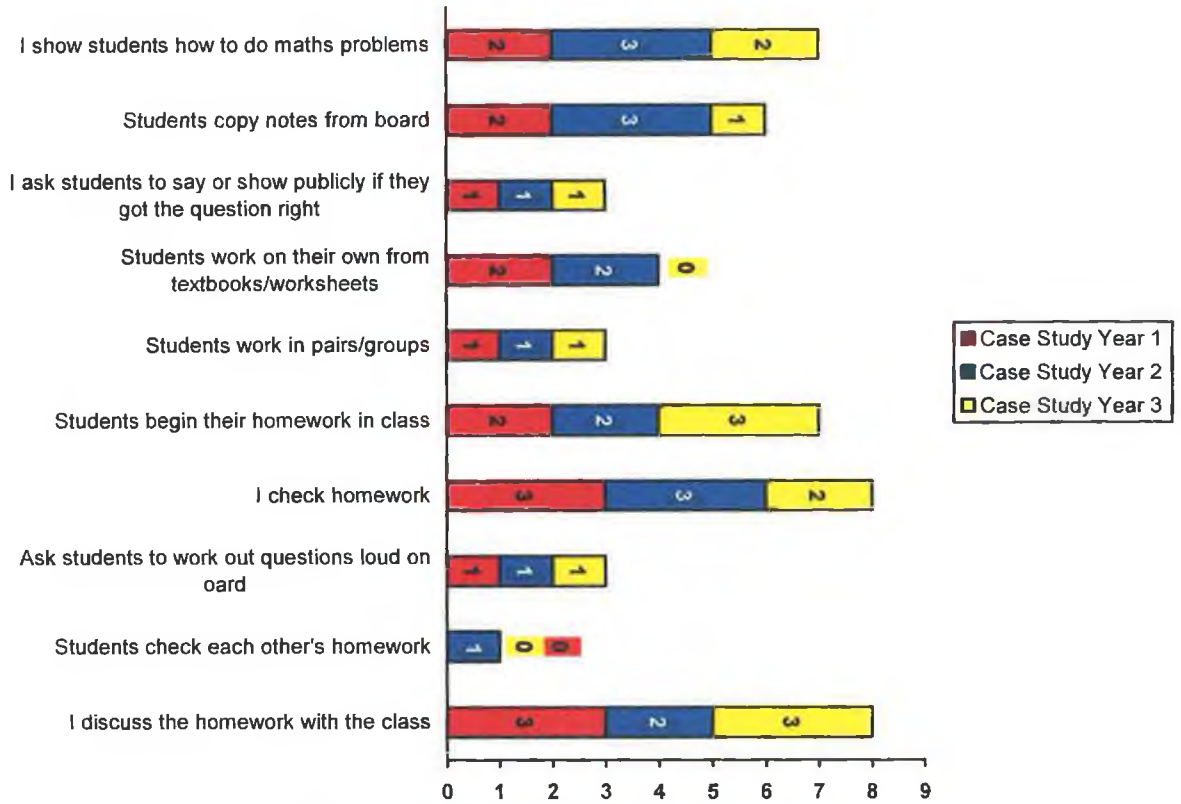
teachers in this study have been involved for the past three years with the researcher in the TL21 continuing professional development project. Clearly the teachers' views of mathematics have changed over the three years, yet their classroom practice still tends to embody features of an absolutist view of mathematics, the focus being on the "know-how" rather than the "why" of mathematics. The questions that have to be posed here are the following: Do the State Examinations which reward well rehearsed collections of techniques play a major role in the decision that teachers make not to move away from traditional instruction? On the evidence presented here the answer would seem to be substantially yes. Secondly are we seeing a paradigm shift in the professional lives arising from their participation in the TL21 developmental project? Again the evidence would suggest yes, but not in a way which leaves old practices definitely behind.

Figure 7.25 presents the teachers' reports from questionnaires on their level of usage of particular methodologies in their Junior Cycle lessons. These findings provide a context for the qualitative accounts that follow. From the findings presented in Figure 7.25 it is clear that discussion of the previous night's homework is a standard feature of every class. This is good if it provides a focus to resolve difficulties, on the other hand if it is just part of a predictable routine it adds to the drill and practice style of teaching and learning mathematics. Teachers' accounts of their work in this respect confirm the findings from the video recordings. Figure 7.25 highlights another finding from the video recordings and that is that the teachers' practice is tending towards memorization of facts and procedures. So despite their changing views this is further evidence that their practice tends to be pulled towards memorization of facts and procedures which is rewarded in the State Examinations.

The claims of teachers from Figure 7.25 regarding the nature of the public work in class resonate with what was observed in the lessons. While appraisal through questioning in demonstration/student practice phases is conducted publicly, the teachers are less likely to ask students to do questions on their own out loud on the board, show publicly if they got the question right or check each other's homework. It is as if the teachers fear students displaying publicly to the whole class their errors, that somehow this is failure. Real advances could be made in the quality of learning mathematics if mistakes could be investigated collectively and publicly. If students could dismantle their defensiveness, not be afraid to say that they were wrong, and instead could say: "let me learn from my mistakes". This would enhance the students' self esteem and confidence with the subject.

Figure 7. 25: Teachers' reports about whether events happen in "every lesson" or "most lessons"

3=All three teachers in the Case study
 2= Two out of three teachers in the Case Study
 1=One out of Three teachers in the case Study



The mathematics problems in the classes observed tended to be broken down routinely into small parts, with a range of students being asked to offer correct solutions at different stages and the teachers in general tried to avoid any incorrect solutions. Finally work by students on their own happened frequently but work in small groups happened only in one recorded lesson at Chestnut Hill. The data would suggest that there is a tension between what the teachers earnestly believe about the nature of mathematics and mathematics teaching (see Figure 7.23 and 7.24) on the one hand, and on the other hand what their accounts of practice actually reveal. (Figure 7.25). While the teachers' beliefs and attitudes may have changed in Year 3

their practices in many respects have not. A very serious question poses itself here: Regardless of what the teachers believe, or what might be good practice, does the proximity of the State Examinations cast an ominous dark cloud on the horizon for these teachers?

In the semi-structured interviews with the three teachers they were asked:

1. if any students had fallen or made progress back since they came into Third Year;
2. if students ask for help if they need it;
3. if it is possible, to get students to a level or do they have to have to have that sort of innate ability;
4. if they could describe their own approach to teaching mathematics in the classroom;
5. if their approach to teaching mathematics had changed over the years, from the time they had started teaching;
6. if their method of teaching varies in the way they would teach Ordinary Level and Higher Level;
7. if they could identify the main problems associated with the teaching of Junior Certificate mathematics.

To explore teachers' views of learning the first question asked if they could identify any students who had "made progress" over the course of the school year and equally if they could identify students who "had fallen back". The teachers from Chestnut Hill and Kenmore could identify students who had made progress during the year. The teacher from Kenmore explained this in terms of two students who had "matured, settled down, gained confidence and were more focused now that they were doing examination papers". The teacher from Chestnut Hill attributed the progress to a traffic light system students had been using for two years to assess their own work and to an increase in "self-esteem". All three teachers identified students

who had fallen back, and felt that this was due to students' lack of work and or ability in the case of Honours students. (See Appendix G (i)).

When asked if they felt students ask for help if they need it, the responses varied. The teachers from Kenmore and Riverside were aware that some students do not, simply because they are "shy". The teacher from Chestnut Hill felt that the students do ask for help but from each other first before asking him for help. (See Appendix G (ii)). When asked if they felt that they could get students to a particular level of mathematics or if students have to have a specific innate ability to succeed in mathematics the responses did not vary. All three teachers felt that they could help students realise their full potential, but to reach Higher Level students had to have an innate ability. (See Appendix G (iii)).

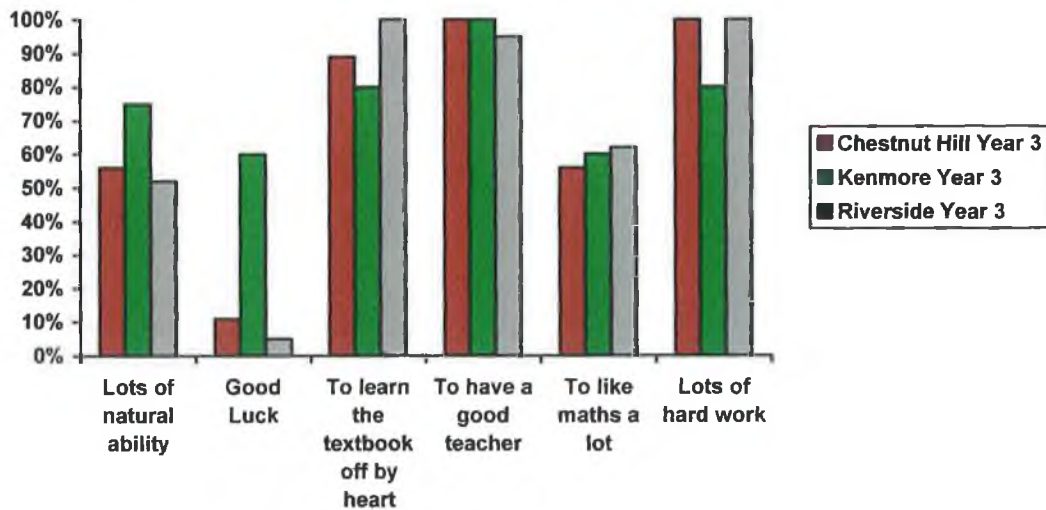
When asked to describe their approach to teaching mathematics the teachers seemed to find it difficult to articulate what their approach is. This is significant because in Year 1 and 2 of this study the teachers were very able to do this and described their approach in quite traditional terms. The teachers, now in Year 3, in response to this question, showed a greater critical awareness of what their teaching was like (i.e. largely traditional) and detailed how it had now changed, such as "teaching for understanding, greater emphasis on geometry, and experimenting with different methodologies such as group-work and wait-time". The three teachers all attributed these changes largely to their experiences with the TL21 project. (See Appendix G (iv) and G (v)). They all pointed out that the way in which they teach Foundation, Ordinary and Higher Level Junior Certificate would vary. (See Appendix G (vi)).

All three teachers could identify problems associated with teaching Junior Certificate mathematics, such as students “afraid of getting the wrong answer, giving up easily, dependency on the teacher, not reflecting on answers”. They tended to attribute these problems to the students, to the Primary school, or to a consequence of mathematics teaching in general as opposed to their own teaching specifically. As the teacher from Chestnut Hill put it “Where the responsibility for that lies, I don’t know”. (See Appendix G (vii)).

Students’ Perspectives

We will now consider the students’ perspectives on learning mathematics, and in particular, their experience of learning mathematics in class. In questionnaires issued to all students in the three Third Year classes, they were asked to indicate their level of agreement or disagreement with six statements about what is required for success in school mathematics. The responses to the questionnaire were then followed by focus groups interviews with students in each of the three schools. Figure 7.26 shows the level of students’ agreement with each of the statements.

Figure 7.26: Students' views of requirements for success in school mathematics
(Base: 51 students Year 3)

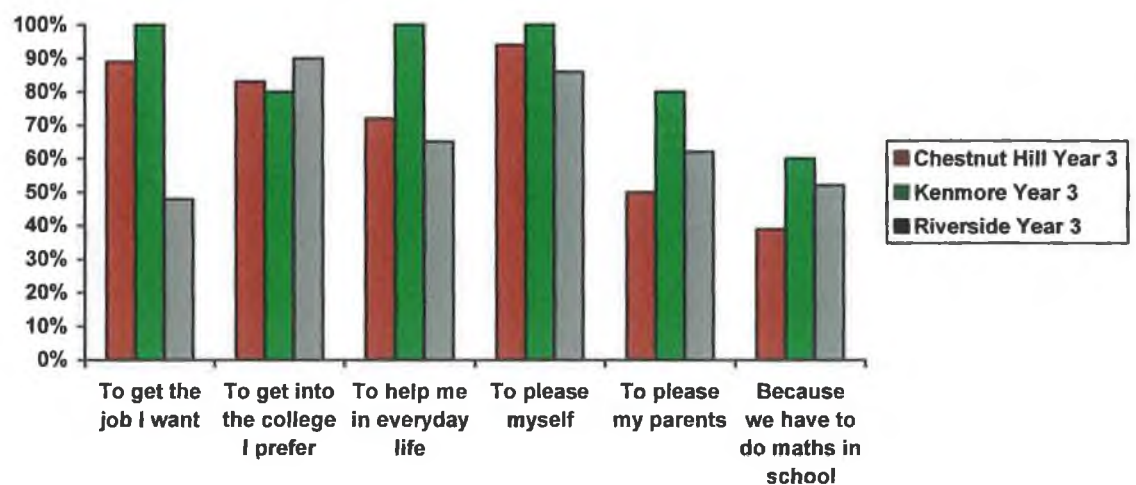


“Having a good teacher”, “lots of hard work” and “to learn the textbook off by heart” received the highest level of agreement among students in all three classes. Most students also agreed “lots of natural ability” and “to like maths a lot” was important. Few regarded “good luck” as a requirement for success in Chestnut Hill or Riverside however a significant number of students in Kenmore (a Foundation Level class) felt this was important. A significant finding for the Year 3 is the emphasis placed by so many students in all three classes on the importance of learning the textbook off by heart. On this account it would appear that students have an absolutist view of mathematics. Overall, the majority of students felt that success in mathematics is the outcome of good teaching, memorization and hard work.

In the questionnaires students were also asked for their views on “why they need to do well in school”. They were asked to indicate the extent of their agreement or disagreement with a number of statements. The results are shown below in Figure

7.27 and would suggest that students showed the same awareness in Third Year as they did in the First and Second Year of this study of the importance of mathematics in their lives. The students' responses were very positive about the value and importance of mathematics. They realise mathematics is important for employment, although noticeably less so in the case of Riverside students, as was the case with these students in Year 2. The importance of mathematics for further education is also acknowledged. Its importance in everyday life is acknowledged by students at Chestnut Hill and Kenmore but noticeably less so in Riverside as was also the case with them in Year 2. The short term views on why they need to do well in mathematics in school are more about pleasing themselves than pleasing their parents. Whilst only one third of the students from Chestnut Hill feel that they need to do well in mathematics in school because it is compulsory, fifty per cent of students in Riverside and sixty per cent of students at Kenmore feel that they need to do well in mathematics at school because it is compulsory.

Figure 7.27: Students' views of why they need to do well in school mathematics (Base 51 students, Year 3)



The questionnaires were used to explore students' own views on their classroom experience of learning mathematics. (See Appendix H 1-4 for construction of scales, the four areas examined and Appendix I: Table 2 for the findings). As data was available from the ten case study schools from *Inside Classrooms* (2002), the findings from the three schools in the current study are presented in comparison with the schools in *Inside Classrooms*, giving a total of thirteen schools and the comparisons of the findings of this study with *Inside Classrooms* can be found in Appendix J: Table 3 (i) and (ii).

One of the most striking findings from the questionnaires in Year 3 of the current study was that statistically the attitudes of both Chestnut Hill and Riverside towards mathematics have disimproved (not significantly) and Kenmore has improved significantly. The "attitude towards learning mathematics", of students in Chestnut Hill largely reflected the schools from Lyons' study and would be classified by Lyons et al. as being a "negative" one. Kenmore and Riverside students' attitudes did not fall into this category and would be categorised in Lyons' terms as occupying an interim place between being positive and negative towards learning mathematics. Another important finding from Year 3 of this study is the improvement of students' self image from Kenmore (Foundation Level) and the converse in Chestnut Hill (Higher Level). Of the three classes Chestnut Hill had the lowest self-image and ranked last in this category when compared to the findings of this study with *Inside Classrooms*.

Two scales were constructed to examine the Year 3 students' experiences with the class teacher; these were for positive and negative classroom interactions. Positive interaction with their teacher was measured along several items such as: the students' perceptions of the frequency of their interaction with their teacher and the level of reward by the teacher for achievement in class. Within the three schools in this study the students from Kenmore and Riverside (ranked 5th and 7th in comparison with schools from *Inside Classrooms*), reported higher levels of positive interaction with their mathematics teacher than students from Chestnut Hill (ranked 12th in comparison with schools from *Inside Classrooms*). The differences between Kenmore and Riverside on the one hand and Chestnut Hill on the other were significant. However, the students in Chestnut Hill, Kenmore and Riverside all reported receiving less negative attention i.e. correction, sanctioning by their teacher for poor work or bad behaviour.

The data indicate that in general all three classes did not have very positive attitudes to mathematics. However Kenmore (the Foundation Level class) were the most positive of the three schools in this study and their attitude towards mathematics had improved significantly from Year 2. Despite the fact that students from Riverside and Chestnut Hill are both Honours Level classes their attitude towards mathematics has become more negative since Year 2. Riverside students had a slightly higher mathematical self-image compared with Kenmore, although the difference was not very significant. The students at Chestnut Hill had the lowest self-image in relation to maths than in either of the other two schools in this study. They also had a lower self-image than any of the schools in the *Inside Classrooms* study. The experiences of the students at Kenmore and Riverside with their classroom teacher were fairly

positive; however the students from Chestnut Hill reported more negative interactions with their teacher in Year 3 than in Year 2. Given the different levels of ability in the three classes in the current study the students' response to positive and negative interaction suggests that individual teachers play an important role in creating the classroom climate and that the level, whether it be Higher or Foundation, may also impact on students' attitudes towards mathematics, now that they are approaching their Junior Certificate.

When discussing their experiences of learning mathematics compared to Year 2 of this study two of the three student focus groups (i.e. Chestnut Hill and Kenmore) still found it difficult to discuss mathematics as a subject in itself. However the students from Riverside were quite able, indeed enthusiastic, to discuss the subject and its various topics. Students from Chestnut Hill and Kenmore could only identify with the subject as presented by their teacher. Students from all three focus groups identified many areas where they were having difficulty. As in Year 2 of this study algebra, coordinate and synthetic geometry and formulas were still causing problems. Students at Chestnut Hill in particular said they found mathematics "harder" than other subjects, and both students at Chestnut Hill and Riverside (both Honours classes) said that even though one might study hard for mathematics exams one "can never predict what is coming up" on the examination Paper. This was felt by some students to increase their anxiety about the subject. The students from these two focus groups spoke about their difficulties in mathematics examinations along four lines: (i) going "blank"; (ii) failing to spot the questions; (iii) the course is so big compared to other subjects it's difficult to study everything; (iv) examination anxiety. As noted previously in Chapter 5, considerable research (Donady and Tobias,

1977; Tobias and Weissbrod, (1980), Richardson and Suinn, 1972; Hembree, 1990; Wine, 1971) has documented the consequences of feeling anxious about an inability to do mathematics, namely a decline in achievement and a disturbance of recall of mathematics already learned. Students in all classes used similar language to describe their difficulty: “don’t get it” and “something clicked”.

The responses in all three focus groups were very positive when asked about their current experience of learning mathematics. In person-to-person interviews the students were keen to say good things about their teachers. This does not reflect fully the data from the questionnaires, which are anonymous. The students in all three focus groups generally felt that they had made some progress at Mathematics in Third Year and attributed this to “hard work”. Students were also asked what is a good mathematics teacher. Students from all three focus groups spoke about the importance of having (i) a good relationship with the teacher, (ii) a patient and helpful teacher, (iii) a teacher who goes at a “comfortable pace”, (iv) a teacher who is “fair” and doesn’t “snap at you” and (v) a teacher who can explain clearly. Students from Chestnut Hill and Kenmore also spoke about the teacher “breaking it down”.

Another question that arose during the focus group discussions with Year 3 students centred around asking questions in class. In answering this question the students in all three focus groups said that they would ask the teacher for help but preferred to ask the student sitting beside them. The reasons they gave for this were that “you’re better off learning from a mate”, “it’s quicker and easier”. Students seemed more relaxed than in previous years about asking questions in front of the class, although a few students

invariably answered this question by calling attention to their own emotions. Some students said they sometimes felt “stupid” or embarrassed”. Students were also asked if they “liked mathematics”. Generally the students in Chestnut Hill said “Yes” but often qualified the “yes” by adding it was not their favourite subject. Students at Kenmore were less positive. It was at Riverside that the reaction of the focus group to this question was most significant. The students did not give a “yes” or “no” answer but began to discuss various topics they liked and disliked and were quite animated in articulating these likes and dislikes.

Students were also asked if they were praised in mathematics class often. The students felt in Chestnut Hill that their teacher would not “go over the top” in terms of praise and that praise might make you “too confident”. At Riverside the students felt that they were praised vocally to a certain extent, but felt that their teacher was nonetheless wholeheartedly supportive. At Kenmore the students said “sometimes, if we’re good”!

Students were also asked when they remembered feeling “happy or anxious in mathematics class”. Generally the students focused on feeling anxious about the “mock exams” or when they couldn’t understand something in class. Students felt happy when they were getting good results in examinations because this gave them confidence or they had managed to succeed in “getting their head around” a particular topic. (See Appendix K).

Conclusion

Teaching is designed to help young people learn. It is what brings the curriculum into contact with the student and through which the D.E.S aims and objectives of the Junior Certificate mathematics syllabus are to be achieved. It is reasonable to assume that the teacher and teaching make a difference in students' learning. The analysis of the six-recorded lessons provides insights into the direct effects teachers and teaching may have on student learning. In addition to this an analysis of the questionnaires and interviews with the three teachers and their students reveals a number of important issues:

1. how a particular teacher can constrain and/or enable particular kinds of learning opportunities;
2. how students engage in mathematical thinking during the lessons;
3. how the Junior Certificate State examinations can influence what happens in the classroom
4. how the nature and effects of students' anxiety can affect their attitude towards mathematics

In this concluding section, these issues will be considered in turn.

1. The role of the teacher in the classroom and how this can constrain and/or enable students' achievement in mathematics as distinct from examination results.

This study has found that the three teachers were extremely well respected by their students, particularly as revealed in the interviews. This showed itself in their competence in preparing students for and achieving high results in the Junior

Certificate Examinations. In preparing their students for the State Examinations the teachers assumed a role in the classroom to one degree or another of following a script, (i.e. a laid down pattern) which focused on delivering “examples”/problems to their students that would enable them to perform certain types of mechanical procedures to solve particular types of problems in the examinations. However in this study achievement in examinations is distinct from achievement in terms of students’ depth of understanding, and is also distinct from their ability to apply their knowledge to unfamiliar situations. To produce this script the teachers constructed lessons that tended to have three phases, each one varying in length and the number of times they were repeated in any one lesson. In the first phase the teachers delivered, then demonstrated and explained clearly “how” to solve an “example”/problem. The explanations were almost purely procedural. The teacher from Riverside however did seem to structure this phase slightly differently from the other two teachers. Although this phase was used by the teacher at Riverside to accomplish “how” to solve problems as in Chestnut Hill and Kenmore, there were differences: (i) the teacher spent a considerable amount of time in doing so and each of the recorded lessons only contained one episode of this phase (a total of three examples of moderate procedural complexity in the two recorded lessons), (ii) the explanation promoted: thinking, understanding, and a sense of the coherence of mathematical ideas. The teacher solved the “example”/problem by encouraging the students through careful, well-judged questioning to collaborate and explore with her not only “how” to solve a particular problem but also considered “what and why” in an attempt to increase the students’ understanding of the problem. The teacher left “wait time” before and during her questions with students; she also tackled incorrect answers positively, using them as an aid to learning and encouraged students to try

things out, even if they might be wrong. In this way during phase one of each lesson, students at Riverside were provided with the opportunity to make sense of the problem they were trying to solve and thus increased their relational understanding. In contrast, at Chestnut Hill and Kenmore, this first phase tended to concentrate on “how” to solve the “example”. The explanations were almost purely procedural and, in comparison to Riverside, questioning moved at a swift pace which did not stimulate the kind of learning opportunities for their students that students at Riverside were afforded. A significant number of “examples”/problems were presented during the recorded lessons and each teacher completed at least six first phases in any one lesson.

In the second phase, or application phase, students practised solving similar exercises to the “examples” just demonstrated on their own while the teacher helped individual students who were having difficulties. The structure of this phase was distinctively different at Riverside to Chestnut Hill and Kenmore. At Riverside the teacher assigned a set of problems for students to work on privately; these problems also constituted their assigned homework. During this phase the teacher helped students who were having difficulty, checked students’ progress and corrected individually and privately the previous night’s completed homework. At Chestnut Hill and Kenmore during this second phase students were given a single problem similar to the “example” to work on for a clearly definable period of time. The teacher then corrected this problem publicly for the whole class. The emphasis in these two classrooms appeared to be on how well and how quickly students could replicate the procedures in the “example”. When this single problem was corrected the teachers reverted back to a phase-one style again to deliver another “example” that was

slightly different from what students had been practising (Chestnut Hill) or, as was the case in Kenmore, a completely different topic was introduced in each new “example”. The third phase included the setting of homework assignments and in all three classes students were observed starting their homework in class. Homework, whether it be correcting a previous nights’ homework, setting new homework or starting homework in class, played a central part in all three classrooms and took up quite a substantial amount of class time.

These different phases seemed to follow from different approaches the teachers followed in the classroom and offered students different opportunities for how they learned mathematics in the classroom: mathematical skills as memorised procedures as opposed to mathematical skills with depth of understanding. These scripts also seemed to define the role of students in the classroom and the kind of mathematical thinking students engaged in during the lessons.

2. The role of the student in the classroom and the kind of mathematical thinking students engaged in during the lessons.

During phase one, in all three classrooms students were shown by their teachers only one way to solve the “example”/ problem and in all classes were given clear explanations by their teachers. Mathematics was taught by the teacher and through the teacher. By doing this, the students’ role in the classroom was to politely listen and acquire the skill or procedure as presented to them. This is not to say that students did not participate in a meaningful way in all recorded lessons, and particularly so in Riverside. Consequently the students did not have, for most of the

time, the opportunity to articulate their own ideas, which would develop their own mathematical thinking. The expectation was for students then to practise in phase two of the lesson these routine procedures and consolidate the standard techniques from the “example”. Such practices are praiseworthy insofar as students need to be able to execute basic techniques accurately and speedily. However the students’ role in their own learning, in both phase one and phase two, did not involve any high expectations on behalf of the teacher for the students. Students were not deeply engaged in the hallmarks of the special forms of reasoning involved in solving mathematical problems that distinguishes mathematics from other school subjects (National Research Council 2001, p. 73). These special forms of reasoning include deduction, conjecture, generalization or the use of counter-examples (National Research Council 2001, p. 193). This is not to say that these types of mathematical reasoning were not evident in the recorded lessons. They were, but it was the teacher who mainly experienced them in the demonstration of the “example”/problem. The teachers were observed in the recorded lessons inventing the solution, developing the problem through a chain of logical steps to its solution and sharing this with the students. The teachers mainly dominated the class work for too long without allowing students to engage with mathematics and find out the solutions to difficulties for themselves.

The role the teacher assumed in phase one and two (helping students with difficulty as soon as they arose) did not provide students with many opportunities or the environment to engage in higher forms of mathematical thinking of applying their knowledge and skills in unfamiliar situations other than for the most part the reproduction of procedures, even in the case of Riverside. The teachers generally did

not expect or challenge students to think for themselves and to develop their own solutions, which would create a positive atmosphere in building students' confidence. However, in Riverside, the teacher used "wait time" to allow students the time to think when they were asked a question and this did create a positive atmosphere in the learning environment and gave students the confidence to answer questions in class. Students were not observed being encouraged to discuss, debate or deliver their own solution methods publicly, nor were they asked to invent, examine, or present alternative solutions. Students were not given the responsibility on their own to recognise the power of mathematical thinking and make connections for themselves with mathematical ideas, facts or procedures. It is not surprising then that students in the interviews said that to do well in school mathematics one primarily needed to: work hard, learn the notes and textbook off by heart and have a good teacher. These qualities are hardly to be criticised however, it is reasonable to say that, although the teachers were effective in explaining mathematics to the students, for the students mathematics has to a large degree become an extended series of procedures. The result of this may be that the students lack the ability to reason and the confidence to discover solutions for themselves in unfamiliar contexts and questions.

3. The influence of the Junior Certificate State examination in the classroom

Teachers' attention to covering the syllabus, practising examination questions and paying attention to the details of the different types of problems that could appear on examination papers are major factors in students' success in the Junior Certificate Mathematics Examinations. This is sometimes however at the expense of developing students' ability to apply their mathematical knowledge. We have seen how two of

the three teachers in this study had already started in January 2007 teaching students how to approach and solve examination questions. In Chestnut Hill the teacher referred to the Pre/Junior Certificate on numerous occasions: “For that you would get seven marks out of ten which isn’t too bad”, “When you see that question in the exam you should be automatically thinking...” Again in Kenmore similar language was used: “So remember that’s the way the question will appear in the Pre exam as well as the Junior Certificate”. Teachers are not blameworthy for doing this. This illustrates the point that the proximity of the examinations pulled the teachers’ practices back to traditional patterns despite the change that occurred in the teachers’ beliefs (Figure 7.23). At Riverside the teacher made no reference to the Junior Certificate and was still covering the course, as already noted in point 1 above it was at Riverside where students were given an opportunity to engage in depth of understanding. All lessons both the two earlier and the two later recording were taken at the same times of the year.

At all levels in Junior Certificate Mathematics Examinations questions typically on the papers have three parts with:

- an easy first part which tests recall or very simple manipulation
- a second part of moderate difficulty which tests the choice and execution of routine procedures or constructions
- a final part of greater difficulty which tests application

(An Roinn Oideachais agus Eolaiochta, Junior Certificate Mathematics Syllabus, p. 35).

The design of the questions largely supports “drill and practice” and it is acknowledged that even students taught this way can achieve very good results in

their Junior Certificate. Some of the instructional practices observed in the recorded lessons which presented mathematics as a collection of arbitrary rules and procedures, allied to a narrow range of learning activities which did not engage students in real mathematical thinking, are a consequence of a narrow focus on meeting examination requirements by “teaching to the test”. Although students are able to do very well in the examinations they may not be able to apply their knowledge independently to new contexts. The Chief Examiner’s Report for Leaving Certificate (2005) found this to be the case. When questions required students to display effective understanding and problem-solving they failed to do so but showed great competency in mathematical techniques.

4. The nature and effects of students’ anxiety and attitude towards mathematics.

Three key factors had an impact on students’ anxiety and attitude towards mathematics: (i) the role of the teacher in the classroom and how a particular teacher can constrain and/or enable particular kinds of learning opportunities; (ii) the role of the student in the classroom and the kind of thinking students engage in during the lessons; and (iii) the influence of the Junior Certificate State examination in the classroom. This study found that there was not a clear positive relationship between a stronger liking for mathematics and higher achievement. Indeed students at Chestnut Hill (Honours Level) and at Riverside (Honours Level) although less so, did not feel positive about mathematics (See Appendix J Table 3 (i)). Conversely students at Kenmore (Foundation Level) were the most positive of the three groups. This may be attributed to the fact that from the focus group discussions students from Chestnut Hill felt that mathematics takes up quite a lot of time at home, compared to other subjects. A synthesis of comments made by students from both Chestnut Hill and Riverside during the interviews highlighted that they felt mathematics requires a lot

of hard work (i.e. learning methods off by heart) and was probably the hardest subject in school with so much to remember.

An important issue to emerge from the focus group discussions with students was their anxiety over mathematics tests. A combination of the comments made by the students in this study show that this anxiety was quite high in the two Honours classes and was specific to mathematics (See Appendix K (viii)). Students from Chestnut Hill and Riverside were very open in expressing how they felt about mathematics tests. Some expressed fears that they would “go blank, get confused, forget, or freak out” in the Junior Certificate examination. Students in both these classes said that their anxiety stemmed from the fact that “you don’t know what’s coming up” (i.e. questions which test application and understanding). This sharp focus on the examination, with textbooks written to match examination questions, and practice papers produced as effective tools for revision and preparation, does not promote mathematical enquiry. Unless teachers promote genuine mathematical enquiry and understanding in classrooms the subject becomes reduced to a collection of techniques for passing examinations.

In the next and final chapter we will consider a range of recommendations arising from significant issues and findings in this thesis.

Chapter 8

Looking Ahead

1. Cultivating richer learning environments for students

The first part of this section reviews the significant features of mathematics teaching and learning environments that have emerged in this study and then analyses why we need to promote qualitative improvements in these environments. This review builds on the findings in the thesis and in the research literature. The next part of this section explores at close range some distinctive features of meaningful teaching and learning environments that seek to promote learning for understanding among students, as distinct from the learning of memorised procedures.

As we have seen in chapter 1, as far back as the 1960s leading practitioners in Ireland were voicing complaints that rote learning and drill were over-emphasised (Oldham, 1980 (a), p. 54). In chapter 2 we saw how in the seventies teachers were again urged to be innovative (*Rules and Programmes*, 1974/75 p. 64) and more recently in the 1990's the TIMSS international study (Beaton et al., 1996), made criticisms of teachers' priorities in Irish mathematics classrooms. In chapter 3, we noted that *The Chief Examiner's Report for Leaving Certificate Mathematics 2001* referred to the practices in Irish mathematics classrooms of "mimicking well rehearsed examples" as being at the root of many difficulties experienced by students. As recently as 2002

O'Donoghue pointed out that mathematics teachers focused more on getting students to pass examinations than on learning mathematics.

This main finding of this study is, that pedagogical practices in Junior Cycle mathematics are largely traditional, didactic in nature and based on a transmission style. Students in all three schools in this current study, but less so at Riverside, were involved in learning which focused on "how to do mathematics" rather than the "why". This "how to do" style of mathematics teaching may be more disabling than enabling for students. This current study found that students were frequently dependent on their teachers, and tended to view their teacher as the expert who ratified their answers. All three teachers in this study identified student dependency as a problem in teaching mathematics. When students are not engaged in their own learning they begin to feel disconnected and powerless in relation to their subject and this leads to anxiety and fear. This current study also found, as does a study by the ESRI Smyth et al., 2007, that students perceived mathematics to be the most difficult of school subjects. While students are aware of the importance and usefulness of the subject, there seems to be a tension between their recognition of a necessity to achieve well in mathematics on the one hand, and their experiences of learning mathematics on the other.

The same ESRI study (2007) also found this tension to exist in relation to mathematics among third year students. It states that: "mathematics appears to emerge as an area of concern for many students. Over half of the students find mathematics difficult in third year and are more likely to receive grinds in

mathematics (than in other subjects) while a significant proportion of students would like extra help with the subject” (2007 p.90). These findings raise important issues for teachers, curriculum advisors and policy makers about what is taught and how it is taught. In this current study students were most anxious about mathematics out of all their subjects, and mathematics tests in particular caused considerable anxiety, as did asking questions in mathematics class. In light of our findings on students’ experience of mathematics and how the subject is taught at post primary level, the current NCCA initiative “Project Maths: Developing Post-Primary Mathematics Education” (www.ncca.ie) is timely. It appears to have the essential attributes that are needed to improve the teaching and learning of mathematics in Irish post-primary schools. In the long term however its success lies in being well-resourced, both in its initial implementation in twenty four post-primary schools and its eventual roll-out to all schools.

The research in Chestnut Hill, Kenmore and Riverside indicates that students were for the most part involved in the reproduction of learnt skills and procedures. This current study established that the consequences of involving students in these practices in terms of the quality of the learning experience are unproductive. In the analysis of the video recordings we saw that students were learning for most of the time “how to do sums” and were memorising step-by-step mechanical procedures and “fixed plans” for performing tasks. Students were shown in Chestnut Hill, Kenmore and Riverside single approaches to the solution of problems. As a result students from all three schools largely equated success at mathematics with learning procedures off by heart. Their knowledge of mathematics became limited to narrow low-order questions and getting the “right answer quickly”. This also contributed to

their learning to become dependent on the teacher and to a feeling of uneasiness and pressure when faced with new issues in mathematics. As the classrooms studied in some detail in the current research were among the more progressive and successful among Irish post-primary schools, there is reason to believe that the problems identified here are more pronounced in post-primary maths classrooms more widely.

In chapter three we saw that, in comparison with current international trends such as in Holland (RME), the USA (problem solving), Northern Ireland, England and Wales (modelling and investigation), Irish post-primary mathematics syllabuses (analysed in chapter one and two) have remained largely formalist, abstract and conservative. The current trends in Holland, the USA, Northern Ireland, England and Wales require different teaching approaches from the traditional didactic transmission style associated with the “new mathematics” of the sixties. The shortcomings we have been reviewing in this current study clearly underline the case for a wide-scale promotion of new, non-didactic approaches to the teaching of mathematics in Ireland.

Notwithstanding the difficulties and shortcomings we have just been considering – indeed arising from the very analysis of such difficulties – it is possible to make some promising suggestions for the teaching and learning of mathematics in Irish post-primary schools. These suggestions would provide students with the kind of opportunities and learning environments that would enable them to engage with and learn mathematics in a more meaningful way with understanding.

From the investigations undertaken in previous chapters it is clear how teachers can hinder the clarity of the lessons for students. As the international research on assessment for learning continually emphasises, stating the goals of a lesson helps students identify the key mathematical points of a lesson. However the three teachers at Chestnut Hill, Kenmore and Riverside spent a minimal amount of time doing this and in some lessons did not state the goals at all. A second kind of aid to help students recognize the key ideas covered in a lesson are summary statements. Summary statements highlight points that have been studied in a current lesson. They are statements that occur near the end of the public portions of the lesson and summarise the mathematical point(s) learned in a lesson. Their use is not common among Irish Mathematics teachers. According to the TIMSS 1999 Video Study, Japan and Hong Kong, two countries that scored highly in the TIMSS (1995) study place a high value on both goal and summary statements.

What is particularly evident from the current study is the way in which the teachers could constrain or provide learning opportunities for students simply by the way they used the different episodes of classwork and seatwork. Compared to the top performing nations in the TIMSS 1995 Study and TIMSS 1999 Video Study the classwork and seatwork episodes in this study were used to accomplish different and sometimes more mundane purposes. The teachers at Chestnut Hill and Kenmore, and to a lesser extent Riverside, used the public whole-classwork episodes to deliver content and explain examples to students while the students' role in turn was chiefly to absorb what the teacher was transmitting to them. By doing this the teachers took much of the learning initiative and experience of thinking away from the students and effectively sheltered them from the kind of stimulus necessary to become better

at true understanding and problem-solving. During seatwork episodes students were mainly expected to practise routine procedures or methods which were practically the same as the example the teacher had demonstrated during the classwork episode (Figure 5.4). In Chestnut Hill, Kenmore and Riverside, the yardstick was how well and how quickly (though to a much lesser extent at Riverside) the students could replicate routine procedures.

Practice of course can consolidate standard techniques and such practice is creditable insofar as students need to be able to execute basic techniques accurately and speedily. However, being able to reproduce demonstrated routines does not mean that students necessarily understand. Indeed Hiebert & Carpenter (1992) and Ma (1999) tell us that reproducing demonstrated routines does not promote relational understanding and that students taught in this way cannot think of a problem in another way than what they have been shown by their teachers. Using seat work episodes as they were used in Chestnut Hill, Kenmore and Riverside primarily for executing procedures with lower-order goals for students (i.e. not to think and reason but to solve problems by explicitly using the one-solution method that had been presented publicly by their teacher) constrained the learning opportunities for students at Chestnut Hill, Kenmore and Riverside.

There are some compelling theoretical arguments, along with empirical data (TIMSS 1999 Video Study) to suggest that students can gain a deep understanding from both developing for themselves alternative solution methods and being allowed some choice in how to solve a problem (Brophy 1999, American National Research

Council 2001). In order to provide greater learning opportunities for their students during seatwork episodes teachers need to place a higher expectation on their students, particularly to take responsibility for the development of alternative solution methods. During classwork episodes students need to be afforded the opportunity by their teachers to actively participate by presenting and discussing their own solution methods publicly. Students in Chestnut Hill, Kenmore and Riverside were rarely asked to discuss or present their own different (different to the teacher's) solution methods. Opportunities were thus bypassed for encouraging higher-order discussions and enquiries with the students. It is important for teachers to avail of such opportunities for at least two important reasons: if students were involved in discussing and presenting alternative solution methods then they could more readily realise (a) that there is more than one way to solve a problem and (b) that they need not be dependent on the teacher to find a valid solution. In this way both classwork and seatwork episodes could be used by teachers to enable students to engage systematically and meaningfully with mathematics as learners.

The three teachers in this study initiated over 90% of all public interactions in the classrooms. These interactions were significantly high on quantity but relatively low on quality: they were made up of content elicitations for the most part, requiring short answers from students. Students were rarely asked at Chestnut Hill and Kenmore the type of questions that gave them the opportunity to produce thinking-out-loud answers or articulate their own ideas. However students at Riverside were asked at times the type of questions that gave them such opportunities. The teacher at Riverside concentrated questioning on both content elicitations (that tends to limit students' involvement to lower-order activities) and more sustained and thoughtful

development of key ideas that engaged students in using their own ingenuity.

Students at Riverside, when afforded these opportunities, were observed generating new knowledge for themselves. Questions need to invite fruitful interactions between teachers and students and among students themselves. Students need to be asked to explain a solution method (that they may have discovered themselves), give a reason why something is true or not true more than assimilating facts and procedures (memorisation).

At Riverside one promising methodology used by the teacher was to allow students time to process the question and formulate a response. Another fruitful finding at Riverside was that students were less likely to have their answers evaluated as right or wrong by the teacher. This contrasted with Chestnut Hill and Kenmore. Instead of evaluating answers as either right or wrong the teacher at Riverside tried to pursue the wrong answer and it was encouraging to see that when this happened, and the students were given the time, they often abandoned their misconceived answers and began to adopt more valid ideas and answers. Teachers need to be more alert to giving students time to answer a question. Teachers also need to pursue, as did the teacher at Riverside, incorrect answers as distinct from just dismissing the incorrect answer and asking another student for the correct one. This was commonly witnessed at Chestnut Hill and Kenmore and is the more usual pattern in mathematics teaching. Waiting for an answer can increase students' self esteem and confidence in class. It can also have the opposite effect but the teacher at Riverside used her judgement, and when "wait time" was not having the desired effect she rephrased her question so that the student would have an answer. Teachers generally did not ask questions that determined the students' current level of understanding or the students' progress.

This is something that needs to be remedied so that teachers become better informed on how to plan their teaching for subsequent lessons.

Teachers at Chestnut Hill, Kenmore and Riverside seemed to fear students making mistakes publicly. The teachers from Chestnut Hill and Kenmore tended to help students as soon as a difficulty arose. However at Riverside the teacher did allow students find out most of the difficulties for themselves. Teaching practices need to avoid this common tendency of helping students as soon as a difficulty arises, because the message that is being sent out here is that errors equal failure. Real advances can be made in the quality of learning mathematics if mistakes are investigated constructively, collectively and publicly. If students and teachers could dismantle their defensiveness and not be afraid to say that they are wrong, that mistakes provide unforeseen learning opportunities, this would be another way to enhance students' self-esteem and confidence with the subject. It is important for students to realise it is okay to make errors and that errors are expected by the teachers as way stations. Once the classroom is no longer dominated by concerns of right or wrong, opportunities for learning through focused and frank discussion increase.

The connections between mathematical symbols and procedures and the underlying mathematical concepts that teachers take for granted are not always apparent to students. Research tells us that the use of concrete models help students learn the abstract ideas of mathematics with understanding (Fennema & Franke, 1992, p. 154). With three exceptions, such concrete models were absent in the learning

environments observed over the course of the current study. The teachers relied mainly on practice and more practice to memorise formal procedures. Practice of this kind does little however to connect symbols to what would give them meaning within the experience of the learners.

One of the ways to resolve this dilemma is for students to link mathematical procedures and symbols to concrete models that are meaningful. Models can be introduced by the teacher or constructed by students. In this thesis examples of models being used were found at Riverside in Year 1 of the study (chapter 5) where the teacher used geostrips for exploring relationships between angles and the students then constructed their own models from cardboard and built up individual geometry folders. The second example was found at Kenmore in Year 1 (Appendix F to chapter 5), where the teacher used algebra tiles, albeit for a few minutes. The third example was found at Chestnut Hill in Year 2, where the teacher used a clinometer to find the height of a clock on the wall and then used trigonometry to find the same height and compared the two answers. According to Carpenter & Lehrer (1999, p. 25) and Romberg & Kaput (1999, p. 10) the use of models as a means of simplification is a crucial stage in understanding. With appropriate guidance from teachers, these models can successfully lead students into more abstract mathematical reasoning. Again, although tools can facilitate understanding, it is important that tools are not seen as giving the answers but as models to think from, to bridge the gap between a concept and the symbol. Tools are simply a means to solve problems with understanding and a way to communicate problem-solving strategies.

The scientific calculator was used by all students in Chestnut Hill, Kenmore and Riverside in Year 2 and Year 3, but was infrequently used in Year 1. The use of calculators in Junior Cycle has become increasingly widespread since their introduction in 2000 with the revised Junior Certificate mathematics syllabus. While students were observed by the researcher using calculators in class they were most often used for checking answers and doing routine procedures. Teachers did not exploit the calculator to its full potential. Failure to do so may reinforce negative attitudes towards it on the part of teachers (Surgenor et al., 2007, p. 8). Professional development courses have been found to have a positive effect on teachers' beliefs about the potential of the calculators as a tool for exploration of mathematics (Schmidt, 1999). In the light of the empirical evidence from this thesis and Surgenor et al., (2007), greater attention needs to be given in mathematics teacher development initiatives to the judicious use of the calculator for developing understanding of specific concepts and procedures, estimation skills and use of real-life data; not simply for checking answers.

So far we have analysed the critical features of mathematics teaching and learning environments that have emerged in this current study and reviewed why we need to promote qualitatively richer learning environments. We have also explored some distinctive characteristics of meaningful teaching and learning environments that seek to promote learning for understanding among students. In the section that follows we will consider practical approaches of cultivating richer learning environments.

2. A Practical Example of Cultivating Richer Learning Environments

An overriding theme in all elements of this study has been the need to secure higher quality teacher participation in curricula and pedagogical reform in mathematics. This study has shown that mathematics students are typically disempowered and poorly engaged. It may also be the case that the student experience is replicated among the teachers. The case studies described in chapter 2 draw attention to the centrality of teacher engagement in curricula reform. The main issue to emerge from an overview of these case studies in this regard would appear that they are unlike most traditional mathematics professional development models used in Ireland to date (as discussed in chapter 1 and 2). Some of the characteristics of Lesson Study, such as teacher-led professional development, professional development that takes place in the classroom and having students at the heart of the activity, would make Lesson Study quite unique in an Irish context and could be especially promising for Irish mathematics teachers. Le Metais (2003, p. vi) warns however that policymaking can fall into a “quick fix” trap if an approach is simply transplanted from one country to another. Careful attention needs to be paid to professional cultures and local social circumstances in all such attempts at emulating best practice in other countries. Le Metais (2003) also points out that cross-national studies best help policy-making when care is taken to include: (i) informal self-review; (ii) clarification and refinement of learning goals (2003 p. vi-vii). Conway et al. (2005), amongst others such as (Kelly & Sloane 2003, Lewis 2002, Stevenson & Stigler 1992), point out that Japanese performance in international comparisons is based not only on high quality Lesson Study but also on other cultural, social, religious and family factors.

Bearing this advice in mind we will now review the main challenges to launching Lesson Study as a form of professional development for mathematics teachers in Ireland. In the course of reviewing these challenges we will also offer a justification and a rationale for promoting Lesson Study in an Irish context. We have already noted above the advice of Le Metais about transferring successful approaches from one country to another. Yet there is little about the essentials of Lesson Study that hasn't already been recommended by educators in Ireland. Reasons for promoting Lesson Study in Ireland are many. Lesson Study is a coherent and systematic way of putting the features of best practice of professional development together (identified by OECD Report 1998, p. 50 and discussed in chapter 2): e.g. it is teacher-led, it is collaborative, it builds a body of professional knowledge over time, and it fits well with the officially adopted "3 I" teacher education policy teachers for Ireland (chapter 2). So much for good reasons, but the chief challenge here is that of developing in practice the structures that would meet the aims of the "3 I" policy.

A second challenge is the question of time. Teachers need time to conduct Lesson Study on an ongoing basis (Willis 2002 p. 2), as distinct from engaging in CPD as a series of once-off events. Time for collaborative practices and ongoing development has not been a strong feature of CPD for Irish teachers (discussed in chapter 1 and 2). However teachers could, with strategic planning and the support of their school leadership, make some provision for Lesson Study within the normal working week. Dedicated time for CPD within the school calendar year is needed if innovations like Lesson Study are to be properly embedded into the cultures of teaching.

Thirdly, another potential challenge to successful implementation of Lesson Study in Ireland is the peer observation component of Lesson Study. This happens when one teacher from the group teaches the lesson and the other teachers who have co-planned the classroom lesson observe it being taught. This necessitates the teachers leaving their own classrooms to observe the lesson. In an Irish context this would entail getting substitute teachers to supervise the unattended classes which may not be possible. Willis (2002 p.10) suggests that using video and audiotapes, and gathering copies of students' work, is the best way of representing the teaching and learning process so that the lesson can be studied later without teachers leaving their classrooms to observe the lesson. This would lend an added dimension to Lesson Study; it would provide teachers with the opportunity to listen to learners' misconceptions and misunderstandings.

While the success of Japanese mathematics education may have much to do with practices such as Lesson Study, researchers like Chokshi & Fernandez (2004) point out that while there is no formal proof that Lesson Study will improve student performance in summative assessments (Chokshi & Fernandez, 2004), Lesson Study can provide rich formative assessments of student performance. Given the current focus in Ireland on student achievement in high stakes state examinations (chapter 7) the fact that there is no formal proof that Lesson Study will improve students performance in examinations may represent a fourth challenge to its successful implementation in Ireland. But data collected during the Lesson Study class could provide a rich picture of students' understanding - even more than what standardized tests can offer. Lesson Study provides ongoing information on a range of relevant aspects of learning that allows teachers to continually tailor their teaching to their

students' learning needs. By gathering concrete particulars on student understanding (such as transcripts of student discussions, examples of student problem-solving strategies, lists of questions students ask, and so on), teachers are essentially compiling assessment portfolios for their students, and these can provide valuable, evidence-based insights into students' classroom performance and conceptual understanding. Lesson Study thus provides a portal through which teachers can judge their students' work, improve their own teaching practice, and document their own as well as their students' progress. Such rich documentation can also be used to show to others how Lesson Study is being used to improve student performance.

A fifth challenge to promoting Lesson Study in Ireland is that Irish teachers are largely insulated and isolated from their professional colleagues (OECD 2004, Table D4.3) so they may feel anxious about publicly teaching lessons for their peers to study. A crucial point to note here however is that, unlike traditional classroom observations, Lesson Study is generally conducted in a way that shifts the focus from evaluating the performance of the teacher to the "how" and the effects of teaching. In addition, ownership of the lesson is diffused across the group of teachers who planned the lesson. Consequently, teachers are less nervous than they would typically be in such a public situation.

In the light of this analysis of the challenges that Japanese lesson study might present it is clear that an adapted form of it would be necessary for its introduction to post-primary mathematics teachers in an Irish context. Such adaptations could be developed with distinctive features that meet the needs of particular national or more

local circumstances. For example Stewart and Brendefur (2005) have created a blended model of lesson study to fit an American context. For their model they based their work on Stiggins (2001), Fullan (2001), and DuFour and Eaker (1998) and drew up their design on the belief that they “needed a model that would get to the classroom level as quickly as possible and start deep conversations about curriculum, instruction and student learning” (Stewart and Brendefur, 2005, p.682).

We will now consider some further benefits to be gained from Lesson Study for teachers. To fuel deep conversations about practice that leads to more effective Lesson Study work it is important in the first place for teachers to have a rich understanding of mathematics (Ma, L. 1999). By a rich understanding we do not mean an instrumental understanding, but a relational understanding of the subject (as discussed in chapters 3, 5, 6 and 7), where teachers are at home in their subject and are able to move about imaginatively within the subject without difficulty. Not all mathematics teachers have this knowledge, as we saw in chapter 2. However Choski & Fernandez (2004, p. 2) point out that shortcomings in content knowledge do not prevent the work of Lesson Study. In fact they argue that Lesson Study can serve as a vehicle by which teachers can deepen their understanding (relational) of the content. Their research found that because of the collaborative nature of Lesson Study it allowed teachers to "fill in the blanks" for one another, especially as the activity of planning a lesson together creates many opportunities for teachers to learn from their colleagues. More important, the content knowledge developed during Lesson Study is learned in an embedded context, because the task of learning the content is closely intertwined with the authentic activities of teaching and can be immediately applied to the classroom.

Research studies have shown that a further benefit to be gained from engaging in the formal process of Lesson Study is that the effects of the intensive work on just a few lessons can be quite far-reaching. By engaging in the formal process of Lesson Study, teachers often carry an informal "Lesson Study mentality" into their daily practice (e.g., paying greater attention to anticipating student responses, greater emphasis on relational understanding). Teachers who have participated in Lesson Study can also develop and apply more widely the general teaching principles they extract from this collaborative examination of practice. They might, for example, pose higher order of questions to provoke students' thinking or employ better methods of dealing with incorrect responses from students; features which we saw in chapter 5, 6 and 7 were not to the fore in the practices of Irish mathematics teachers. The experience of engaging in Lesson Study tends, moreover, to cultivate in teachers a disposition toward continual improvement of their teaching.

Lesson Study is just one example of a number of innovative approaches to promote advancements in teaching and learning. All such approaches need to be monitored and developed further by well-designed support structures. The final section of the chapter will explore the issue of such support structures and make some key recommendations.

3. Supporting and Sustaining Innovative Practice

Among the findings of earlier chapters were that innovative attitudes and practices were not a notable feature of mathematics teaching in Irish post-primary schools.

Recurring shortcomings even among those who would be regarded as good teachers have been analysed in this study. *The Chief Examiner's Report for Leaving Certificate Mathematics* (2001) has also recognised such shortcomings and has acknowledged a need to transform teaching practices. This study has shown in chapter 2 that there is a significant problem in student achievement in mathematics in Ireland. In the light of these concerns, and of the evidence from this thesis, it is clear that there is a pressing need to improve the attitudes and practices of mathematics teachers and students through support programmes. The typical forms of CPD provision and strategies in Ireland in the past follow many of the general criticisms of inservice education set out by Fullan (2001) and cited by Smith (2004, p. 119):

- One-shot workshops are a widely used format, but are often ineffective;
- Inservice programmes are rarely directed to the individual needs and concerns of participants;
- Follow-up support of ideas and practices introduced during inservice programmes is rarely provided;
- Follow-up evaluation occurs infrequently;
- Most programmes involve teachers from a number of different schools but the potential different impact of positive and negative factors in individual teacher's local environment is typically not factored in to the programme.

These criticisms imply that a successful model of CPD requires first of all, the active participation by teachers. There must also be continuity between CPD sessions, as opposed to one-off events. Colleagues need to share with each other the practices learned at CPD workshops.

The French model of CPD (IREMs) described earlier contains many features which could usefully be adapted to an Irish context. Two features of particular merit of the IREMs are that they include teachers of all levels i.e. university, post-primary and primary school teachers, all working together providing inservice courses on both subject matter (content knowledge) and pedagogy (pedagogical content knowledge). They work in a network as this allows information to circulate constantly between them. Meetings are organised between IREMs every year.

In an Irish context these features, taken in terms of a long term investment in a national infrastructure in a National Centre of Excellence for mathematics, could oversee the provision of subject-specific CPD and other forms of support for teachers of mathematics, both specialist and non-specialist. The National Centre could provide strategy and coordination, together with regional centres providing local support and networking. The regional centres could furnish an opportunity for mathematics advisors to provide back-up and expertise to a support team by tracking best practice from other countries, using formative assessment for learning effectively as well as focusing on an appropriate range of teaching styles and strategies. These mathematics advisors could also assist the support team by developing resources relevant to planned mathematics inservice courses and by researching, designing and publishing suitable mathematics modules. Furthermore they could develop exemplars of best practice using video clips of experienced Irish teachers to demonstrate varying teaching methodologies.

These regional centres could also help to build capacity at school level through sustainable local networks (TPNs and IMTA branches), each with a pool of mathematics expertise. These centres would also liaise with the Primary Curriculum Service and with relevant third level institutions. Closer working links with third-level institutions would open up a number of opportunities for higher education to provide significant new and sustainable support for teachers, for example by increasing teachers' awareness of the extraordinary range of applications of mathematics, supporting teachers through mentoring and supervising advanced degrees, ensuring teachers are well informed about developments in mathematics research and applications.

In France, IREMs that are interested in a particular theme or problem invite local groups to work on them in their classrooms. In this way IREMs undertake their own projects and disseminate this work through publication as well as through meetings and Summer schools. In one or two weeks of intensive work a theme linked to mathematical education is studied thoroughly. These summer schools are directed not only to teachers in secondary schools but to teachers in primary schools, mathematics inspectors etc. In chapter 2 of this thesis it was noted that the IMTA ran summer courses, but that these unfortunately ended in summer 2001. These IMTA summer courses were well attended (though latterly not) and need to be revived. The IREMs appear to offer a model of best practice in terms of promoting sustainable closer links between teachers of all levels and other major educational groups.

As we have noted, writing and disseminating publications is also part of the work of IREMs. Of particular interest is the synthesising of booklets resulting from the work of local groups of teachers centred on topical issues in mathematical education. The richness of these papers lies in the variety of topics dealt with. This would be a very promising practice in Ireland for the pre-service training and continuing professional development of teachers, as well as for advancing a research tradition in mathematics education. There is often a tendency for teachers to view research publications as official documents, remote from their own daily work. If this view is taken then the real potential of research is bypassed by teachers. What is promising from the French model is the fact that these publications are kept alive as they focus on different topics, develop a range of perspectives and heighten awareness of the processes and phenomena involved in the teaching of mathematics, in the teaching and learning process and in the classroom situations that are developing. Such publications enliven the ongoing professional development of teachers with a supply of fresh pedagogical ideas. The fact that teachers actively participate in producing them ensures that teachers' energies are built around these publications. These collaborations are promising for an Irish context and are a very practical way of creating positive energies so that teachers see publications more as an active resource than a readymade package.

As discussed previously in chapter 1 and 2 there is not a strong tradition of participatory CPD among mathematics teachers in Ireland. The French example we have considered has been concerned mainly with the substance, or agenda, for supporting participatory CPD for mathematics teachers. Equally important however are the structures for CPD, and in this instance the example of Scotland is

particularly noteworthy. In Scotland CPD responsibilities and entitlements have been incorporated into a formal agreement, *A Teaching Profession for the 21st Century* (Scottish Government, 2001). Scotland is a country comparable in population size and per-capita income to Ireland. The agreement *A Teaching Profession for the 21st Century* includes the following features:

- Teachers make an ongoing commitment to maintain their professional expertise through an agreed programme of CPD
- There is an additional contractual 35 hours of CPD per annum as distinct from teaching time (as a maximum for all teachers), which consists of a balance of personal professional development, attendance at nationally accredited courses, small scale school based activities. This balance is based on an assessment of the teacher's individual needs, the needs of the school and national priorities
- Every teacher has an annual CPD plan agreed with his/her immediate school principal and every teacher must maintain an individual CPD record

This kind of model would pose a radical change for CPD provision for teachers in Ireland. The Scottish model contains many features of good practice in CPD activities. First, where every teacher has a personal development plan. Secondly this personal development plan is one to which both teachers and school leaders commit to in writing. This places an obligation for CPD on both teachers and school leaders. The Scottish model addresses two key elements in a CPD programme; (i) it is personalised in so far as it addresses the teacher's individual needs and (ii) it involves school leadership in monitoring the effects of CPD.

Concluding Remarks

The picture that emerges in this thesis of teaching and learning mathematics is not as it should be. Neither is the culture of CPD for mathematics teachers as it could be. It is one that is healthy neither for the mathematics teacher or student. As stated in the Introduction (p. 3) the main aims of the current study are:

1. To explore ways of enhancing the quality of mathematics teaching at Junior Cycle.
2. To explore ways of improving the quality of mathematics learning at Junior Cycle.

In pursuit of these aims the thesis:

1. investigated secondary school mathematics from an historical perspective;
2. investigated the provision of inservice support for the Junior Certificate mathematics teacher;
3. investigated the dominant patterns in the attitudes and practices of secondary mathematics teachers at Junior Cycle and their pedagogical consequences;
4. investigated the effectiveness of the work done by Junior Certificate mathematics teachers, as distinct from examination results achieved by their students in state examinations;
5. investigated the relationship between teaching practices and students' attitudes to learning mathematics.

In the course of this study we have analysed and reviewed these issues. In this some key recommendations have been made that would enhance and improve the quality of teaching and learning for students. We have already seen that there have been reforms in syllabus content and the provision of CPD for mathematics teachers and these reforms have not impacted as they should have. The findings of this study indicate a number of possible directions for future developments.

There needs to be a long-term investment in a national infrastructure to oversee the provision of mathematics CPD and other forms of support for teachers of mathematics. These CPD programmes need to be tailored to the needs of teachers of mathematics, both specialist and non-specialist, including leaders in mathematics teaching (e.g. part-time associates). The national support infrastructure for the teaching and learning of mathematics could take the form of a National Centre providing coherence, strategy and coordination in provision of CPD. The National Centre could work together with regional centres (e.g. IREMs) providing local support, networking and continuity between workshops as distinct from once-off events. Lesson Study could be introduced at a local level through regional centres to actively engage colleagues sharing practices and cultivating self-critical reflective practices.

We have seen that the way mathematics is taught by the three teachers in this study is traditional in style to varying degrees. However to understand their approach to teaching mathematics simply as a matter of their personal responsibility is to simplify a complex problem in the teaching and learning of mathematics. The

traditional pattern of teacher demonstration followed by student practice is one that many Irish teachers have inherited. While teachers are autonomous in their own classrooms they are not necessarily authors of their own work. The weaknesses in the provision of CPD for mathematics teachers has resulted in a lack of experience of alternative approaches to teaching mathematics. The State examinations exert a huge influence on how mathematics is taught and often pull teachers away from innovative and creative pedagogies back to the tried and trusted ground of teacher demonstration and student practice.

If we are to improve and enhance the teaching and learning of mathematics in Irish post-primary schools the focus cannot be exclusively on how teachers teach, even though this is of paramount importance. We need to increase teachers' awareness of: (a) the shortcomings of their traditional practices in terms of student learning and (b) more fruitful approaches to teaching and learning. To realise this would require policy makers to invest in a national and regional infrastructure of sustained support that would focus not just on teachers and students in the classroom but an investment in research into mathematics education in Irish post-primary schools on all fronts and at all levels.

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