

A consideration of familiarity in Irish mathematics examinations.

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In this paper, we focus on the idea of familiarity and the differing levels of it that are apparent in Irish mathematics end of school state examination questions. We provide the results of an analysis of recent Higher Level and Ordinary Level Leaving Certificate mathematics examinations in terms of familiarity. Our findings do not indicate any particular recurring pattern evident in the levels of familiarity measured but generally not more than 20% of marks are allocated to unfamiliar questions.

Keywords: familiarity, assessment, mathematics examinations.

Introduction

There is much discussion in the Irish education system about the skills and competencies with which students leave school (HEA and NCCA 2011). Significant criticism has been levelled at the current model of education in terms of the underdevelopment of critical skills and the utilisation of a narrow range of assessment methods, resulting in a reductionist approach to learning (Hyland 2011). There is a growing tension between criticism of the predictability of the state examinations and the public's demand for familiarity in order to be fair to candidates. Hyland's report notes that the media has delivered strong attacks whenever examinations are perceived to deviate from stakeholders' expectations. This is particularly true in the case of mathematics (for example, see Irish Times 2011).

The end of secondary school examination, the Leaving Certificate, serves more than one purpose: it supports learning that aims to prepare students for the next phase of education and it facilitates the selection mechanism for entry into further and higher education. Subjects are normally studied at either Ordinary or Higher Level for the Leaving Certificate (LC) examination, although English and mathematics can also be studied at Foundation Level (DES and NCCA 2000). Foundation Level is not generally accepted for matriculation to third level education and so will not be considered in this analysis. The Higher and Ordinary Level mathematics examinations each consist of two papers covering such topics as algebra, sequences and series, calculus, functions, statistics, discrete mathematics, geometry and trigonometry. The topics that we examine are those of algebra and sequences and series. These topics are common to both levels and account for three questions (two on algebra and one on sequences and series) out of the eight posed in Paper 1.

In the Irish state mathematics examinations, at Higher and Ordinary level, the questions are usually divided into three parts, labelled '(a)', '(b)' and '(c)' respectively (Close and Oldham 2005). Part (a) usually tests recall or the ability to execute simple procedures. Part (b) typically focuses on tasks involving instrumental understanding which are intended to be executed fluently. The part (c) problems aim to address the higher-order objectives of the syllabus, usually testing the ability to apply knowledge in contexts which are unfamiliar or demonstrate the ability to understand concepts and the connections between them. Part (c) is intended to be unpredictable to a certain extent and test the problem solving ability of students at least to some extent.

However, as Close and Oldham note, teachers have attempted to reduce part (c) to the status of rehearsed procedures by practising past examination questions repeatedly. The marking scheme applied usually has part (a) carrying 20% of the marks for the question, and parts (b) and (c) usually carrying 40% each.

It has been noted that the questions on Irish state (mathematics) examinations have an apparent complexity, due mainly to their daunting appearance and layout (Elwood and Carlisle 2003). Elwood and Carlisle's analysis suggests that the questions are quite straightforward and reward students who are well rehearsed in terms of the papers' format. Those who have learnt the formulae associated with the syllabus and made themselves familiar with the way in which such questions expect students to respond are rewarded. This in turn influences how students are prepared for the examinations and how they are taught in the classroom. Teachers are highly dependent on the use of a textbook mirroring the style of the examination questions that their students will eventually sit (NCCA 2005). This can result in a kind of 'learnt helplessness' resulting in students feeling that they cannot tackle even slightly unfamiliar work (NCCA 2005). This problem is not unique to Ireland with similar complaints evident in the UK. 'Examiners are also under pressure to produce predictable [GCSE] papers that will not scare teachers away' (Jones 2010, 66) to competing private awarding bodies. Pope (2011) also points out that GCSE assessment is unable to accommodate innovation due to the high stakes involved and that papers could become unpopular if they used unexpected questions. She suggests that novel items are unable to find their way onto the papers due to the pressures from market forces. As the awarding organisations are all competing with each other in this way, it results in the papers being very predictable. Pope notes that "narrow, predictable assessments enable 'teaching to the test' in England's high stakes performative education culture" (2011, 66).

What is 'familiarity'?

Familiarity has been considered in terms of being a difficulty variable with regards to examinations (Pollitt et al. 1985). It is suggested that the degree of familiarity can be measured in terms of how familiar a particular concept is to students and how central its place is in the syllabus. However, it is not a straightforward process to determine familiarity due to the subjective nature of categorisation. This human element has been addressed by considering problems and exercises as having two components: task and solver (Selden, Mason and Selden 1989). It is necessary to consider not just what skills a task requires but also how the solver views the task. When considering whether mathematical examination questions were routine or non-routine, it has been found that there can be disparate views (Berry et al. 1999). Berry et al. define routine questions as 'those for which students may be expected to execute a rehearsed procedure consisting of a limited number of steps' (1999, 11). The authors categorized questions as routine or non-routine, before asking students to do the same thing. It was found that students only partially agreed with the authors' categorisation of questions, with more agreement on routineness rather than non-routineness.

The reasoning that university students in mathematics have to perform in order to solve examination tasks has been studied by Bergqvist (2007), using Lithner's (2008) mathematical reasoning framework. Imitative reasoning involves the use of memorisation or well-rehearsed procedures and creative reasoning is defined as the use of novel reasoning with arguments to back it up. One such category of imitative reasoning is defined as familiar algorithmic reasoning. A task is defined as being

familiar if it is identified as belonging to a familiar set of tasks that can all be solved by the same known algorithm. Bergqvist also identified similar problems of categorisation to those encountered by Berry et al. (2005). She acknowledged that a task that is possible to solve with imitative reasoning for some students, might require creative reasoning from others, depending on how familiar the task and its solution are to the student. Bergqvist (2007) analysed sixteen examinations in four Swedish universities and found that 70% of the examination questions could be solved using imitative reasoning alone and fifteen of the sixteen examinations could be passed without using creative reasoning.

It is possible that the level of familiarity which students have with a question may affect the level of cognitive demand of that particular question. In examinations, it can be seen that many familiar tasks have a low level of demand cognitively (Doyle 1988). Doyle suggests that this is because many such tasks are dependent on memory, where students must reproduce or recognise information that they have already encountered. For other tasks, they have to use formulae or algorithms to generate answers to a set of problems. While a question may appear to have a higher level demand cognitively, it is important to consider if a similar question has been asked in the past. In an Irish context, it seems reasonable that while a question may appear to be quite demanding, if a student is very familiar with it from practice due to it being a recurrent feature of past examinations, then the real demand on the student has been significantly diminished and the approach to it becomes routine. Familiarity then is an important feature of the context of examination questions.

Analysis of Familiarity in Leaving Certificate examination questions

For our analysis, we created an index involving three levels of familiarity namely: very familiar, familiar, and unfamiliar. The analysis was limited to the six years of examination papers that immediately precede the year of the examination question under consideration. For example when looking at questions in 2011, it was necessary to consider the questions from 2010 to 2005 inclusive (see SEC 2012 for past examination papers). When deciding on the degree of familiarity: we used ‘very familiar’ to represent questions for which similar questions had occurred four, five or six times in the previous six years, ‘familiar’ for one to three times, while ‘unfamiliar’ indicated there were no questions similar to it in the previous six years. When considering the familiarity of a question, we concentrated on whether the questions were testing similar skills or involved using similar methods; as well as how they were presented. In this sense, we did not just consider the task and the intentions of its designer, but also how the solver would view the question and the likely approach used in its solution. We decided on the use of six years because a smaller number such as three may not be very informative or give an accurate indication of the level of familiarity. Similarly, we did not use a larger range such as ten years as, anecdotally, students and teachers tend to place greater emphasis and focus on more recent years when practising questions and thus it seemed reasonable to focus on more recent questions. Bergqvist (2007) judged a task to be familiar if the textbook being analysed contained at least three occurrences of the task.

The following example describes how we classified the examination question: 2008 Higher Level 2 (b)(i) (SEC 2012) α and β are the roots of the equation

$$x^2 - 7x + 1 = 0. \text{ Find the value of } \alpha^2 + \beta^2.$$

We considered the questions from 2007 to 2002 when looking for similar questions, and found that some occurred in the years 2003, 2004 and 2007. With the

incidence of three similar problems in the previous six years, the question was labelled as familiar. The similar questions in the preceding years are as follows:

2007 2(b) (i) α and β are roots of the equation $x^2 - 4x + 6 = 0$. Find the value of $\frac{1}{\alpha} + \frac{1}{\beta}$.

2004 2(b) (ii) The roots of the equation $x^2 + px + q = 0$ are α and β , where $p, q \in R$. Find the quadratic equation whose roots are $\alpha^2\beta$ and $\alpha\beta^2$.

2003 1(c) (i) The real roots of $x^2 + 10x + c = 0$ differ by $2p$ where $c, p \in R$ and $p > 0$. Show that $p^2 = 25 - c$.

Each of these questions tests the students' knowledge of roots of quadratic equations. In terms of the solution, each question can require the student to make use of the formulae for the sum and product of the roots: that is, for a quadratic equation $ax^2 + bx + c = 0$, the sum of the roots $\alpha + \beta = \frac{-b}{a}$ and the product $\alpha\beta = \frac{c}{a}$.

Of these three questions, 2007 is the most similar to 2008 due to the focus being on a specific quadratic equation. 2004 and 2003 are more general, asking the student to find expressions for a quadratic equation or to verify a particular statement. However, all three questions involve the same knowledge of quadratic equations and the use of the same formulae in order to get the desired solutions. The intended higher-order skill involved in solving the question posed in 2003 may be somewhat diminished in the questions asked in later years when viewed in terms of the level of familiarity that the well-practiced student brings to the task.

| Familiarity | 2011 | 2010 | 2009 | 2008 | 2007 |
|---------------|---|---|--|--|---|
| Very Familiar | 1(c) [20] 2(b)(i) [10] 30 Marks | 1(b) [20] 2(a) [10] 30 Marks | 1(c) [20] 2(a) [10] 2(c) [20] 50 Marks | 1(b) [20] 1(c) [20] 4(b) [20] 60 Marks | 1(c) [20] 2(a) [10] 2(c) [20] 4(b) [20] 70 Marks |
| Familiar | 1(a) [15] 2(a) [15] 2(b)(ii) [10] 2(c) [15] 4(a) [10] 4(b)(i) [10] 75 Marks | 1(c) [20] 2(b) [20] 2(c) [20] 4(a) [10] 4(c) [25] 95 Marks | 1(b) [20] 2(b) [20] 4(a) [10] 4(b) [20] 70 Marks | 1(a) [10] 2(b) [20] 4(a) [10] 4(c) [20] 60 Marks | 1(b) [20] 2(b) [20] 1(a) [10] 4(a) [10] 4(c) [20] 80 Marks |
| Unfamiliar | 1(b) [15] 4(b)(ii) [10] 4(b)(iii) [10] 4(b)(iv) [10] 45 Marks | 1(a) [10] 4(b) [15] 25 Marks | 1(a) [10] 4(c) [20] 30 Marks | 2(a) [10] 2(c) [20] 30 Marks | 0 Marks |
| Total | 150 Marks | 150 Marks | 150 Marks | 150 Marks | 150 Marks |

Table 1: Classification of 2007-2011 Leaving Certificate Higher Level questions by familiarity

Discussion

Tables 1 and 2 show the distribution of familiarity for the Leaving Certificate mathematics examination Higher and Ordinary Level papers respectively for the years 2009 to 2011. The questions relate to the topics of algebra and sequences and series. At Higher Level, questions 1 and 2 relate to algebra while question 4 is on sequences and series. At Ordinary Level, questions 2 and 3 examine algebra while question 5 relates to sequences and series. It must be acknowledged that the levels of familiarity in these question areas may not reflect the overall level of familiarity for all questions on the examination. The selection was made because the topics were common to both examination papers and this facilitated comparison between Higher and Ordinary Level. The breakdown is somewhat surprising given the intentions of the (a), (b) and (c) parts described by Close and Oldham (2005). It would seem reasonable to expect the questions to generally follow the format: (a) very familiar, (b) familiar and (c) unfamiliar. This is not the pattern that is evident from analysis. In fact, there is a distinct lack of an overall pattern to the familiarity levels present in the papers at Higher and Ordinary Level. Very often the (c) parts can be very familiar; while the (a) or (b) parts can often be labelled as unfamiliar, something that does not match the expected level of difficulty. Overall, while it would be expected that 40% of the questions would be unfamiliar, this was not the case in our analysis where generally not more than 20% of questions are unfamiliar. There is a lack of an overall pattern in the levels of familiarity but it is apparent that there is a strong tendency towards questions being very familiar or familiar.

| Familiarity | 2011 | 2010 | 2009 | 2008 | 2007 |
|---------------|---|--|---|---|--|
| Very Familiar | 2(c) [20] 3(b)(ii) [5] 5(a) [10] 5(b)(iii) [5] 35 Marks | 2(b) [25] 2(c)(ii) [5] 3(b) [20] 5(a) [10] 5(b)(i) [10] 5(b)(iii) [5] 75 Marks | 2(a) [10] 3(b)(i) [15] 5(a) [10] 5(b)(iv) [5] 5(c)(i) [5] 45 Marks | 3(b)(i) [15] 3(c)(i) [10] 5(a) [10] 35 Marks | 2(a) [10] 3(a) [10] 3(b) [20] 3(c)(ii) [10] 5(c)(i) [5] 5(c)(ii) [10] 65 Marks |
| Familiar | 2(a) [15] 2(b) [20] 3(a) [15] 3(b)(i) [15] 3(c) [15] 5(b) [15] 5(c) [20] 115 Marks | 2(a) [15] 2(c)(i) [5] 3(c)(i) [10] 5(b)(ii) [5] 5(c) [20] 55 Marks | 2(b) [20] 2(c) [20] 3(a) [10] 5(b) [10] 5(c)(iii) [10] 70 Marks | 2(a) [15] 2(b) [20] 2(c)(iii) [5] 3(a) [10] 3(b)(ii) [5] 3(c)(ii) [10] 5(b) [10] 5(c) [20] 95 Marks | 2(b) [20] 2(c) [20] 3(c)(i) [10] 5(a) [10] 5(b) [20] 5(c)(iii) [5] 85 Marks |
| Unfamiliar | 0 Marks | 3(a) [15] 3(c)(ii) [5] 20 Marks | 3(b)(ii) [5] 3(c) [20] 5(b)(iii) [5] 5(c)(ii) [5] 35 Marks | 2(c) [10] 5(b)(iii) [10] 20 Marks | 0 Marks |
| Total | 150 Marks | 150 Marks | 150 Marks | 150 Marks | 150 Marks |

Table 2: Classification of 2007-2011 Leaving Certificate Ordinary Level questions with regard to familiarity

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