Mathematatisation and Irish students: The ability of Irish second-level students to transfer mathematics from the classroom to solve authentic, real life problems.

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Signed Statement

I hereby certify that this material, which I now submit for assessment on the program of study leading to the award of PhD, is entirely my own work and has not been taken from the work of others save and to the extent that such work has been cited and acknowledged within the text of my work.

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Abstract:

This research considers the mathematical performance of Irish second-level students. The author considers the ability of Irish students to utilise the mathematics learned in a classroom situation to solve authentic, real-life problems. It is a mixed methods study involving testing, structured observations and semi-structured interviews. The research participants are Irish second-year, second-level mathematics students and grade 8 students from a school in the state of Massachusetts (both groups share a mean age of 13.5 years). The students from Massachusetts were involved solely at the testing stage of the data collection process in order to consider Irish performance with regard to mathematical performance from students in a different education system.

The observed mathematics lessons provide a valuable insight into the teaching and learning practices used at second-level. The quantitative analysis of the classroom observations highlight patterns and learning theories used in the mathematics lessons observed with interesting results. Two tests were implemented: one traditional in format and based on the Irish Junior Certificate examination; the second consisting of an authentic scenario where students are asked to demonstrate their mathematical comprehension when faced with questions posed in an unfamiliar manner. Statistical analysis of both tests, using a two-sample t-test and a one-way ANOVA, provide the comparison techniques required to consider students performance and highlight various similarities and differences between the test results. The final stage in the data collection process involved semi-structured interviews with the mathematics teachers which provide qualitative data to enrich the findings from the quantitative aspects of the study.

The findings provide an interesting insight into the ability of Irish students to solve mathematics when presented in a traditional, familiar context consisting of closed-ended questions compared with their ability to solve mathematical

questions that are unfamiliar in style and consist of realistic, open-ended, messy questions. This study suggests that an ability to perform well in a traditional examination does not necessarily illustrate an ability to utilise the mathematics learned for examination success when faced with unfamiliar scenarios. The Irish students involved in this research performed at a significantly higher standard in the traditional test given compared with a lack-luster performance in the realistic test. The same pattern held true for the Massachusetts' cohort; however the gap in performance between the two test types was considerably smaller for these students.

An inability to utilise school-learned mathematics when solving real-life problems is a worrying phenomenon and the author hopes that this body of work will engage educators and policy makers in discussion, thus contributing to progress in this field.

1.0 Chapter 1: The Introduction

1.1 Introduction

Mathematics is a key component of the traditional curriculum taught in Irish schools at both primary and secondary level. It is a subject that is studied from the first week in Junior Infants to the last day in second level. Despite the time and effort afforded to mathematics in the Irish education system Irish education is regarded as mediocre at best in international assessments.

This research considers the ability of second-year Irish, second level students to transfer the mathematics learned in the classroom to solve both traditional mathematics problems, and also realistic, authentic mathematics problems, asked in an unfamiliar style. For comparative purposes students from the state of Massachusetts in the United States of America were also involved in the research, and participated in the same tests. The author considers the hypothesis 'that Irish students have the ability to transfer the mathematics learned in the classroom to unfamiliar, realistic, problem-solving situations'. The author gathered data through testing, interviews and classroom observation. From an analysis of this data gathered, the author considers the impact of the findings on the research hypothesis.

1.2 Ireland and International Assessment

Ireland performs disappointingly in the area of mathematics in international studies such as TIMSS (Third International Mathematics and Science Study) 1995 and PISA (Programme for International Student Assessment). This is in contrast to Irish literacy skills which are considered among the best in the world (Ireland placed sixth in literacy skills in PISA 2006). The current recession has had a significant impact on the Irish economy and it is essential that Irish mathematics graduates, and indeed all Irish citizens, have the necessary mathematical skills to compete with the best economies in the world in order to ensure economic recovery. To establish, promote and maintain a knowledge economy it is essential that Irish mathematical mediocrity is addressed as a matter of urgency.

1.3 The Irish Assessment Process

The Irish second level education system is sub-divided into the following levels:

- Junior Cycle: covering the first three years of second level education and assessed by a terminal examination, the Junior Certificate;
- Transition year: which is an optional year, directly after the Junior Certificate examination, offered by most schools. Transition year offers students the opportunity to engage in learning outside of the confines of assessment restrictions. There is no syllabus for transition year; and
- Senior Cycle: covering the final two years of second level education and assessed by a terminal examination. The Leaving Certificate (Established) is the mainstream assessment followed at the end of the primary senior cycle programme and involves the study of academic subjects. Some schools offer one of two alternatives to the mainstream programme; Leaving Certificate Vocational or Leaving Certificate Applied. The Leaving Certificate Vocational Programme combines the academic strengths of the Leaving Certificate (Established) programme

with vocational groupings of mainstream subjects and two link modules: ‘Preparation for the World of Work’ and ‘Enterprise Education’. The alternative Leaving Certificate Applied programme involves students in more practical and less academic subjects. The mathematics course followed in Leaving Certificate Applied is ‘Mathematical Applications’ and involves course work in addition to assessment by examination.

Currently the ‘Mathematical Applications’ course offered in the Leaving Certificate Applied programme involves a significant amount of realistic, authentic mathematical problem-solving scenarios in the style of the questions posed in the international assessments such as TIMSS and PISA. Interestingly this course does not satisfy university entry requirements and the standard of the mathematics is certainly at a less difficult level than those required in the mainstream senior cycle examinations. This would indicate that the Irish education system values realistic, authentic mathematical problems for students who are deemed ‘less academic’, while abstraction appears to be the valued question style for the established Junior Certificate and Leaving Certificate programmes.

In the Junior Certificate examinations in 2010 55,290 students sat the mathematics papers over three levels: higher, ordinary and foundation. Of these students, 8.31% sat the foundation level paper, 46.76% the ordinary level paper and 44.93% sat the higher level paper. Mathematics has a lower up-take at higher level than either of the other core subjects, Irish (48.6%) or English (68.43%). It also compares unfavourably with the up-take in Science (70%). Of the students who sat the Higher Level mathematics paper 47% scored either an A or a B, and 77.7% of students an A, B or C (www.examinations.ie).
The following table displays the Junior Certificate examination results from the first year of examination of the current course, 2003, to the latest examination results, 2010.

Year and Level	Total	A %	B %	C %	D %	E %	F %	NG %
2010(H)	24,840	15.5	31.5	30.7	18.2	3.5	0.6	0.1
2010(O)	25,853	12	32.2	29.9	18.5	5.3	1.9	0.2
2010(F)	4,597	18.2	35.1	28.6	15	2.5	0.5	0.0
2009(H)	23,592	16.7	31.7	29.2	18.6	3.4	0.5	0.0
2009(O)	25,930	11.7	33.4	29.6	17.9	5.4	1.8	0.1
2009(F)	5,186	19.0	32.6	28.4	15.8	3.5	0.6	0.1
2008(H)	23,634	16.6	31.6	31.6	17.4	2.3	0.4	0.0
2008(O)	26,384	12.3	36	28.5	16.4	4.9	1.8	0.2
2008(F)	5,140	18.4	37.5	27.7	12.9	2.9	0.6	0.0
2007(H)	23,804	17.7	29.9	28.1	19.2	4.3	0.8	0.0
2007(O)	27,094	9.3	32.5	31.4	20.2	5.1	1.4	0.1
2007(F)	5,641	16.1	32.6	30.7	16.6	3.3	0.6	0.1
2006(H)	24,205	18	32	28.7	17.2	3.3	0.6	0.0
2006(O)	26,820	13.3	36.7	27.9	16.1	4.4	1.5	0.1
2006(F)	5,941	17.1	37.8	29	13.7	2.1	0.4	0.0
2005(H)	23,388	14.5	30.3	30.8	20	3.7	0.6	0.0
2005(O)	26,518	11.8	32.5	28.7	18.8	5.9	2.1	0.2
2005(F)	5,908	18.0	31.5	27.4	17.7	4.4	1.0	0.0
2004(H)	23,006	16.1	28.4	28.9	20.3	5.2	1.1	0.1
2004(O)	26,347	10.1	34.3	30.8	17.7	5.2	1.9	0.2
2004(F)	6,584	16.4	40.4	29.1	12.0	1.8	0.3	0.0
2003(H)	23,734	17.2	33.6	28.6	17.0	3.1	0.5	0.0
2003(O)	27,383	9.2	31.0	31.3	20.8	5.8	1.8	0.1
2003(F)	7,324	15.4	37.8	29.5	13.6	3.2	0.4	0.0

Table 1: Junior Certificate Results 2010-2003: H=higher level, O=ordinary level, F=foundation level (www.examinations.ie)

As table 1 illustrates, the numbers of Irish students sitting the higher level Junior Certificate mathematics examination trails behind those who opt for the ordinary level paper. This pattern hold true year on year. When the numbers of candidates sitting the ordinary level examination are combined with those sitting the foundation level paper it is apparent that studying mathematics at the highest available level is not attracting students as it should.

1.4 Abstraction

An over-emphasis on abstraction is a possible failing of the Irish education system. Abstraction is the process of solving mathematics questions that are heavily reliant on mathematical skills without particular relevance to real-life knowledge or abilities. Abstract mathematics questions rely heavily on symbols and mathematical notation, with little obvious relation to real-life mathematical situations. In contrast to this other countries, particularly those who are considered more mathematically able, focus on the process of mathematisation. The mathematisation process utilises mathematical skills to solve real-life, authentic mathematical problems. It appears that students who study a syllabus that emphasises mathematisation develop the necessary skills to engage with unfamiliar mathematical problems.

A negative of the Irish mathematics syllabi is the over-emphasis on learning for the terminal examination, as opposed to learning mathematics for understanding and real-world implementation. As discovery and understanding in mathematics are difficult to examine and assess effectively in a terminal examination, these skills are often neglected completely in the teaching and learning that occurs on a daily basis. The Irish mathematics syllabi emphasises assessment techniques to the extent that discovery learning and the ability to apply mathematics effectively in work and life situations can be neglected in favour of examinable mathematical skills, such as procedural learning, which yield a high number of marks in the terminal assessment, the Leaving Certificate or the Junior Certificate. Irish educational achievement is determined by examination success. This is a situation that is not unique to mathematics, it also holds true in most other subjects on the curriculum.

In order to address the poor performance of Irish students in international mathematics assessments it is necessary to first consider factors that may be contributing to a less than stellar performance. It is possible that Irish students perform poorly in assessments such as PISA and TIMSS as they are unprepared for the style of questioning used by such tests.. PISA test questions emphasise real-life preparedness and focus on authentic, realistic questions (OECD, 2000; OECD, 2003; OECD, 2006; OECD,2009). TIMSS assessment questions are more traditional in style but still significantly different to those in the Junior Certificate mathematics examinations. TIMSS questions focus on the following performance areas: knowing; performing routine procedures; using complex procedures and solving problems (Beaton et al, 1996; Mullis et al, 2000; Mullis et al, 2005; Mullis et al, 2008). The author is particularly interested in the non-transfer of knowledge from the mathematics classroom to the real world. Is the poor performance of Irish students in international assessments due to a lack of ability to transfer abstract knowledge to real life problem solving, or a fundamental lack of mathematical knowledge or understanding to begin with? In the Irish examination system students can successfully answer a mathematics question in the examination with no understanding of the underlying concept. There is also the possibility that students may know and have some understanding of the concept but cannot apply this knowledge. If lack of ability to solve realistic, authentic mathematical problems is an issue that particularly affects Irish mathematics students then it is essential that it is addressed if Irish society is to truly engage in meaningful economic recovery by producing able graduates.

Is it possible that the Irish assessment system is a negative contributing factor to students' mathematical development? An over-emphasis on mathematical preparation for examination success can result in the neglect of components of the desired curriculum that are not easily examinable. This can be particularly true when assessment focuses solely on a terminal examination as is the case in the Irish situation. The Leaving Certificate was introduced for the first time in 1924 and it has not changed significantly since that time, despite adjustments within the individual curricula. The Junior Certificate is a very similar examination process for the junior cycle. An assessment process that has only changed marginally in almost ninety years must be considered a factor in the underachievement of Irish mathematics students. The Irish mathematics syllabi rely on the behaviourist method of teaching which in international mathematics education is considered dated and not as forward thinking as constructivist, cognitive teaching and learning methods. The behaviourist model is widely used in mathematics teaching in Irish schools as a result of the Irish assessment model which values reproduction as a key skill. A combined lack of resources and emphasis on the training of teachers on mathematical pedagogy also continues this focus on behaviourism. This results in a situation where generations of Irish teachers are teaching, and learning, mathematics in the same (behaviourist) way – many teachers teach as they themselves were taught.

If Ireland is to compete with other economies it is essential that Irish mathematical skills are of a high and comparable standard. There is much to be learned by considering the curriculum, teaching, learning and assessment methods in other societies, and by examining the success, and contributing factors, of high-achieving countries in international assessments. International comparative studies such as TIMSS (Third International Mathematical and Science Study) and PISA (Programme for International Student Assessment) are valuable in assessing Irish performance in an international context, and in comparing our methods of teaching and learning mathematics with other countries and other curricula. In considering other styles of teaching and learning in mathematics it may be valuable to consider the levels of abstraction versus realistic mathematics education (RME), or mathematisation, in countries that rank both higher and lower than Ireland in international assessments.

1.5 Real-life experience and the Irish Classroom

Conway and Sloane (2005) describe mathematics as a high-yield school subject and one that is simultaneously seen as increasingly important in education and as particularly difficult. They pose the interesting question as to whether it is important that one understands the mathematical task to achieve mastery or can this be done through practicing routine procedures (Conway & Sloane, 2005:78). Conway and Sloane also ask why students do not consider their own experience or common sense when dealing with real-world word problems in a school setting. This raises an issue with regard to the possibility of utilising school mathematics in a meaningful and masterful way in real-life scenarios: be they general life or work situations. Is a student's real-life experience of any value when it comes to mathematics education in the school setting? Is personal life-experience valued and relevant to an Irish student's mathematical school experience? If the real-world is thought to be completely irrelevant to mathematics education in the school setting it should be no surprise that students struggle to apply mathematics learned in the classroom to unfamiliar situations. This raises a very real problem if one considers the ideal scenario of school preparing students for future life and work experiences.

Greeno and Goldman (1998) propose that a student's true abilities are under-utilised by a lack of acknowledgment of their out of school experience, the dismissal of the valuable influence their peers may have, and an over reliance on compartmentalised learning and teaching activities in the classroom. It is possible that this is a shortcoming of the Irish education system: a regimented regime of teaching and learning in order to prepare students to succeed in the terminal examinations but with little value placed on life-experience, which in turn fails to prepare students for future work and life mathematical experiences.

1.6 Changes in the Irish Mathematics Curriculum

In order to consider Irish mathematics education it is important to have an awareness of how the current mathematics curriculum has developed since the introduction of a syllabus in 1878. The following sections outline the main changes to the mathematics curriculum from the introduction of the Intermediate Education Act in 1878 up to the introduction of the most recent syllabus 'Project Maths' in 2010. The author considers Irish mathematics education with regard to the following periods:

- Second level mathematics education 1878-1922;
- Second level mathematics education 1922-1960;
- Second level mathematics education 1969-1973;
- Second level mathematics education 1973-1989;
- Second level mathematics education 1989-2010; and
- The introduction of the 'Project Maths' curriculum 2010.

1.6.1. Second level mathematics education in Ireland (1878-1922)

The Intermediate Education Act in 1878 introduced a formality to Irish second-level education that had not previously existed. Prior to this, schools could decide what, and how, they wanted to teach on an independent basis. The Intermediate Education Board was set up in conjunction with this act and the board published the first formal syllabus in its 'Rules and Programme for Examinations'. A key role of the Intermediate Education Board was to run a public examinations system. The mathematics syllabus outlined in the 'Rules and Programme for Examinations' followed the format and context of the Oxford and Cambridge examinations. This was the foundation for all mathematics syllabi prior to 1922 (MacDonald, 2007).

The 1880 Mathematics syllabus had 3 grades:

- Junior Grade – students under 16
- Middle Grade – students under 17
- Senior Grade – students under 18.

The components in the mathematics syllabus for the Junior Grade were:

- Book-keeping
- Arithmetic;
- Algebra; and
- Euclidean geometry.

For the Middle Grade the mathematics syllabus comprised of the following content areas:

- Arithmetic
- Algebra; and
- Euclidean geometry.

For the Senior Grade the mathematical areas studied were:

- Algebra & arithmetic;
- Plane trigonometry;
- Elementary mechanics; and
- Euclidean geometry. (MacDonald, 2007).

All mathematics courses could be studied at pass or honours level. To gain a pass in mathematics two mathematical subjects were required but for girls arithmetic was considered to be worth two mathematics subjects. This was a stable syllabus with no significant changes or curriculum development during this period (MacDonald, 2007).

1.6.2. Second Level mathematics in Ireland 1922-1960

After the formation of the Free State and Northern Ireland the Intermediate Education Board was replaced by the Commission on Secondary Education.

This was formed in 1921. The Department of Education was set up in 1924. Second level education was non-compulsory during this period. The published mathematics syllabus (1924-1925) followed the syllabus format recommended by the Commission on Secondary Education and the earlier syllabi of the Intermediate Education Board. Recommendations made by the Commission on Secondary Education included:

- Mathematics was compulsory for school-goers at junior level;
- Practical mathematics was eliminated;
- Mathematics was made easier for girls than for boys;
- Junior, Middle and Senior Grades were replaced with the Junior Leaving Certificate and the Senior Leaving Certificate.

(MacDonald, 2007).

The new mathematics syllabus of the time had two options:

- Programme A: To include arithmetic, algebra and geometry. This was presented as a unified subject and treated as one in class terms.
- Programme B: To include geometry and trigonometry. Initially this was to be transitional with the aim that programme A would eventually become the core syllabus – in effect the opposite occurred.

(MacDonald, 2007).

The two-programme syllabus (1924-1925) was replaced with a one-programme syllabus (1934-1935). This course was offered at two levels for the intermediate certificate:

- - 'Elementary Mathematics (for girls only)'; and
- - 'Mathematics'.

Each of these syllabuses were divided into three sections:

- Arithmetic;
- Algebra; and
- Geometry.

The introduction of 'Elementary Mathematics (For girls only)' led to a reduction in the number of female students following the more difficult 'mathematics' course. Syllabus 1 (1942-1968 examinations) was considered a stable syllabus and no changes occurred in content during that time period. There were three, 2 hour examination papers: Arithmetic, Geometry and Algebra.

1.6.3. Intermediate Certificate Mathematics 1969-1973

The second Intermediate Certificate mathematics syllabus was first introduced in 1966 and was sent to the main teaching organisations including the IMTA for perusal. It was first examined in 1969. It was last examined in 1975. Three major changes occurred according to Mac Donald (2007):

- The new curriculum eliminated 'Elementary Mathematics (For girls only). Instead there was a lower and higher course for both male and female students. For the first time there was to be no academic distinction between boys and girls in mathematics;
- The new Intermediate Certificate mathematics syllabus was the first syllabus available for implementation in both secondary and vocational schools. Prior to this the Intermediate Certificate was available for secondary schools only;
- The mathematics syllabus was available for the additional students that were entering second-level as a result of free education and free transport. Due to these changes there was a huge influx in students attending post-primary education during this period.

In addition to the changes outlined above by MacDonald (2007) the 'Report of the Irish National Committee' (1976) also places emphasis on the major modernisation of mathematical content. These modernisations included the introduction of the popular, modern mathematics of the time:

- Sets and set algebra;
- Relations;
- Functions;
- Linear programming, and;
- Co-ordinate geometry.

The 'Report of the Irish National Committee' (1976) also notes that for the first time it was not necessary to pass mathematics in order to achieve a pass in one's Intermediate Certificate examination.

The 'Report of the Irish National Committee (1976) for SIMS (second international mathematics study)' discusses the issues and trends that were an impetus in designing a new curriculum in the 1960s. These included:

- The introduction of free education in 1967. This led to an increased demand for school places at second level. The numbers of students attending second level education doubled during the 1960s from 100,000 to 200,000 students. As a result of the increased level of participation in post-primary education teachers were dealing with a situation where there were a greater number of less academic students than they were previously accustomed to. This led to a greater need for mixed-ability teaching which increased the strain on teachers who were unaccustomed to catering for a broad academic range.
- A drop in the average age of students entering second-level led to a situation where teachers at this level were teaching much younger pupils for the first time. This, combined with the raising of the school-leaving age to 15 in 1972, necessitated teachers to teach mathematics within a broader age range.
- A wider range of subjects were introduced. Secondary schools were introducing technical, as well as academic subjects, for the first time. There was greater cohesion between secondary and vocational schools

in terms of subjects taught – this had not previously been the case. Prior to these changes there was a distinct difference in the ethos of secondary and vocational schools, and the subjects taught in both. Teachers in vocational schools were also not required to hold the same academic qualifications as those in secondary schools. Secondary school teachers were required to hold a higher diploma in education, in conjunction with their primary degree.

- During this period the first comprehensive schools were also introduced.
- In 1967 the Intermediate Certificate was introduced to vocational schools for the first time. Prior to this it was restricted to secondary school students.

(Report of the Irish National Committee, 1976).

The above trends initiated a period of great change in Irish education and necessitated a change in the mathematics syllabus as a result. Initially post-primary school teachers greeted the wave of 'new mathematics' with enthusiasm but for many reasons this enthusiasm was not sustained. A primary factor was possibly the fact that teachers were under considerable strain due to the increased participation in second level education. This had the knock-on effect of a broader range of subjects being taught in all second-level schools, a wider age range attending second-level and a greater ability range academically – all of these factors challenged teachers in a way they had previously not being subjected to (Report of the Irish National Committee, 1976).

There was a significant change in the assessment style also – a section of objective test questions was included in each paper. On the lower course 15 questions (30% of the marks) on each paper and on the higher course 20 questions (33.33% of the marks). Candidates were required to select one of four possible answers and write their selection of (a), (b), (c) or (d) in the box opposite the question number. Not all of the 'long' questions were of equal marks and the first of the long questions was compulsory so candidates were required to answer one question with a choice of 3 out of 6 others. The mark distribution on the lower course was 25, 20, 20, 20, 25, 25, 30 and on the higher course: 40, 40, 40, 40, 50, 50, 60.

The change in geometry was significant with Euclidean geometry gone and the entire geometry course restructured as a result. Set algebra was introduced in an effort to unify algebra and geometry. A problem resulted from the fact that teachers were unfamiliar with the 'new' geometry. This led to subsequent difficulties with geometry, and with the high level of choice in the long questions (3 out of 6) much of the geometry could be sidelined or avoided.

1.6.4. Intermediate Certificate Mathematics 1973-1989

Revisions were made to the 1966 syllabus in 1973. This syllabus change was examined for the first time in 1976, and remained the mathematics syllabus until the Intermediate Certificate examinations in the summer of 1989. The 'Report of the Irish National Committee' (1976) describe the revised curriculum of 1973 as merely consolidating the changes introduced in the previous curriculum and explain that no new mathematics were introduced except in geometry. The 1976 report also explains that:

'*an attempt was also made to revive an emphasis on calculation and algebraic manipulation, which, as mentioned earlier, were felt to have suffered some neglect in the first rush of enthusiasm for the modern subjects'* (Report of the Irish National Committee,1976:14).

MacDonald (2007) believes that this curriculum, introduced in 1973, was one of the most controversial curriculum moves in Irish mathematics due to the changes introduced in geometry. This further invoked a move from traditional mathematics towards the modern mathematics of the time. The higher level course was very challenging and the examination papers consisted of mathematical content at a very high level.

According to Oldham (2005) the changes implemented in 1973 strongly followed the modern mathematics of the 1960s. Teachers expressed dissatisfaction with the abstract nature of the ordinary level mathematics course and felt that the needs of weaker students were not adequately catered for by the lower-level intermediate syllabus.

The objectives for the Intermediate Certificate Programme suggested that the following skills should be acquired by studying the Intermediate Certificate mathematics course:

- Understanding, accuracy and efficiency;
- An understanding of mathematical facts and concepts;
- A logical understanding of the nature of proof;
- An ability to utilise mathematical skills to discover generalisations and applications;
- **An understanding and association of mathematics and their role in everyday life**;
- The development of appreciation, confidence, initiative and independence; and
- The development of skills that will lead to independent progress in mathematics.

(Report of the Irish National Committee,1976:18).

The Curriculum and Education Board (CEB) identified the following issues with the mathematics curriculum in 1986:

- A high failure rate among students studying the lower level mathematics course;
- The length of the higher level mathematics course;
- The high numbers of students opting for the lower level course, and a concern that many of the students taking the lower level course would be capable of succeeding in the higher-level course.

(Oldham, SMEC, 2005).

1.6.5. The 1987 Revision to the Intermediate Certificate

A major amendment to the Intermediate Certificate syllabus occurred as a result of the 1987 revision (first examined in the summer of 1990). This was the introduction of a third, lower level syllabus known as Syllabus C. This was introduced to facilitate the many students who were failing the 'lower level' course. The course names were changed from 'higher level' and 'lower level' to Syllabus A and Syllabus B respectively with the addition of a third, easier syllabus, Syllabus C. The need for Syllabus C was apparently justified when over 10,000 students sat the Syllabus C paper in the introductory Intermediate Certificate examinations in the summer of 1990. The ratio of the number of students sitting the three syllabi in the examination was in the ratio of 2:3:1 for Syllabus A, B and C respectively (Oldham, 2005). This suggests that the introduction of Syllabus C was a necessary amendment to the Intermediate Certificate mathematics course.

1.6.6. Junior Certificate Mathematics

The Intermediate Certificate syllabus was amended in 1989 and renamed the Junior Certificate programme. The 1987 syllabus changes were adopted for the Junior Certificate without any amendments other than a change in name. Syllabus A, B and C were renamed the higher, ordinary and foundation level syllabuses respectively.

The aims of the Junior Certificate syllabus were to:

- Contribute to the personal development of the students; and
- Help to provide them with the mathematics knowledge, skills and understanding needed for continuing their education, life and work (Junior Certificate Mathematics Syllabus, 2000).

1.7 The Introduction of Project Maths

'Project Maths' is the name given to the newest curriculum to be introduced in second-level schools in Ireland. It is described by the NCCA (National Council for the Curriculum and Assessment) as the most significant curriculum change in Irish mathematics education since the 1960s. 'Project Maths' will be implemented in phased increments, commencing in all Irish schools in September 2010, for Junior Certificate assessment in 2013 and for Leaving Certificate assessment in 2012. Prior to the whole school role out of 'Project Maths' in 2010, 24 schools in Ireland were involved in a pilot study which commenced in September 2008 (www.ncca.ie).

The emphasis in 'Project Maths' is to increase the relevance of the mathematics studied in the Irish classroom to real-life scenarios. The Junior Certificate curriculum states that:

'*In each strand, and at each syllabus level, emphasis should be placed on appropriate contexts and applications of mathematics so that learners can appreciate its relevance to current and future life*' (Department of Education and Skills, 2010:10).

'Project Maths' was designed as a response to the following concerns:

- Ireland's relatively poor performance in PISA (Programme for International Student Assessment);
- The relatively small number of students sitting the higher level mathematics examination in the Leaving Certificate;
- The difficulty that students show in coping with mathematics at third level;

- Evidence that students could not apply mathematics learned in the classroom except in the most practiced and familiar ways;
- Employer’s complaints that Irish students have good mathematical knowledge but poor understanding and problem solving skills; and
- A need to produce more able graduates for the knowledge economy.

(www.educationmatters.ie)

The aims of ‘Project Maths’ include:

- Enhancing the student learning experience;
- Developing student problem-solving skills;
- Greater levels of achievement in mathematics for students of all abilities;
- Making mathematics more meaningful for students and relatable to their own life experience;
- Allow students to appreciate how mathematics relates to real-life and work;
- Develop student skills in logical reasoning and argument; and
- Develop skills in applying mathematical knowledge to solve familiar and unfamiliar problems.

(www.ncca.ie)

The mathematics syllabus will change in phases, with the following topics introduced in sequential order:

1. Statistics and Probability,
2. Geometry and Trigonometry,
3. Number,

4. Algebra, and
5. Functions.

The Department of Education and Skills (2010) defines a good mathematician as someone who:

'*will be able to compute and then evaluate a calculation, follow logical arguments, generalise and justify conclusions, problem solve and apply mathematical concepts learned in a real life situation'* (Department of Education and Skills, 2010:6).

It is anticipated that the 'Project Maths' curriculum will enable the development of skills as students' progress through the curriculum. These skills will be linked continuously to skills developed at earlier stages in the students' mathematical career. These skills include:

- Application of mathematical skills;
- Problem-solving abilities;
- Integrating and connecting mathematical concepts;
- Reasoning;
- Implementing; and
- Understanding and Recalling.

(Department of Education and Science, 2010:10)

Differentiation is considered to be essential to the new 'Project Maths' curriculum. An appreciation of the fact that all students learn in different ways, and at different work rates, is key to its success. For Foundation Level students this involves studying large elements of the ordinary level course. Strands 1 and 2 'Statistics and Probability' and 'Geometry and Trigonometry' have the same learning outcomes at both ordinary and foundation level. This will allow foundation level, Junior Certificate students to study ordinary Level in the Leaving Certificate if they wish (Department of Education and Science, 2010). This upward flow of movement is a new development in Irish mathematics and is facilitated by the introduction of the 'Project Maths' curriculum. The common introductory course followed in first year also ensures that all students have one year of covering the same strands at the same rate. This allows students to adjust to the vigour of secondary school and catch up on any mathematics they have missed in primary school, for whatever reason. It also allows for easier upward movement at a later stage if the student is demonstrating the necessary mathematical ability.

There is a sense of apprehension among Irish mathematics teachers regarding the implementation of Project Maths. Teachers are hesitant to implement a new syllabus that they do not fully understand themselves and many feel that it is being rushed out without the necessary consultation with the mathematics teachers who will be implementing these changes in the classroom. Lubienski (2011) highlights the concern of Irish teachers she interviewed regarding the implementation of Project Maths and refers to the fact that Irish teachers care enough about their profession and the students under their care to engage in heated arguments regarding the syllabus changes. Lubienski raises issues that may impede the effective implementation of Project Maths in Ireland including:

- The examination system and the pervasiveness of exams in the classroom;
- Textbooks: Irish teachers are used to using a textbook and are nervous about working without a textbook to refer to or rely on;
- The challenge of teacher change: and the need for more teacher support as they begin their Project Maths journey.

1.7.1. Project Maths and the Junior Certificate

‘Project Maths’ anticipates examining the new syllabus for the first time in all schools in Ireland in the Junior Certificate 2013. Strands one and two, ‘Statistics and Probability’ and ‘Geometry and Trigonometry’ will be introduced in all Irish second level schools in September 2010 for examination in 2013. Strands 3 and 4, ‘Number’ and ‘Algebra’ will be introduced to first years in 2011 for Junior Certificate examination in 2014. Strand 5 will be the final strand to be implemented and this will happen for all first years in September 2012 for examination in the Junior Certificate in 2015 (www.ncca.ie).

‘Project Maths’ promotes cumulative mathematical learning, with students developing a hierarchy of knowledge. The Junior Certificate will be firmly linked to mathematics previously studied by students in early childhood mathematics and in the mathematics studied at primary level. The new Junior Certificate and Leaving Certificate curricula are also being developed simultaneously, resulting in a strong relationship being created between the mathematics strands at both junior and senior cycle. In this way it is anticipated that students will develop an understanding of the connectivity between all levels of mathematics, from the most basic mathematical concepts in early childhood mathematics to the most complicated mathematics students will encounter at Leaving Certificate level. Problem solving is a key component of mathematical learning at all stages in the new curriculum. For the first time in Irish mathematics education at second level the ‘Project Maths’ curriculum will promote mathematics in relation to other subjects in the second-level curriculum. Connections to other subjects will foster an appreciation of mathematics as a subject not learned in isolation (Department of Education and Science, 2010).

Connections with other subjects in the Junior Certificate include:

- Science: linking quantitative learning methods in both subjects;
- Technical Graphics: recognising the geometric principles linking mathematics with the 2D and 3D drawings in technical graphics;

- Geography: ratio is used in both subjects;
- Home Economics: money judgments and budgeting in home economics necessitate the use of mathematical skills;
- Business Studies: mathematics is used in budgeting, consumer education and reporting on accounts;
- Music: recognising the historical links and practical relationship between mathematics and music. Pythagoras uncovered mathematical relationships in music as early as the 5th century B.C.; and
- Art: geometric skills are utilised in art.

(Department of Education and Science, 2010:7).

Changes to teaching and learning of Junior Certificate mathematics as a result of the introduction of the ‘Project Maths’ curriculum include the following:

- A more investigative approach to mathematics learning;
- A bridging framework is currently being developed which aims to link primary school mathematics strands with the Junior Certificate syllabus;
- A common introductory course for study in first year mathematics will be introduced. This will allow students to delay their choice of syllabus (subject level) until second year;
- A targeted uptake of 60% of students sitting the higher level Junior Certificate paper, with an aim of this following through and increasing the uptake of higher level mathematics at Leaving Certificate; and
- Initially the foundation level syllabus will be offered at both Junior and Leaving Certificate level, but it is anticipated that the new ‘Project Maths´ curriculum may negate the need for this level of study. foundation level will be kept under review.

(www.ncca.ie/overview)

The aims of the new Junior Certificate mathematics course, as outlined in the 'Project Maths' curriculum, include:

- The development of mathematical knowledge, skills and education that are needed for future education and real-life experience: in both one's personal life and in the world of work;
- The development of the necessary mathematical skills to deal with context and applications in a competent manner. Problem-solving skills will also be developed; and
- To foster a positive attitude to mathematics in all students.

(Department of Education and Skills, 2010)

The objectives of the 'Project Maths' Junior Certificate course include:

- To develop the ability to recall mathematical facts;
- To foster the ability to 'know how' (instrumental understanding) and physical co-ordination skills that are necessary to mathematical learning;
- To develop relational understanding in students so that they 'know why';
- To develop students' ability to apply their mathematical knowledge competently when faced with problems in familiar and unfamiliar situations;
- To improve students analytical skills;
- To encourage students to think creatively; and

- To foster an appreciation of mathematics in students of all abilities.

(Department of Education and Skills, 2010)

1.8 Conclusion

In this chapter the following have been noted:

- Mathematics is a key component of the second-level syllabus in Irish schools;
- Irish students perform relatively poorly in international studies;
- The percentage of Irish, second level students who study mathematics at higher level is low;
- Abstraction is a key component of Irish mathematics education while mathematisation is not. This may be a possible failing of the Irish system;
- Mathematics teaching and learning in Ireland is grounded in the theory of behaviourism and has not embraced real life experiences and situations;
- The behaviourist style of teaching and learning has been a key feature of the various mathematics courses prescribed for junior cycle students in Ireland from the initial 1880 mathematics course, through the various syllabus changes from 1924, to the current 1987 revision still being taught in schools;
- The introduction of the new ‘Project Maths’ syllabus which will be fully implemented by 2013, for the first full Junior Certificate examination in 2015, is designed to address the perceived failing of the past and prepare the students of the future to assist, and participate fully, in the knowledge economy.

The following chapters consider mathematics education, and the issues pertaining to it, both at a national and international level.

2.0 Chapter 2: Issues in Mathematics Education

2.1 Introduction

This chapter considers the major issues in mathematics education and the literature surrounding these issues. The author provides a review of the literature pertinent to the current issues in mathematics education. The topics considered include the learning theories supporting the teaching and learning of mathematics, realistic mathematics education, the impact of gender and ethnicity on mathematics education and social factors affecting mathematical performance.

2.2 Learning Theories in Mathematics Education

The philosophies and epistemologies supporting the teaching and learning of mathematics education focus on the concept of learning theories. The following section considers the major learning theories in mathematics, and the influences these theories have had, and continue to have, on Irish mathematics education.

2.2.1. Absolutist versus Relativist Perspectives

In the most general of divisions it can be said that there are two primary concepts underpinning mathematics education and the underlying theories. Lyons et al (2003) describe these two perspectives as the Absolutist and Relativist epistemological approaches to mathematics. The absolutist theory of learning draws on behaviourism, positivism and objectivism, and defines mathematics as an *'objective, value-free, logical, consistent and powerful knowledge-based discipline which students must accept, understand and manipulate'* (Burton, 1994).

Absolutism promotes a didactic style of teaching and relies on the principles of rote-learning, repetition and reinforcement. Teachers present the information to the students and the students are taught how to produce the correct answers, without necessarily having to understand the underlying concepts. The relativist perspective draws on constructivism and the cognitive theories of learning. Under the relativist perspective *'all knowledge is regarded as culturally, historically and politically situated'* (Lyons et al, 2003:3).

Irish mathematics education focuses on the absolutist perspective, with little social or cultural value traditionally given to mathematics, in the Irish classroom. Mathematics is seen as abstract and devoid of social context. Lyons et al (2003) describe the Irish mathematics curriculum as remaining relatively uninfluenced by international advances in mathematics, with little regard or interest in following current moves and advancements in mathematics education, and describe the Irish mathematics education system, prior to 2003, as being located in the absolutist tradition. This is currently being challenged by the introduction of the new 'Project Maths' curriculum, as outlined in the Introduction, which is introducing the concept of placing value on each student's social experience in a way that has not been attempted previously in the Irish mathematics education system.

Tims (1994) explains that the Objectivist theory traditionally associated with mathematics tends to favour male over female students. Society traditionally socialised females towards connectedness and focusing on others, whereas males are socialised towards independence. As objectivism in education is concerned with separating the learner from their social environment in a manner that tends towards the notion of separate knowledge, the focus is on the male notion of separated learning and independence. Whereas female learners perform at a higher level when knowledge is connected to what they already know and the social environment and context in which it is placed, male students perform successfully in a context-free scenario. Tims (1994) explains that when this concept is applied to assessment in mathematics, the socialisation of female learners leads to an attempt to connect with both the assessor and the assessment through the context, whereas male students perform successfully in an abstract environment. The abstract nature of the current Junior and Leaving Certificate mathematics examinations would therefore suggest that male students are favoured by the context free questions.

It is interesting to note that Irish female performance in the Junior Certificate mathematics examination follows a different pattern than the other two core subjects, Irish and English. In 2011, male students performed better than female students in higher level mathematics at the highest level – 17.8% of male students sitting the paper obtained an A grade compared to 17.0% of females. This is significantly different to the pattern in English where 13.8% of higher level females obtained an A compared to 7.4% of male students, and Irish where 12.4% of female higher level candidates obtained an A compared to 7.7% of male students (www.examinations.ie). This pattern hold true for all other years of the established Junior Certificate (male higher level mathematics students out-perform females at the A grade level in every year recorded).

PISA mathematics results favour male students for most countries, including Ireland. This is interesting as the assessment type is authentic and realistic which the above research would suggest favours female students. In the PISA 2009 mathematics assessment male students outperformed females in 35 countries, with female students performing better than males in just 5 (OECD, 2010). Male students report higher self-efficacy and lower anxiety about mathematics than female students which perhaps can provide a justification for their performance in an assessment that is unfamiliar in style – it stands to reason that students who are more confident in their mathematical ability will perform better under pressure. This may explain why female students are not reaping the benefit of contextual questions. Students who identify with having high self-efficacy in PISA mathematics achieve a significantly higher mean score than students with low self-efficacy (Close and Shiel, 2009). Female students reporting lower levels of self-efficacy would probably benefit from the opportunity to familiarise themselves with authentic problem solving before the stressful situation provided by an assessment such as PISA. The opportunity to study and practice predictable examination questions for the

Junior Certificate mathematics examination possibly goes a significant way towards narrowing the achievement gap between male and female students in terms of Junior Certificate performance while lack of practice of PISA style questions may impede female 15 years olds who tend to be more focused on their studies than their male colleagues.

2.2.2. Behaviourist Learning Theory

Ryan and Williams (2007) explain that a behaviourist defines learning as a change in a person’s behaviour, and for this reason behaviourist learning theory focuses on the extrinsic and the visible. Behaviourist theory was initially guided by empirical studies, first tested on animals and then on people. These empirical studies demonstrated that behavioural reinforcement can accelerate learning. In behaviourism the focus is on reinforcing behaviour to obtain the results you want. Positive reinforcement is considered to be very effective, with sporadic reinforcement also thought to have some value as long as behaviour is established. The focus on the extrinsic elements results in the neglect of intrinsic factors such as pride or personal enjoyment in solving a mathematical problem. If the learner is to be in control of his/her own learning then rewards must be intrinsic to the activity itself. The focus in behaviourism lies with the teacher being in full control of the learning rather than the student. The role of the student is to master the bite-size objectives fed to him/her by the teacher (Ryan and Williams, 2007:154-156).

Behaviourists claim that education is an observable change in behavior (Eisenberg, 1975). The emphasis in behaviourism is on observable progress, which ideally is permanent. There are three basic concepts for learning in the behaviourist traditions:

1. Tasks are broken down into small and manageable components, to their simplest possible form;
2. The basics are taught first, with tasks forming hierarchical steps going from the basic to the most complex;

3. Observable progress is reinforced and rewarded.

Criticisms of behaviourism include the fact that it does not individualise instruction but may in fact hinder it; there is an assumption of a learning hierarchy that does not always exist in reality; and only observable changes in behaviour are measurable – intrinsic, heuristic changes do not exist (Eisenberg, 1975).

Lack of authenticity is also acknowledged as a downfall of the behaviourist movement as the focus is on teaching the basics by decomposing the task into parts; this negates its relativity to real-life, authentic tasks somewhat as students are not introduced to the problem as a whole. The student's social and cultural background is not considered relevant in behaviourism and this was one of its appeals initially, as it was believed students would benefit from a consistent programme of learning for all students, regardless of their background (Conway & Sloane, 2005).

Behaviourist teaching focuses on the teaching of basic tasks first and then moves on to considering their more complicated components, assuming that students can apply the basic skills to the most complicated tasks. Assessment is an important aspect of behaviourism. Each component is assessed individually, with the assumption being that once lower-order tasks are mastered the skills learned in these tasks can then be applied to more complicated, higher-order skills. Each skill is assessed prior to moving on to a more difficult task. Conway and Sloane (2005) also discuss how behaviourism emphasises the development of self-regulation and self-instruction. Self-regulation, as defined by behaviourist theory, is

'self-instruction with attention to identifying reinforcements that will strengthen desired behaviour' (Conway & Sloane, 2005:85).

The underlying assumption in behaviourism is that once a student is familiar with the basic, underlying mathematics they will then have an innate ability to apply these fundamental skills to solve more complex, higher-order mathematical problems. There is an assumption of an innate ability to transfer knowledge from one mathematical context to another. The behaviourist movement promotes direct teaching, involving exposition, where the teacher conveys information in a hierarchical manner, focusing on the basics before moving onto higher-order tasks or skills. Self-regulation is promoted, with the standard to be met agreed with the teacher in advance, and reinforcement by the teacher throughout, particularly on completion of the task, to reinforce positive behaviour. Tasks are assessed before progressing to more complicated, related tasks. Mathematics problems are broken-down into component parts by the teacher which leaves little room for the analysis of the problem as a whole. This may lead to difficulty in transferring mathematical knowledge from the classroom to real-life situations as students may not have developed the necessary skills to recognise mathematical tasks in terms of complete, authentic problems which can exist in a non-classroom environment. This may also leave students ill-prepared for non-hierarchical type mathematical assessment.

The Irish tradition of mathematics learning embraces many aspects of the behaviourist tradition, with a clear emphasis on observable progress as demonstrated by the complete reliance on the Junior Certificate and Leaving Certificate examinations as a means of determining and regulating educational progress in mathematics and other subjects. The tradition of terminal examinations in Ireland, and the associated points procedure for third-level entry, values ‘observable progress’ over all other. Societal factors are not considered, and little value, if any, is given to real-life experience in the mathematics examination. As a result Irish society encounters a culture where students learn for the examination rather than for understanding, and teachers teach to the examination rather than educating students in the true sense of the word.

2.2.3. Constructivism

Constructivism is an assumption that *'what we know is a direct reflection of what we can perceive in the physical world'* (Resnick et al, 1991). Leonard (2007) describes the constructivist movement as one in which it is assumed that to build new knowledge individuals must connect it to what they already know. Delaney as in Gates, 2001, describes constructivism as a process by which the learner must be active in their own learning, and construct meaning for oneself, rather than being a passive participant in the learning process. Nickson (2000) refers to a central theme of the constructivist approach being

'*an acceptance of the fact that the reality of one individual is different from that of another and that individuals construct their own mental representations of situations, events, and conceptual structures'* (Nickson, 2000:4).

Leonard (2007) describes the constructivist approach as one which benefits all students, regardless of societal variables, in acknowledging the importance of culture and life experience. Leonard believes this in turn leads to more successful problem solving efforts as students' learn to incorporate their experience outside of the classroom in their mathematical activity and the development of problem solving skills.

Constructivism is a theory of learning where the child is considered the agent for learning rather than the teacher (as proposed under behaviourism). The intrinsic process is considered as being important, in addition to valuing the extrinsic and visible characteristics of learning. The emphasis is on active participation rather than passive learning. Constructivist theory is based on the work of the Swiss psychologist Jean Piaget (1896-1980). Anghileri (2005) describes Piaget's constructivist theory as follows:

- Children are actively involved in constructing knowledge from their own experience and learn through an active process of self-discovery;
- Students make mental connections in an active manner;
- The learning outcome for any one child can vary depending on the individual's framework for understanding; and
- Students create and observe situations for themselves rather than being told facts and figures.

Jaworski (1994) explains that Piaget was of the opinion that introducing a mathematical concept before the learner was already in possession of some element of self-discovered knowledge regarding the new concept damages the student's learning process. Piaget held the belief that each student should be given the opportunity to create the concept for themselves, and hence develop a greater understanding of the task at hand.

2.2.4. Criticisms of Constructivism

Ryan and Williams (2007) explain that Piaget's constructivist theory was not without its critics. Criticisms include the fact that children can reason in more advanced ways than Piaget believed possible, as long as the context is meaningful for them. This underestimation of what children know and what they can do is believed to have arisen from difficulties adult researchers may have had in communicating with children in clinical interviews. It is also noted by Ryan and Williams (2007) that Piaget only considered cognitive learning theories in relation to children from socially privileged groups, and as a result he did not take into account varying social influences and the affect that they may have on learning. Anghileri (2005) also refers to criticisms regarding Piaget's proposition that there is a close association between age and 'logico-mathematical thinking' and this may lead to restrictions for students in terms of what it is believed that they are capable of. Jaworski (1994) expresses a belief that Piaget's theory of constructivism may in fact negate the value placed on the mathematics teacher as the emphasis is so firmly focused on the individual and the individual as agent for learning. Piaget very much focused on the individual as a learner rather than as a cultural participant and ignored the social and cultural implications the tasks may have on a learner's thinking.

2.2.5. Bruner's Constructivist Theory (Cognitive Learning Theory)

Jerome Bruner (1915-present) is an American psychologist who further developed Piaget's constructivist theory. Bruner's constructivist theory is commonly known as cognitive learning theory. Anghileri (2005) explains that Bruner also shared the view that children are active participants in the learning process, but he placed more emphasis on language, instruction and communication. This led to the introduction in constructivist theory that the teacher, and all adults, have an important role to play in prompting the correct actions and responses in order for children to turn their creative thoughts into meaningful, symbolic outcomes.

2.2.6. Critical Mathematics Education

Critical Mathematics Education is an educational philosophy that was developed by Skovsmose. Skovsmose expressed a belief that

'if educational practice and research are to be critical, they must address conflicts and crises in society' (Skovsmose, 1994:22).

Skovsmose promoted mathematics education as a means of developing 'critical citizenship' and believed that mathematical knowledge brought power to the individual in an increasingly technological society. In commenting on

Skovsmose's Critical Mathematics Education, Nickson (2000), describes this philosophy as being similar to that of Realistic Mathematics Education (RME), but with a stronger emphasis on the reflective. Nickson (2000) describes how Critical Mathematics Education not only asks students to reflect on the mathematics involved, but also to spend time reflecting on the following:

1. The social issues that arise in the situation which forms the context of the mathematics project;
2. How the judgments they have made have been informed by the mathematics they have engaged in; and
3. The consequences of their actions (Nickson, 2000:8).

2.2.7. Apprenticeship and Situated Learning

Cognitively Guided Instruction (CGI) is a program which encourages students to use informal or invented problem solving strategies (Leonard, 2007:24). CGI involves the use of methods such as modeling (the use of fingers, counters etc.), counting strategies and number facts to solve realistic word problems. Nickson (2000) defines Cognitively Guided Instruction as

'*a project in which a particular approach is embedded that focuses on the identification of the strategies children use as opposed to an interpretation of how children learn'* (Nickson, 2000:5).

Nickson explains that CGI values the importance of children's informal knowledge, and utilises this knowledge to provide teachers with this knowledge as a framework for their teaching. Nickson describes the following four underlying assumptions of the Cognitively Guided Instruction program:

1. Children construct their own mathematical knowledge;
2. Mathematics teaching should facilitate children's construction of knowledge;

3. The development of children's mathematical ideas should provide the basis for sequencing topics of instruction, and;
4. All mathematical concepts and skills should be taught in relation to, and with the prioritising of, understanding and problem-solving (Behr et al., 1992: 325).

2.2.8. Cognitive Apprenticeship and Situated Learning

The Cognitive Apprenticeship model, developed by Brown et al. (1989) considers knowledge as

'*situated, being in part a product of the activity, the context, and culture in which it is developed and used'* (Brown, 1989:32).

In the Cognitive Apprenticeship model it is the role of the teacher to facilitate and encourage students to make connections between what they are learning in class and their personal, informal, life-experience. Nickson explains that

'*a major implication of this perspective for learning and teaching mathematics is that to be meaningful, new mathematical knowledge and skills are most effectively learned in situations where they are applied'* (Nickson, 2000:6).

Situated Cognition considers one's lack of ability to transfer knowledge from a particular situation to another. The basis of the theory behind Situated Learning is the difficulty an individual may have in performing a task, that they have mastered in one particular setting, in an unfamiliar situation. Research undertaken by Nunes, Schleimann and Carraher (1993) demonstrates the difficulty street children in Brazil, working as street vendors, had in transferring the skills they had developed in selling their goods, to the mathematical concepts needed for problem-solving in the mathematics classroom. It was found that these children had developed complex algorithms in the process of their work as vendors but when presented with similar tasks in symbolic form in the classroom they were unable to perform at the same level (Conway & Sloane, 2005; Leonard, 2007).

Conway and Sloane (2005) argue that the inability of these children to utilise their real-life skills in the mathematics classroom is due to several factors, including:

1. Underutilisation of the students out-of-school mathematical knowledge by the school and teachers;
2. The children had developed complicated algorithms to meet their needs as street vendors, independent of in-school direction; and
3. Basic computation was learned within more complex problem solving activities as it was needed.

Leonard (2007) concedes that perhaps the students inability to transfer the skills in which they are proficient as street vendors to the classroom is due to the

'language, written text and symbols, or the abstract nature of the assessments'

(Leonard, 2007:24).

Lave and Wenger (1991) propose that learning does not merely take place in an individual's mind, but rather that the learning process takes place in a participation framework, and they consider the kind of social engagements necessary for learning to take place. The authors propose that learning could be viewed as a special type of social practice, with the participation frame designated Legitimate Peripheral Participation (LPP). LPP is the process under which one acquires the skill to perform by actually engaging in the process. Under Legitimate Peripheral Participation one participates in the practice of an expert, but only to a limited degree, and with limited responsibility. Noyes (2007) explains that research on the use of mathematics by adults workers in the workplace determines that there can be significant mathematical content in real-life contexts: he determines this as situated mathematics. Resnick (1991) argues that

'the social context in which cognitive activity takes place is an integral part of that activity'.

2.2.9. Cognitive Learning

Conway and Sloane (2005) discuss the importance of cognitive learning in relation to current trends in mathematics education. They focus on four areas of cognitive learning that have a particular relevance to mathematics teaching and learning today:

1. The idea of active learning;
2. Cognitive challenge and its place in all forms of mathematics teaching and learning;
3. The introduction of competent problem solving in mathematics; and
4. The production of literature proposing the teaching of self-regulated learning.

The focus of cognitive learning is on the students being active in their own learning process.

'Knowledge is made as learners engage with and experience the world' (Conway & Sloane, 2005:87).

Cognitive learning acknowledges that we are not asocial beings and that prior life experience can effect an individual's learning within the classroom. Conway and Sloane (2005) believe that a large element of the appeal of cognitive learning, at an international level, is a desire to move away from a didactic form of teaching. Cognitive learning also provides a model for competent problem solving in mathematics, and methods by which to teach problem solving. Cognitive methods also emphasise the use of 'think-sheets' and 'think alouds' as prompts for students in the process of problem solving. Conway and Sloane (2005) discuss the fact that previous research has shown that teachers rarely use such prompts in the mathematics classroom, a fact they refer to as troubling as such methods have been shown to assist lower-achieving students. This leads to questions regarding the position of a lower-achieving student in the typical mathematics classroom: is the mathematical progress of each individual student valued equally? Does an over-emphasis on examination success detract from educating and encouraging mathematical achievement at all levels?

Dewey (1933), a proponent of active learning, states that '*the complete domination of instruction by rehearsing second-hand information, by memorising for the sake of producing correct replies at the proper time'* (Dewey, 1933:201) is a negative aspect of didactic learning and leads to bad practice by students. Assessment in cognitive learning focuses mainly on authentic problem solving and the consideration of the mathematics problem at hand as a whole. Cognitive challenge is a significant focus of cognitive learning with Bloom's Taxonomy of the Cognitive Domain (Bloom et al, 1956) recognising six levels of increasing cognitive demand. These cognitive levels, in increasing order, are:

1. Knowledge;
2. Comprehension;
3. Application;
4. Analysis;
5. Synthesis; and

6. Evaluation.

The revised Taxonomy (Anderson, 2002) has six levels of cognitive process, slightly altered from the original:

1. Remember;
2. Understand;
3. Apply;
4. Analyse;
5. Evaluate; and
6. Create.

Leonard (2007) discusses how teachers with a cognitively based approach use word problems for introducing addition and subtraction. The author believes that this realistic approach, and the grounding of skills in realistic stories, benefits the students by making mathematics relevant to their lives. Teachers without a cognitively based approach place more emphasis on memorising facts and low-level skills, which require little effort or thinking, and do not encourage active learning.

2.2.10. Self-Regulated Learning

The role of educators should be to prepare students for success in life and in future academic endeavors. A key component of this success is the ability of students to regulate their own learning. Self-Regulated Learning is a response to this need and is positioned within the theory of Cognitive Learning which embraces students being active within their own learning process. Self-regulated learners do not require a large amount of external regulation; they are self-motivated and internally regulate their performance and progress; and they appear to be confident regarding what they know and feel in relation to study

(Boekaerts, 1997). Zimmerman (1990) describes self-regulated learners as those who approach learning with confidence, diligence, and resourcefulness.

It is essential that teachers and educators have a clear understanding of the value of self-regulated learning so that they can endeavor to promote this type of learning through their teaching. Zimmerman (1990) stresses the importance of students assuming personal responsibility for their own learning in order to achieve long-term success.

2.2.11. Socio-Cultural Theories

Socio-cultural theories are consistent with the constructivist view of education. Vygotsky (1978) defined socio-cultural theories as those that assert that the mind originates through the culture that the person inhabits. This acknowledgment of one's culture as having influence is a move away from the behaviourist and cognitive views of an asocial learner. Socio-cultural theories focus on the notion of communities of learners

'*which provide not only opportunities for cognitive development but also the development of students identities as numerate members of knowledge-building communities'* (Conway & Sloane, 2005).

Brown (1994) outlines a set of principles that underlie this idea of a 'community of learners':

1. Learning is active, strategic, self-motivated and purposeful;
2. The classroom can be a setting for many areas of development with structured support from the teacher, peer-group and technological assistance;
3. Individual differences are acknowledged and legitimised;
4. The development of communities of discourse and practice; and

5. The teaching of deep conceptual content that is aware of students knowledge and skills in particular subject areas (Conway & Sloane, 2005).

Brown (1997) used the five principles above to foster the emergence of communities of learners in classroom settings. This led to the construction of the concept of 'Fostering a Community of Learners' (FCL). Brown believed that:

1. It was essential to encourage and develop the capacity of students to 'think about thinking' or metacognition. Students, she believed, should be taught methods of self-monitoring;
2. Brown believed in teaching students at the upper levels of their perceived competence;
3. FLCs (Fostering a Community of Learners), according to Brown, should place value on the learners cultural perspective;
4. Learners should be encouraged to engage in an active discourse around mathematics as Brown believed that higher-level thinking is an internalised dialogue; and
5. Teaching deep conceptual content to students, without underestimating their mathematical ability (Conway & Sloane, 2005).

The socio-cultural perspective of teaching and learning in mathematics education, according to Conway and Sloane (2005), places emphasis on the importance of teaching basic skills, but only within the context of relevant, authentic, realistic problems. This is in complete opposition to the behaviourist method of teaching mathematics which was hierarchical in nature, with an insistence that the basics are taught before anything more complicated can be considered. Behaviourist theory dictates that mathematics problems are broken down into their component parts resulting in it becoming very difficult for the learner to see the mathematics problem as a whole, therefore losing relatability and authenticity.

The socio-cultural perspective focuses on active learning, and a move away from passive learning and didactic teaching. The emphasis is on teaching and learning in authentic situations so that the students are learning skills which in turn will make them valuable members of society. There is a focus on communication and working together to find solutions: both essential skills for problem solving in the workforce, and in everyday life. This type of teaching and learning has most in common with the constructivist point of view, but moves away from the idea of the student working as an individual, and more towards the idea of a student working within, and as part of, a community.

2.3 The Realistic Mathematics Education Movement

The author believes that an understanding of the concept of 'Realistic Mathematics Education' is important if an honest critique of Irish mathematics education is to be undertaken. The following section considers techniques used to encourage relatability between mathematics in the classroom and mathematics as an essential human activity.

2.3.1. Realistic Mathematics Education (RME)

Hans Freudenthal was the promoter of the Realistic Mathematics Education (RME) movement. He developed RME in response to his strong opposition to mathematical advancement in education during the 1950s which strongly emphasised abstraction. Freudenthal felt that the abstract nature of problems used in mathematics education was a weakness as

'*It is wasted on individuals who are not able to avail themselves of this flexibility'*(Freudenthal, 1968).

Freudenthal stressed the social and cultural role of mathematics and its valuable place in society. Nickson (2000) describes Freudenthal's Realistic Mathematics Education as a process of 'guided reinvention'. The Realistic Mathematics Education (RME) movement is described by Van den Heuvel-Panhuizen (1996) as the design of assessment and learning opportunities that are genuine problems and open to mathematisation. Van den Heuvel-Panhuizen encourages the use of practical application problems as part of the RME movement, as opposed to artificial word story problems which he felt were often boring and unappealing. He emphasises the notion that true problems rarely have only one solution and it is important that RME problems follow this pattern with multiple possibilities for correct answers. Van den Heuvel-Panhuizen (1996) also determines that in solving a realistic problem it is important that students place themselves in the context given and draw on their own life experience. Van den Heuvel-Panhuizen encouraged the use of real-life tools including graphical information, newspaper clippings etc. to solve the given problem. Conway and Sloane (2005) explain that a significant RME strategy is the presentation of mathematics problems, in a real world setting, necessitating skills that the students have not already been taught therefore enabling the students to work independently to develop the necessary techniques to solve the problem.

Freudenthal emphasised the distinction between vertical and horizontal mathematisation. Freudenthal (1991) described vertical mathematisation as the process in which '*symbols are shaped, reshaped, and manipulated, mechanically, comprehendingly, reflectingly*'.

Freudenthal placed curricular emphasis on horizontal and vertical understanding and provides a four-level framework for doing so: mechanistic mathematics, which focuses on routine mathematical drills; empiricist, which emphasises horizontal mathematising, moving from real-life situations to working with symbols; structuralist, which is vertical mathematising and focuses on symbolic manipulation, and; realistic, which necessitates both horizontal and vertical learning (Conway & Sloane, 2005). This four-level framework emphasises the dual nature of mathematics. Conway and Sloane (2005) summarised Realistic Mathematics Education in relation to reality and the related notions of rich pedagogical context, the mathematising cycle, and the four-level framework, shown above, for classifying curricular emphasis in mathematics education. Conway and Sloane emphasise that RME focuses on mathematics for all, and does not determine that either horizontal or vertical learning are more suitable to particular students or types of learners. RME emphasises the dual nature of mathematics and encourages mathematics, and the study of mathematics, as a human activity for all.

Streefland (1991) determines five educational principles implicit in Realistic Mathematics Education as follows:

1. Reality is the source of concept formation;
2. Pupils are given the opportunity to be actively involved in constructing the problem;
3. The learning process is interactive with students discussing the inherent mathematics and collaborating with other students where necessary;
4. Both vertical and horizontal mathematisation can take place; and
5. The students use various mathematical tools to describe what they have discovered for themselves.

2.3.2. Mathematics with Real World Connections

Leonard (2007) found, in carrying out her own research in mathematics education, that using real-world connections to involve students in mathematics projects improved their interest and motivation for learning mathematics. This would indicate that there is significant value in increasing the relatability of mathematical tasks for students. By including the everyday and students’ real-life experience in the mathematics classroom, one would expect that the converse is also true: that students would develop an awareness of how mathematics can influence their reality.

Sethole et al (in Clarke et al, 2006) argue that the inclusion of the everyday in the teaching and learning of mathematics will assist in the promotion of mathematics as a discipline which is central to human activity and important for day-to-day life. The author explains that some tension can arise when considering what items to incorporate from the everyday in producing realistic mathematics problems. Dowling (1998) explains that there are two types of tasks when it comes to categorising everyday mathematics:

1. *Esoteric and Descriptive tasks* which explain mathematics in purely conventional mathematical terms. The mathematical context can be from the mathematics itself or the everyday; and
2. *Expressive and Public tasks* which use non-mathematical language.

Clarke et al (2006) found that retaining authenticity in realistic mathematical problems may result in an overly complicated question. For this reason inauthentic data is often used as it may be easier to demonstrate the objective to the students. Clarke et al found that relatability for the learner increases when the data is ‘near’. ‘Near’ data is data that is considered to be familiar to the learner. ‘Far’ data is less relatable as the information is unfamiliar to the students, and bears little relation to their own personal, real-life situation. Information used for school activities can not merely be determined as ‘near’ or ‘far’ by taking age and school experience into account; students of the same age, in the same class may differ in the topics that relate to their life experience. There can be many reasons for a differing appreciation of ‘near’ and ‘far’ data to include, but not limited to, cultural references, gender and socio-economic background.

Clarke et al (2006) also consider the value of tasks being authentic or inauthentic, and describe four contexts when modeling real-life mathematical problems:

1. ‘*Authentic, Near Context*’: The context and the data are genuine. Learners can relate the context and data to their own real-life experience.
2. ‘*Authentic, Far Context:* The context is genuine and the data used is real, but the information does not resonate with the learners immediate life-experience;
3. ‘*Inauthentic, Near Context*’*:* The data and context resonate with the learners experience, but the context and data are not genuine, and are made up.
4. ‘*Inauthentic, Far Context*’: The data is made up and the context is not genuine. There is no resonation with the students’ real-life experience, and the data and context are unfamiliar to the learner.

The examination of authentic versus inauthentic data, combined with the concept of near and far contexts used in Clarke et al (2006) would suggest that data that resonates with a student’s own life experience, be it authentic or inauthentic, is more successful in preparing students for real-life mathematical problems, and their ability to solve them.

2.3.3. Freudenthal

The legacy of Freudenthal, as discussed in Conway & Sloane (2005), is the significant impact Realistic Mathematics Education has had on mathematics education in general, and in particular with respect to contemporary mathematics education. The Freudenthal Institute in the Netherlands is at the forefront of mathematics development and it is interesting to note the high scoring of the Netherlands in the 2003 PISA mathematics assessment with a ranking of 4th out of the 40 countries considered (there were 41 participating countries but the United Kingdom results are considered invalid as they did not meet the respondent rate required). The Netherlands was the highest ranking European country. In the Trends in International Mathematics and Science Study (TIMSS) 2003 the Netherlands had a mathematical ranking of 7th (out of 46 countries) in the eighth grade assessment, with a mean scale score of 536. The Netherlands was the second highest scoring European Union country in the assessment, at the eighth grade, with Belgium (Flemish) scoring marginally higher with an average scale score of 537. The TIMSS 2003 international average in this assessment was considerably lower at 467 (Mullis et al, 2004). The Netherland did not participate in TIMSS 2007. In the latest PISA results available, PISA 2009, the Netherlands scored significantly above the OECD average and ranked 12th out of 68 participating countries (OECD, 2010).

One has to consider the impact that Freudenthal, his progressive attitude to mathematics, and the work carried out by the Freudenthal institute has had on the comparative success of the Netherlands in terms of international mathematics success. The PISA (Programme for International Student Assessment) tests are very much in keeping with the aims of Realistic Mathematics Education, its hope to provide a mathematics education that will prepare students for participating in society, and a need for a more socially embedded mathematics education for all.

2.3.4. Mathematisation

Important to the development of mathematics education in Ireland is the mathematisation process. It is an essential aspect of Realistic Mathematics

Education. Mathematisation is the process of developing mathematical skills to solve real life problems. There are five steps in the mathematisation process:

1. The first step involves considering the real-world problem;
2. The problem is then organised according to mathematical contexts;
3. The problem is pared down to what the solver considers the most important aspects;
4. Using mathematical skills, the problem is solved;
5. The solution of the mathematical problem is then considered in terms of the real situation.

Mathematisation provides a skill by which students can develop a process to address realistic mathematical problems.

Nickson (2000) summarises the process of mathematisation by explaining that it can be carried out within mathematical or everyday situations, that the problems may be non-contextualised mathematics questions or everyday problems and that generalising and formalising play a significant role. Mathematisation, according to Nickson, involves searching for mathematical problems, organising subject matter and solving mathematical problems. Mason (1999) explains that the essence of learning mathematics is in the doing of the mathematics itself.

2.4 Issues in Mathematics Education

The following section considers issues that influence mathematics education. These range from social factors to gender. The author considers it important to consider the various issues that arise when discussing mathematics education, and the influence they have on mathematics and mathematical success.

The NCCA (National Council for Curriculum and Assessment) in conjunction with the ESRI (Economic and Social Research Institute) carried out a longitudinal study concerning students' experiences in Irish schools at Junior Cycle. While this study does not relate directly to mathematics there are many interesting and relevant findings. Particularly of interest to the author is phase 2 of the research which focused on the experiences of second year students (Smyth et al, 2006). There were particularly interesting findings with respect to the streaming of classes which is common place in Ireland: the authors found that streaming benefits more able students but sacrifices the needs of those who are less able. Streaming is particularly prevalent in mathematics classes in Ireland and there possibly needs to be serious debate around this. Smyth et al (2006) also found that didactic teaching is more common in lower ability classes, where students also tend to be male and working class. The following section considers various other factors that have a direct influence on mathematics education.

2.4.1. Social Factors and Mathematics Education

Zevenbergen (in Gates, 2001) speaks of the commonly held assumption, that mathematics is culturally and socially neutral. This would be considered true under the Behaviourist point of view as mathematics was seen as being distinctly asocial and therefore of equal advantage to students, regardless of socio and economic backgrounds. The constructivist point of view challenges this assumption and considers social factors as being important.

If one considers that there is an intrinsic value in considering an individual's social and economic background then it is important that the evidence for this is considered. Shiel et al (2007) consider the mathematics results in PISA 2003 in relation to socioeconomic background. Parental occupations were categorised according to the International Socioeconomic Index (ISEI) and given a high, medium or low status. It was noted that students with high status significantly outperformed those with medium and low socioeconomic status. Students who lived in lone-parent families performed at a significantly lower level (by 34 points) than the mean score of students living in two parent families. Students with low educational resources in the home (a desk, a quiet place to study, access to books) performed 24 points lower than the mean of students with median educational resources in the home, and 44 points lower than students with high resources. Those students who lived in a home with books also performed higher than those without, with those students who had 10 or less books in the home scoring almost 100 points lower than those students with 500 books or more. Students with a full school attendance record in the two weeks prior to the PISA assessment outperformed students who were out of school for one or two days by almost 20 points, and students who were absent for 3 or more days by 50 points (Shiel et al, 2007).

Shiel et al (2007) considered the number of Irish students who were entitled to the fee-waiver for the Junior Certificate examination due to a financial inability to pay the fees. They found that students who attended schools with a high proportion of students who were entitled to the fee-waiver performed at a lower level than students in schools with a higher proportion of students from financially affluent backgrounds. Leonard (2007) notes that NAEP (National Assessment of Educational Progress) results in the United States show that there is a significant difference in the mathematical and educational attainment between poor and affluent students, favouring the affluent.

Zevenbergen (in Gates, 2001) refers to the many studies that have been carried out that consistently show that a student's social and cultural background is deeply influential in determining how successful he/she will be in mathematical assessments. The author refers to studies such as those carried out by Lamb (1997), Marjoribanks (1987) and Secada (1992), and remarks that results of studies that focus on the correlation between social background and success in mathematics have consistently shown a strong, positive co-relation, for the last thirty or forty years, between those from higher socio-economic backgrounds, and academic success in mathematics. Zevenbergen argues that it is the language used in the teaching and assessment of mathematics that marginalise some social and cultural groups of students, while favouring others. The argument put forward by the author is that despite students being native English speakers, they may still be alienated by the unfamiliar mathematical language and terminology used in mathematics in the classroom. This is particularly the case for students from lower socio-economic (or working class) backgrounds.

'*The language used by some students positions them as marginal within the context of contemporary mathematics classrooms'* (Zevenbergen, in Gates, 2001:40).

Zevenbergen suggests that rather than particular social or cultural groups being deficient with regard to success in mathematics, it would appear that through the use of language mathematics education is acting as a type of social filter.

Leonard quotes Gutierrez (2002) as saying

'*We will know that equity has been achieved when demographic variables such as race, ethnicity, language, and socio-economic status can no longer be used to identify high and low achievers in mathematics'* (Leonard, 2002:5).

Leonard considers CRT (Critical Race Theory) and its acknowledgement that there is a relationship between skin colour and access to power, privilege, and status in society.

'As beliefs about race become entrenched in society over time, systems of privilege and marginalisation become institutionalised' (Leonard, 2002:7).

The influence of the home on schooling varies between working-class and middle-class homes (Zevenbergen, in Gates, 2001). In studies carried out by Zevenbergen it was observed that there was a stronger negative attitude to mathematics ability, and a sense that it was unlikely that the child could be good at mathematics as the parent never was, in working class families. It is also noted that there is a strong gender bias, favouring men, in relation to mathematical ability: this gendered attitude does not appear to exist to any significant extent in middle-class families. Zevenbergen notes that there is a strong 'can do' attitude in middle-class families, as opposed to an inherent sense of academic and mathematical inability in working-class families. This can often lead to a lack of support of a child's education due to a sense of inadequacy on the part of the parent. Zevenbergen also notes that access to educational resources: books, stationary, computers, a study desk, a quiet place to study; vary between social groups with working-class families less likely to have access to these tools due to financial restrictions. As noted in the PISA 2003 observations, a student's access to educational tools and resources has a significant impact on their level of success in mathematical assessment.

Leonard (2007) puts forward findings by Kitchen (2007) which show that teachers in schools in the United States where a high proportion of the students come from lower socio-economic backgrounds place more emphasis on low-level skills in their mathematics lessons.

'While there is a plethora of reasons for underachievement in mathematics, what happens in the teaching-learning context in mathematics classrooms influences students' success or failure' (Leonard, 2007:40).

Leonard (2007) suggests that students who are viewed positively by the teacher will have educational advantages over those who are perceived negatively. Leonard explains Good and Brophy's theory on self-fulfilling prophecy in the classroom:

1. The teacher expects certain, specific academic achievements from particular students;
2. The teacher then relates to those students according to those perceptions;
3. The teacher behaves in such a way as to show students what is expected of them;
4. Students, in turn, internalise the teachers expectations and behave accordingly. Students aspirations can be impeded;
5. The student's behaviour and attainment levels become more closely aligned to what is expected of them by the teacher over time;
6. As a result students' academic success, and possibly their economic success, is affected by the teachers' initial perception, and perhaps the teachers' small-mindedness.

If society and culture are shown to affect a student's mathematical success it would be unwise to follow mathematical theories such as behaviourism or cognitive methods (as discussed earlier) where the learner is assumed to be asocial. The refusal to consider cultural or social influences would suggest that these two theories of teaching and learning mathematics are inferior when compared to methods which incorporate a socio-cultural influence and place value on a students' real-life experience. This is particularly worrying in the Irish context as there is little or no acknowledgement of real-life, informal, personal experience in the current mathematics syllabus. This leads to a distinctly behaviourist leaning in the curriculum, and particularly in the teaching of mathematics.

In the TIMSS 2003 assessment the eighth grade participants (those students who were most likely to have completed eight years of formal schooling) were also asked about their social background. It was discovered that students with parents who had completed high levels of education were more likely to achieve higher results in the TIMSS assessment. This held true for almost all participating countries. It was also noted that students who had an expectation of going to a third-level educational institute in the future were more likely to perform well in the TIMSS tests. At both eighth and fourth grades there was a strong positive relationship between the number of books in the home and student achievement. Students who also had computers in the home, and reported usage of these computers, scored higher in the eighth grade assessments than those without. Participants who spoke the same language at home as that used in the assessment always had a higher achievement than those who spoke the language used in the test less frequently outside of the school environment (Mullis et al, 2004).

School principals were asked to determine the percentage of students in their schools that came from economically disadvantaged backgrounds as part of the TIMSS 2007 study. At eighth grade the international average of students attending schools with few economically disadvantaged students was 22 percent. In Chinese Taipei, Japan, Kuwait, Malta, Singapore, the Ukraine, and the Basque Country in Spain, more than half the students assessed in these countries attended schools that had low numbers of students from disadvantaged backgrounds. Students in Algeria, Colombia, Egypt, El Salvador, Ghana, Indonesia, Lebanon, Morocco, the Palestinian Authority, Thailand, Tunisia, and Turkey, had a high percentage of students attending schools with a large number of disadvantaged students. TIMSS 2007 saw a strong, positive relationship between schools with low numbers of disadvantaged students and strong mathematical performance in the assessments; conversely students attending schools with a large number of disadvantaged students scored significantly lower (Mullis et al, 2008). Mullis et al (2008) noted that, at both the fourth and the eighth grade, almost three-quarters of students attended schools where the vast majority of the students had the test language as their native language.

2.4.2. Culturally Relevant Learning

Leonard (2007) puts forward a convincing argument for the incorporation, and acknowledgement of the importance, of culture and real-life experience in the teaching and learning of mathematics. Leonard considers the situation in mathematics education in the United States with respect to culture and mathematical thinking, learning and teaching. Leonard argues that there is a very real need for the teachers of

'*diverse student populations to possess a pedagogy that will enable them to motivate, engage, and teach these students what they need to know in order to have access to higher education and economic success'* (Leonard, 2007:xiii).

Leonard explains that culturally relevant teaching in mathematics classrooms can be defined as recognition that mathematics has been present in every culture for as long as that culture has existed, and an acknowledgement of mathematics on that culture and its people.

Cooper, in Gates (2001), considers the link between mathematical success and social status in the English situation. Cooper argues that

'*in England, the degree to which mathematics has been related to everyday tasks and anticipated working lives has been related to the social class of* *pupils, with working class pupils typically being seen as those who would benefit most from this approach'* (Cooper, in Gates, 2001:245).

Copper believes that studies carried out by his research team suggest that a student's social class may affect how he/she responds to realistic mathematical problems.

Arnot (1993), explains that 'in the name of equality of opportunity, schools were encouraged to

'*treat all alike' in order to overcome social disadvantages – even though those disadvantages were built into the social fabric'* (Arnot, in Arnot and Weiner, 1993:198).

2.4.3. Gender and Mathematics Education

According to Shiel, Perkins, Close and Oldham (2007) males significantly outperformed females in mathematics literacy in 21 of the 29 OECD (Organisation for Economic Co-operation and Developments) countries, including Ireland, in PISA (Programme for International Student Assessment) 2003. Male Irish students also outperformed females in the four content areas: Shape and Space, Change and Relationships, Quantity, and Uncertainty. In the PISA assessment more males than females also scored at the highest levels, level 5 and 6 (13.7% compared to 9%). Females also scored lower at the lowest levels with 18.7% of females scoring at Level 1 or below compared to 15% of males. Males reported higher self-efficacy levels than females in all countries, including Ireland. In all countries, except Poland and Serbia, tested in PISA 2003 males reported having significantly lower levels of anxiety than females.

In the English context, Paechter (in Gates, 2001) discusses the reasons behind the low uptake of mathematics after the GCSE (General Certificate of Secondary Education) examination by females. Paechter puts this down to a lack of emphasis on conceptual learning and collaborative working; both forms of learning that appear to benefit the female way of approaching learning and the female desire for context. This would indicate that authentic, realistic tasks benefit female students and encourage relatability between classroom mathematical tasks and real-life situations. In contrast, the teaching and learning methods underpinning the way in which mathematics is taught within the English system prioritise speed and individualised competition. The strong masculine association of mathematics also, Paechter asserts, discourages female participation after the compulsory level as this is a life stage for many females when they are seeking to assert their femininity, and therefore shy away from subjects that may negate from this. The poor uptake of mathematics by females at A-level (advanced level) is particularly disappointing as girls perform just as well as boys in GCSE mathematics: in 1999 48% of 15-year old boys achieved grades A*-C, compared with 49% of 15-year old girls (Paechter, in Gates, 2001). This is particularly significant as in 1980 the average scores for boys of fifteen were higher than those for girls in all topics tested, and in the top 10% of pupils boys outnumbered girls by three to two (Askew and Wiliam, 1995:32).

Arnot et al (1998) found evidence that girls are more attentive in class, more willing to learn, and perform better than boys on tasks that are '*open-ended, process-based, related to realistic situations, and require pupils to think for themselves' in their examination of the English education system'* (Arnot et al, p.28, 1998:28). Could the introduction of a more realistic style of mathematics education negate the current gender imbalance that was reported in PISA as existing in the Irish educational system? Boys are considered, by Arnot et al, to be more adaptable to traditional approaches, and to excel at memorising facts and rules. Boys appear to have less of a need to understand fully the task at hand, and are comfortable with solving questions in mathematics without full comprehension. The authors also recognise that boys' contributions in mathematics classes, as in other subjects, are more prominent, both physically and verbally, than those of their female colleagues. This results in a situation where boys receive more feedback, both positively and negatively, than girls in their interaction in mathematics lessons from an early age.

Lees (1993) discusses the fact that when girls are successful in their academic performance at school they are regarded by their teachers, and very often their parents, as 'hard workers', whereas successful males are regarded as naturally clever.

In considering the Irish education system and mathematics performance in particular, Lyons et al (2003) determine that where gender differences do exist in mathematics achievement they are increasingly linked to economic and social status, rather than gender. This study also noted previous research performed by Harker (2000) which found that when academic ability and social background is controlled there are few significant gender differences in mathematics performance. The male domination of mathematics lessons is explained by Lyons et al. (2003) by considering the pedagogical style of the teachers, and an acceptance of answers that are 'called out' which is something male students are more likely to do. Taber's (1992) research, as considered by Lyons et al. (2003), notes that research on educational performance in physics classes found that boys received more attention than girls from the teacher because boys called out answers more frequently than girls.

Arnot et al (1998) agree that English students do not perform particularly well in international comparisons and feel that gender-related patterns in England are similar to those in comparable countries. It is noted that performance differences in mathematics in England are non significant until the late teen years, at which stage there is a low uptake of mathematics by girls for A-levels and boys also begin to outperform girls. Arnot et al recognise that in the TIMSS study the performance differences between boys and girls in mathematics in Year 9 was negligible.

In TIMSS 2003 there was no significant gender difference, with a similar number of countries where girls outperformed boys to those where boys outperformed girls (Mullis et al., 2004). At the eighth grade girls significantly outperformed boys in mathematics in Serbia, Macedonia, Armenia, Moldova, Singapore, the Philippines, Cyprus, Jordan and Bahrain. Boys were better performers at the eighth grade in the United States, Italy, Hungary, Levanon, Belgium (Flemish), Morocco, Chile, Ghana, and Tunisia (Mullis et al., 2004).

Arnot (1993) considers the introduction of co-educational education in the United Kingdom. The author asks if

> *'the principle of 'proximity equals equality' really worked in the case of gender?'* (Arnot, as in Arnot and Weiler, 1993:199).

Arnot (1993) is concerned that co-educational schools may be a method of providing the correct conditions for introducing women into a sex segregated labour market rather than genuinely reducing the differentiation between the sexes. Are co-educational schools assimilating female students into a world of male educational values?

2.4.3.1. Gender and assessment

Tims (1994) discusses the relationship between assessment in mathematics education and gender based performance. She explains that females often struggle to perform effectively in abstract mathematical assessments as they have difficulty with context-free situations. This leads back to, Tims suggests, the objectivist nature of male learners versus the cognitive nature of female learning where female socialisation leads to a situation where females need context in order to connect with both the assessment questions and the assessor. Males perform more successfully in context-free situations as they have not developed a need for the same level of connectedness. Boys internationally have demonstrated an advantage in multiple-choice style questions, while girls struggled to perform as well in context-free assessment formats (Bolger and Kellaghan, 1990).

2.4.3.2. Single-Sex Schooling and Performance

Arnot et al (1998) consider the findings of a study carried out by Steedman (1985) on students who were born in 1958 and passed through English secondary schools in the mid-seventies. This research implied that the students that entered single-sex schools were already performing at higher than average levels across the board before they entered their single-sex secondary school. Steedman found that *'very little in their examination results was explained by whether schools were mixed or single-sex, once allowance had been made for* *differences in (their) intakes'* (Steedman, 1985:98). Arnot et al (1998) point out that in more recent English studies, such as Nuttall et al (1989), only small differences between co-educational and single-sex schooling have been found, but those that do exist favour single-sex schooling for girls. The authors also consider Daly (1996) who examined single-sex schooling in Northern Ireland and who found that there was not a significant difference in the standard of educational performance between co-educational and single-sex schools, but that the slight disadvantage that did exist was for co-educational schools. Arnot (1993) considers the introduction of co-educational education in the United Kingdom. The author asks if '*the principle of 'proximity equals equality' really worked in the case of gender?'* (Arnot, as in Arnot and Weiler, 1993:199). Arnot (1993) is concerned that co-educational schools may be a method of providing the correct conditions for introducing women into a sex segregated labour market rather than genuinely reducing the differentiation between the sexes. Are co-educational schools assimilating females students into a world of male educational values?

Hannan et al (1996) carried out a significant study in Ireland in the 1990's which considered gender equality in education in relation to co-educational and single-sex schools. This study used large sample sizes, and single-sex schools made up almost half of the total schools examined leading to a very accurate study. Hannan et al (1996) found that more females than males sat the Leaving Certificate in Ireland, and also that female students performed better than male students in these terminal examinations. However '*girls remain significantly under-represented in mathematical, scientific and technical subject areas*' (Hannan et al, 1996:4). Hannan et al found that differences in performance in the Junior Certificate Examinations were more likely to be due to social-class background than to any significant gender imbalance.

Arnot et al (1998) determine that studies carried out in the USA in relation to academic performance in single sex schools versus co-educational schools show that single-sex schooling is more effective for both males and females (Riordan, 1990). Single-sex schools in the USA tend to be private, therefore this performance difference could also be due to social-class and economic wealth. Research in Australia shows no significant benefit due to single-sex schooling: *'research on achievement effect has established no clear superiority of either co-educational or single-sex schooling for girls, once other factors are controlled for'* (Yates, 1993:94).

2.4.3.3. Gender and the Irish Situation

Lyons et al (2003), in their 'Inside Classrooms' study, found little difference with regard to gender in the Irish mathematics classroom when gender is controlled. This contradicts the PISA assessment findings which found that boys performed at a higher level than girls in mathematics assessments. Lyons et al (2003) found the following situations to hold true in Irish schools:

- An equal proportion of males and females took the higher level paper in state examinations in 1992 (30%);
- The proportion of girls taking higher level mathematics in 1996 was slightly higher than boys, 37% versus 35% respectively;
- Girls in single-sex schools are less likely than boys in similar schools to take higher level mathematics;
- In co-educational schools girls are more likely than boys to take higher level mathematics;
- Girls have a 30% lower uptake of mathematics at foundation level (the lowest level of mathematics in the Irish system) than boys;
- Girls in single-sex schools are slightly more likely to take foundation level mathematics than boys in the same sector;

- Boys consistently achieve a slightly higher proportion of A grades in higher and foundation level mathematics; and
- An equal number of A's are achieved by both males and females at ordinary level (this holds true for the years 1992 to 1996).

Lyons et al, 2003.

The table shown in Appendix I displays the Junior Certificate mathematics examination results for both male and females students for the years 2003 to 2009 (at the time of submission the 2010 gender results were not yet available). The percentage of male and female Junior Certificate candidates obtaining each grade is displayed.

The most noticeable finding from an analysis of the Junior Certificate mathematics examination results in Appendix I is that a greater proportion of female than male students opted for the higher level, Junior Certificate paper in each of the years considered (2003 to 2009). As a result, a greater proportion of male than female students sat the Ordinary and Foundation level papers in each Junior Certificate examination (2003 to 2009). The fact that this situation holds true for every Junior Certificate year that data is available for suggests that this is a significant trend.

There is no particular trend with regard to gender and Junior Certificate mathematics results within the three levels (higher, ordinary and foundation) but it is worth noting that in the four out of the seven years considered, male students outperformed female students in terms in the number of A-grades obtained at higher level. As the attainment of A-grades is so closely distributed between male and female students yet female students have a greater participation rate in higher level Junior Certificate mathematics for each of the seven years, one would imagine that female achievement in Junior Certificate mathematics at the highest levels is greater than that for males.

2.4.4. Ethnicity and Mathematics Education

Kassem (in Gates, 2001) notes that pupils from ethnic minority groups have been of concern in matters relating to mathematical success in education for the last thirty years or more. Issues of racism in education are still a problem, Kassem believes, with African-Caribbean boys over-represented in exclusion figures from schools in England with black children being '*nine to thirteen times more likely to be excluded than white pupils*' (TES 9/7/99). Leonard (2007) remarks that NAEP (the National Assessment of Educational Progress) results in 2006 in the United States for 8th graders show '*59% of Black and 50% of Hispanic 8th grade students scored below basic levels in mathematics compared to 21% of whites*' (Leonard, 2007:3).

Leonard (2007) remarks that in the United States statistics show that poor students are more likely to be from minority ethnic groups including American-Indian, Alaskan-Native, Black and Latin/o (information from the Child Poverty Sheet, 2001). Leonard explains that the majority of African-American children still attend schools which remain segregated, both economically and racially. This is due to the fact that public schools are tied to the neighbourhoods in which they are located, therefore schools in wealthier neighbourhoods will have students that are economically better off, and the opposite also holds true. Leonard believes that culturally relevant teaching, and an acknowledgement of the value of that culture, is an effective way to meet the intellectual and social needs of students of colour. Leonard explains that '*in order to help African-American children develop mathematics socialisation and identity, they must realise that mathematics may be found in many aspects of African-American life and culture*' (Leonard, 2007:162). In considering the work of Ladson-Billings (2006), Martin (2000), Taylor (2004) and others, Leonard (2007) comes to two conclusions regarding the connection between mathematical underachievement and students of colour:

1. A history of generational underachievement, due to past discrimination and economic restraints; and
2. Institutional barriers.

Leonard speaks of findings by Harmon (2002) that speak of the experiences of academically gifted African-American students who were bused to predominantly white schools in better areas. Harmon found that these African-American students found that their teachers '*did not attempt to teach concepts in a culturally responsive way nor did they use visual nor tactile methods of teaching during instruction'* (Leonard, 2007:152).

Leonard believes it is essential that students of colour are exposed to strong academic and intellectual role-models of their own ethnicity in order to achieve academic success. It is through making mathematics relevant to the students lives, and to each student's cultural references, that they too believe that they can succeed, both academically and economically.

2.4.5. Self-Esteem and Anxiety

In PISA 2003 students rated, on a 4 point scale, how they were feeling in regard to their mathematical ability with options ranging from 'very confident' to 'not at all confident'. Based on these results students were then placed in low, medium and high 'self-efficacy in mathematics' categories. Higher assessment scores were achieved by students who had a high self-efficacy level. The mean score difference of students with high and low self-efficacy levels was 108 points. Irish self-efficacy levels were at a similar level to the OECD average and were higher than those in some higher-achieving countries such as Korea and Japan. Culturally it may be considered more appropriate to define oneself as confident in some countries than others.

Paulos (2001) is of the opinion that anxiety about mathematics plays a major part in underperformance in mathematics. He reasons that fear has a major part to play in the mathematical experience for many individuals, and describes how the most intelligent and academic of people often embrace innumeracy. Paulos states that *'part of the reason for this perverse pride in mathematical ignorance is that its consequences are not usually as obvious as are those of other weaknesses'* (Paulos, 2001:4).

People are unaware of the everyday importance of a high level of numeracy: in decision making, in understanding budgetary constraints, in estimating how many bricks you will need to build a wall etc. Paulos believes that 'math anxiety' can be counteracted using simple yet effective techniques:

- The explanation of the mathematical concepts underlying the problem at hand;
- The use of smaller numbers to illustrate the point;
- Examining easier, but related problems;
- Working backwards from the solution;
- The comparison of the problem, or parts of the problem, to problems one already understands and are familiar with; and
- The familiarisation of oneself with as many different, similar-type problems, and related examples, as possible.

2.4.6. Mathematical Equipment and Textbooks

The use of a calculator is optional during the Programme for International Student Assessment (PISA). According to Shiel, Perkins, Close and Oldham (2007) those who reported using a calculator during the PISA 2003 assessment scored 20 points higher than those students who opted not to use same. Mullis et al (2004) noted that the use of calculators varied greatly from country to country. In TIMSS 2003 '*at eight grade, in 10 countries nearly all the students (98% or more) were permitted to use calculators. In contrast, less than half were permitted to use calculators in seven countries'* (Mullis et al, 2004:10).

In TIMSS 2007 most of the countries involved in the study had a policy about calculator use as part of their mathematics curriculum. Roughly half of the countries involved in the study permit widespread use, and almost all countries permit calculator use, to some extent, for the majority of their students. According to Mullis et al (2007) teachers asked students to use their calculators for the following purposes:

- solving complex mathematical problems (31%);
- checking answers (26%);
- for routine computations (25%); and
- to explore number concepts (16%).

TIMSS 2007 posed questions on computer usage in the teacher questionnaire, and found that about one-third of the eighth-grade students, on average internationally, had computer access as part of their mathematics lessons. Use of computers as part of the regular mathematics lessons was rare, even in countries with high availability (Mullis et al, 2007).

The use of mathematical textbooks as a teaching tool is common in Irish mathematics teaching. Many teachers rely heavily on the prescribed textbook to establish not only the order in which topics are taught, but also the style in which each mathematics topic is addressed. Is this because of a fundamental lack of mathematical knowledge on the teacher's part or merely teachers wishing to give responsibility for the content and order in which their class unfolds to some invisible, third-party? A reliance on the textbook may be problematic with the introduction of the 'Project Maths' curriculum. 'Project Maths' is designed for teaching and learning to occur without the reliance on a sole textbook but rather using many resources from various websites, real-life sources and indeed textbooks. The lack of a prescribed textbook may prove to be an issue for teachers who are used to the comfort of following one. Inevitably there will be some resistance to such a significant change to Irish teaching methods and it will be interesting to see how successful this proves. It remains to be seen if the lack of a prescribed textbook will prove a stumbling block too far. Will teachers demand and follow a textbook regardless? If so will this affect the success of the new 'Project Maths' curriculum?

Hodgen and Wiliam (2006) suggest that some mathematics textbooks over emphasise the practicing of familiar procedures through completing predictable exercises, but they propose that with some adjustment textbooks could be used as a starting point for formative teaching. Currently the over-use of textbooks in Ireland over-promotes the skill of reproduction – with some adjustments the textbook may become a valuable tool for discovery learning.

Conway and Sloane (2005) consider the significant study of textbooks carried out by TIMSS (the Third International Mathematics and Science Study) which considered the use of textbooks in and from 48 countries. Valverde et al (2002) found that textbooks could impede or create learning opportunities for students. In particular, Valverde et al. (2002) focus on the impact textbooks can have in academic achievement in different countries. Conway and Sloane (2005) consider the TIMSS textbook analysis and the five measures of textbook characteristics that are considered:

1. *'The nature of the pedagogical situation posed by the textbook';*

2. *'The nature of the subject matter in the textbooks – not in terms of mathematics but rather in terms of the topics included';*

3. *The 'Sequencing of Topics';*

4. *'The physical characteristics of the textbooks' and*

5. *'The complexity of student behaviour textbook segments are intend to elicit'* (Conway and Sloane, 2002:25).

Conway and Sloane (2002) found that there is a connection between the textbook size and success in the TIMSS assessment, with countries that had light and compact textbooks tending to score highly in TIMSS. Conversely the USA was noted for having very large mathematics textbooks, and it is a country with a relatively low performance ranking in TIMSS 1995.

In TIMSS 1999 a mathematics textbook was reported by students, at both fourth and eighth grades, as the primary basis for their mathematics lessons. The three most common teaching methods used were teacher lecturing, teacher-guided students practice, and students working on mathematics problems on their own. Of the teachers of students at the eighth grade, 45% reported using lesson time, in at least half their lessons, to encouraging students to decide on problem-solving procedures (Mullis et al, 2004).

2.4.7. School Characteristics

In an analysis of PISA 2003, carried out by Shiel, Perkins, Close and Oldham (2007), Irish schools were characterised as:

- small (1-40, 15-year olds enrolled);
- medium (41-80, 15-year olds enrolled); or
- large (81 or more, 15-year olds enrolled).

Students in large Irish schools significantly outscored students in medium sized schools with scores of 509.5 versus 491.5. The mean score for students in small Irish schools was 471.5 but this is not considered to be a statistically significant result as there is a large standard of error reported with small schools. Students attending Irish secondary schools scored 40 points higher than those students attending vocational schools, and 17 points higher than those attending community or comprehensive schools. This could be due to the fact that traditionally vocational, community and comprehensive schools were viewed as providing a more rounded educational experience with access to vocational subjects that would prepare students for trades, while secondary schools focused more on academic subjects, while offering some vocational subjects. In Ireland DEIS (Delivering Equality of Opportunity in Schools) status is accorded to schools which are designated as disadvantaged based on the socio-economic status of the parents of students attending. According to www.education.ie

> *'The DEIS initiative is designed to ensure that the most disadvantaged schools benefit from a comprehensive package of supports, while ensuring that others continue to get support in line with the level of disadvantage among their pupils'*

Students in DEIS schools achieved a mean score that is 35-points lower than those students attending non-DEIS schools in PISA 2003. Shiel et al (2007) report that the Irish educational system is relatively uniform with respect to mathematics achievement, with less variation in performance, at 17%, between schools than those in other countries such as the USA, which has a between schools variance value of 25.8% and Germany which has a variance value of 52.4%. The OECD (Organisation for Economic Co-Operation and Development) average variance for between school performance was 33%.

Zevenbergen (in Gates, 2001) remarks on the differences, in the English system, in educational resources due to a schools socio-economic status. Zevenbergen remarks that, more often than not, schools in middle-class areas have access to more resources and better technology. She puts this down to strong parent committees who are willing to fund-raise to obtain the necessary funds for such equipment. In middle-class neighbourhoods, Zevenberger notes, it is more likely that parents will have the confidence, ethos and skills to participate in this kind of fund-raising. As a result schools in less financially stable areas are more likely to be lacking in equipment that may benefit the learning and teaching experience for students and enhance their educational experience.

2.4.8. The Culture of the School and Performance

The TIMSS (Trends in Mathematics and Science Study) assessments also consider social factors, such as school culture, and '*'TIMSS collects a rich array of contextual information about how mathematics and science learning takes place in each country'* (Mullis et al, 2004). This is achieved by asking school principals, mathematics and science teachers of participating students, and those students themselves to complete questionnaires designed by the TIMSS organising committee. These questionnaires consider curriculum, the schools themselves, classrooms, and mathematics and science instruction. In considering TIMSS 2003 Mullis et al (2004) note that the average mathematics achievement at eighth grade was 57 points higher for students in schools where fewer of the students came from homes that are economically disadvantaged, than for those students where more than half the students are from disadvantaged homes. TIMSS 2003 also noted a strong, positive relationship between schools with principals who viewed the school climate positively and student performance in the assessments. This held true at both fourth and eighth grades. Mullis et al (2004) remark that when school teachers are asked the same seven questions in the TIMSS assessments as the school principals their answers are not quite as positive. Nonetheless, the teachers impression of school climate still relates strongly, and positively, with student achievement in both mathematics and science assessment. There was also a noted relationship between the perceived safety of the school (based on questions asked of the teachers in each of the schools) and mathematics achievement (Mullis et al, 2004).

2.5 Teaching and Mathematics Education

2.5.1. Mathematics Teaching in Ireland

'Inside Classrooms' is an Irish video study of 20 different mathematics lessons in second year classrooms. This study is of particular interest to the author as not only is the investigation focused on teaching and learning in Irish mathematics classrooms, but it also investigates second year mathematics classes specifically which is also the focus of the author's research. The 'Inside Classrooms' study found Irish teaching methods to be traditional in nature, with emphasis placed on preparation for the Junior Certificate examination. The lessons observed tend to be formal in nature with the following steps normally adhered to:

1. The teacher explains a concept;
2. Followed by the students having the opportunity to ask questions, and;
3. Then some practice time in which the students repeat the format they have learned.

The 'Inside Classrooms' study determined that the mathematics classrooms that were observed in the course of their research are isolated from real-world contexts.

Shiel et al (2007) examined Irish teaching methods in mathematics with respect to the PISA assessment, focusing on PISA 2003 in particular. They report that less than 5% of class time in a typical Irish mathematics classroom was spent on real-life mathematics and mathematisation. However, 60% of Irish mathematics teachers felt that an understanding of how mathematics is used in the real world is important. Almost all teachers reported assigning mathematics homework in most or all lessons, with homework given less frequently to Foundation level students than to those studying at Higher and Ordinary levels. In their examination of PISA 2003, Shiel et al (2007) considered how Irish mathematics teaching compared to the teaching methods outlined in the PISA framework. At ordinary and foundation level the majority of Irish teachers interviewed emphasised the recall of basic mathematical facts, with emphasis at higher level on applying this mathematical knowledge. Just 4% of class time was spent on transferring mathematical knowledge to solve realistic mathematical problems. The role of mathematics in culture and society, and the history of mathematics tended to be neglected. These topics are currently not examinable topics in Irish mathematics examinations. Worryingly, 40% of teachers felt that there was no correlation between understanding how mathematics works in the real world and performing well in mathematics at school. This may be due to the fact that Irish teachers prioritise examination success as it is considered the sole indicator of educational attainment.

A particular characteristic of the current state examinations, the Junior Certificate and the Leaving Certificate, is the high level of predictability. The structure, format, content and actual question type in the examination papers are extremely similar year-on-year. If any element of this was changed it would have a serious impact on results. It is interesting to see how the new 'Project Maths' curriculum will cope with this tradition when it comes to assessment:

for the new syllabus to be effective this predictability must be broken as it is such an impacting factor on how mathematics is taught in the classroom, however there is bound to be resistance to this from both teachers and students.

2.5.2. PISA recommendations for Teaching and Learning in Ireland

Shiel et al (2007) suggest the following recommendations for applying the PISA approach to the teaching and learning of mathematics in Irish schools:

1. Emphasise a more interactive, hand-on approach in which students are fully engaged in actively participating in the solving and discussion of problems,
2. Emphasise reproduction less and to ensure the development of greater conceptual understanding by encouraging the full range of cognitive processes,
3. Implement a more balanced mix of context-free and real-world questions,
4. Encourage discussion in mathematics lessons and encourage the use of correct mathematics language,
5. Engage in the process of mathematisation, and
6. Ensure that higher ability students are given full opportunity to reach their potential by providing them with challenging work.

Despite the higher level course at both the Junior Certificate and Leaving Certificate level being considered very challenging, the over-emphasis on abstraction and preparation for examinations through reproduction possibly leads to the most able students not being fully challenged, especially with regard to higher-order thought processes. An over-emphasis on reproduction can lead to the development of rigor and careful copying of what has already been seen demonstrated by the teacher to the detriment of creativity. Students lacking in mathematical creatively are unlikely to succeed at the very highest level as mathematical innovation is both artistic and creative.

2.5.3. Teacher Qualifications and Experience

The mathematics teachers of the students assessed in TIMSS 2003 tended to be very experienced mathematics teachers, with an average of 16 years experience. The majority of these teachers held a minimum of a university degree; 76 percent of the eighth-grade teachers and 65 percent of the fourth-grade teachers. Most teachers involved in the TIMSS assessment had studied mathematics as a major subject at university (70% of those teaching eighth-grade mathematics and 54% had a qualification in mathematics education). Morocco was unusual in that 72% of students involved in the TIMSS 2003 study were taught mathematics by teachers whose education did not continue after secondary school. Schools participating in TIMSS 2003 reported a strong belief in the importance of continuing professional development for their staff members, and more than 80% of the eighth grade students involved in the study were taught by teachers who had completed some further professional training in either teaching skills or improving content area. 88% of the eighth grade students involved in TIMSS 2003 were taught mathematics by fully certified teachers, and 85% of the fourth grade teachers had full certification (Mullis et al, 2004).

Mathematics teachers of students involved in the TIMSS 2003 study reported frequent discussion with their colleagues regarding instructional issues, with at least 80% of teachers reporting weekly or monthly interaction regarding mathematics education issues. Observing colleagues' mathematics lessons, or having one's own lessons observed by others, was not common practice in the countries participating in TIMSS 2003, with 60% of teachers reporting that observation never happened. (Mullis et al, 2004).

Shiel et al (2007) found that the vast majority of Irish mathematics teachers held a bachelor's degree, with 88% holding a higher diploma in education. Other post-graduate qualifications, such as masters and Ph.D. qualifications were less common. The majority of mathematics teachers who responded to the questionnaire provided had a mathematics component in their primary degree, with one-third having studied mathematics as part of their higher diploma.

A report carried out by Ni Riordáin and Hannigan (2009) in conjunction with the National Centre for Excellence in Mathematics and Science Teaching and Learning (NCE-MSTL) in 2009 looked at a survey of mathematics teachers and principals in a representative sample of Irish schools. The report focused on out-of-field teaching and the following observations were among their findings:

- 48% of second level mathematics teachers do not have an appropriate qualification in mathematics (most degrees held by these teachers are in business or science);
- Younger and weaker students are more likely to be given unqualified teachers;
- 65% of mathematics teachers over 35 years of age are appropriately qualified to teach mathematics, with only 40% of mathematics teachers under 35 years holding a qualification in mathematics;
- Qualified mathematics teachers are more likely to be given higher level classes, classes in examination years and senior cycle class groups;
- 76% of unqualified mathematics teachers would avail of continuing professional development if it was available; and
- 88% of principals would encourage mathematics teachers to seek further training in mathematics.

(Hannigan and Ni Riordáin, 2009).

2.5.4. Mixed Ability Teaching

Mixed-ability teaching, according to Sullivan and Clarke (1991), should provide equal and challenging learning opportunities for all students, without disadvantaging anyone. They suggest that a selection of tasks, meeting the following criteria, should be chosen to suit all abilities:

1. All students should be able to attempt the task;
2. Students should be given the opportunity to discuss any fears they may have in a whole class discussion;
3. Students should be actively involved with working out the problem, with minimum assistance from the teacher;
4. Whole class explanation and review of tasks should be teacher led;
5. Extensions should follow on for the most able students;
6. There should be a minimum of direction from the teacher;
7. Where possible there should be more than one correct solution, and several ways of attempting the problem. This is to encourage the learner to engage with mathematical tasks that are more authentic, and more relatable to skills he/she will need in the real world.

In many countries the notion of mixed-ability teaching is the norm, with the underlying assumption being that all students have equal rights with regard to the curriculum. In France the constitution determines that all students have equal educational rights: Dunne (1996:50) explains that *'it is not essential for a child to demonstrate a well-defined knowledge of a topic before moving onto the next'*. In Hungary there is a commonly held view that all pupils will eventually understand the concept and that '*grouping by ability within classrooms is discriminatory'* (Stevenson 1999: 117).

The introduction of 'Project Maths' to the Irish mathematics curriculum will increase the provision of mixed-ability teaching within classrooms as for the first time it is determined by the curriculum guidelines that students in the first year of second level will follow a common course, and not be streamed into classes decided on by their mathematical ability. This is being implemented for first years in Irish second level schools from September 2010 and will continue to be the case for the following first year groups under the ‘Project Maths’ curriculum. Classes may be streamed into ability groups for all other year groups at the discretion of the school.

The implementation of mixed-ability teaching for first year groups will require further in-service and the up skilling of teachers. There is no mention of the availability of such in-service workshops from the ‘Project Maths’ team at the time of writing. A fundamental problem with mathematics’ in-service through the years has been a focus on mathematical content with little or no attention to pedagogy or different teaching methods and approaches. The ‘Project Maths’ in-service workshops to date are more innovative in this regard than the previous in-service available, however this must be expanded on to fully upskill teachers with all the necessary skills.

2.5.5. Questioning and Mathematics Education

Sullivan and Clarke (1991) propose that if mathematics is to benefit students, and provide them with the opportunity to become truly educated in mathematical terms, then it is essential that they engage in classroom activity that most resembles mathematical activity in the real world. Sullivan and Clarke believe that mathematical problem solving in the classroom is the activity that most resembles authentic, real-life mathematical situations. Rigid, conventional teaching leads to an inability by students to access the mathematical skills they already possess when faced with unconventional mathematical problems. There is, the authors believe, an over emphasis on the recall of facts, skill acquisition and the practicing of routine procedures to the detriment of the exploration of relationships between mathematical skills and concepts. The process of quality questioning is an important component of engaging students in realistic mathematical activity.

'The quality of students' responses is dependent on the quality of the questions asked' (Sullivan and Clarke, 1991:45).

Sullivan and Clarke (1991) consider whether the quality of mathematics education would be enhanced if questions were asked that required the learner to think and analyse at a deeper level. Sullivan and Clarke discuss a study carried out by Sullivan and Leder (1990) in which forty-six lessons were coded and analysed. The research found that 58% of all classroom events consisted of questions and instruction, but that over half of these questions required the recall of information only, a further 30% were closed questions with only one possible answer, and that only 5% of all questions posed required the learners to think independently. Sullivan and Clarke consider two types of questions:

1. Fact questions which only require basic recall skills; and
2. Higher-order questions where independent thinking is required.

Gall (1984 as in Sullivan & Clarke, 1991) believed that fact questions were better suited for disadvantaged students, with higher-order questions more suitable for the more academic student. Sullivan and Clarke (1991) speak of research carried out by Tobin (1984) where the time available for a student response is considered. Tobin suggests that teacher-pupil communication is dramatically improved if there is more time provided for the student to give a response. This would suggest that a rapid series of questions puts students, particularly less confident students, in a disadvantageous situation. Clarke suggested that increasing the possible response time led to students giving longer responses and more appropriate answers with more explanation. Sullivan and Clarke (1991) also suggest that with fewer, but more varied questions asked by the teacher, weaker, less academic students could improve.

Sullivan and Clarke (1991) promote the idea of quality teaching influenced by better quality, open ended questions. The authors suggest that both teaching and learning could be improved if questions are identified which require higher-level thinking, and are accessible to the vast majority of students: both the very academic and those that are deemed to be academically weaker. It is suggested by Sullivan and Clarke that the use of fact questions can lead to a situation where students may have the ability to solve routine mathematical problems, but may not understand the topic at hand. Lack of understanding may lead to an inability to utilise the mathematical skills known in unfamiliar situations.

Sullivan and Clarke (1991) identify good questions as follows: questions that require more than a basic recall of facts; active learning where the students learn by actually doing the task, and the teacher may learn from how the students attempt the problem; and open-ended questions where there may be several correct answers. This lends to Socrates' belief that anything could be taught through the use of carefully selected questions. Sullivan and Clarke (1991) emphasise the importance of pre-planning with regard to higher-order questions and they state that for good questions, that require independent thinking, to work effectively it is important that the teacher plans such questions in advance, and furthermore that they are pitched at the appropriate difficultly level for the students in question. It is imperative, for a question to be considered a good question, that all students be able to attempt the question at the very least.

Is this a possible downfall of the Irish education system as it currently stands? Mathematics, for both the Junior and Leaving Certificate, demands recall and practiced routines rather than questions that call for the students to think independently. There is a complete absence of open-ended questions in the Irish mathematics syllabus, which in turn leads to a situation where Irish students have no awareness that in mathematics there is not always only one correct solution, and that in fact it is more probable in the real-world, and in authentic situations, that there may be several correct answers. Is this type of syllabus and assessment impeding the students' ability to learn in a realistic fashion and in a method that may be of benefit to them in future work and life situations?

2.5.6. Assessment

Hodgen and Wiliam (2006) define assessment for learning as *'any assessment for which the first priority in its design and practice is to serve the purpose of promoting pupils' learning'.* They explain that assessment can be used to improve learning if the information discovered is used in a productive manner to modify current teaching and learning methods. Hodgen and Wiliam define formative assessment as assessment where the information discovered is used to adapt teaching to meet the learning needs of students. A review, by Hodgen and Wiliam, of Black and Wiliam, 1998 (Assessment in Education: Principles, Policy and Practice) implies that formative assessment can be paramount in raising the standards of student achievement. They determine three types of feedback as being essential to formative assessment: from student to teacher; from teacher to student; and between students.

Hodgen and Wiliam (2006) recognise five principles of learning as being essential to effective mathematics education. They determine the principles of learning as follows:

1. The first principle of learning is a recognition that it is essential to 'start from where the learner is';
2. That learning is an active process in which students must be fully involved, doing the learning rather than it being done for them by the teacher or others;
3. Students must use mathematical language to discuss and express their mathematical ideas,
4. Students must understand the underlying ideas of the mathematical problem in order to fully learn from solving it; and

5. Feedback from teachers and assessment must suggest methods for improvement to students (Hodgen and Wiliam, 2006:4-5).

Hodgen and Wiliam (2006) insist that students must be challenged by activities that encourage them to think mathematically, and to talk in mathematical terms about their ideas, if they are to truly learn and understand mathematical concepts. A focus on learning without understanding, and an overemphasis on mathematical procedure and learning by rote, leads to a curriculum where even the most able students struggle when faced with familiar mathematical problems in new or unusual contexts. Hodgen and Wiliam (2006) propose the use of mathematical problems with more than one solution and the continuous process of challenging students' mathematical presumptions as a means of drawing on student understanding of mathematics and provoking discussion.

Sullivan and Clarke (1991) promote ideas put forward, regarding assessment in mathematics, by the California Assessment Program, 1989. The California Assessment Program suggests the following:

1. Students should be given the opportunity to think for themselves and thus they are espousing the notion of independent thinking;
2. Students should be encouraged to construct their own response rather than selecting a single answer, from a closed question; and
3. Students should be given ample opportunity to demonstrate the level of their understanding regarding the mathematical concepts at hand.

It is essential that teachers '*employ assessment strategies which recognise the multi-dimensional nature of mathematical activity'* (Sullivan and Clarke, 1991:43). Sullivan and Clarke determine that assessment is concerned with the exchange of information, and therefore it should be a two-way path.

2.6 Conclusion

In this chapter the author has summarised various learning theories in mathematics education and issues which arise in contemporary mathematics education. The author has placed the Irish mathematics situation within this context and considers the issues in mathematics education which influence and affect mathematics education in Ireland. Within this context, the author is firmly of the belief that Irish mathematics education follows the behaviourist learning model.

The author is of the belief that this situation of teaching and learning based on the behaviourist model is brought about by the structure of the syllabus which is reinforced by a rigid, predictable, terminal examination at the end of each cycle. The author considers how this has influenced educational performance and achievement, and believes that as a consequence the problem-solving skills of students when faced with either an unfamiliar situation or a real-life problem are often not developed enough to adequately address the issue. The author investigates this issue in more detail in Chapter 7.

3.0 Chapter 3: International Comparative Studies

3.1 Introduction

It is not possible to consider Irish mathematics education without considering what is happening in other countries throughout the world. No education system stands in isolation and if Irish graduates are to compete on an international level it is imperative that education levels in Ireland are of a comparable standard to those elsewhere. In this section the author considers international mathematics education and the large-scale, comparative studies that have been implemented.

3.2 International Comparisons in Mathematics

International comparative studies are an important means of assessing the effectiveness of education within and between countries. This is done through the comparison of academic performance of students, at the same educational level, across different education systems. International comparisons provide opportunities for considering educational practice in other countries, and for developing an awareness of factors which may improve mathematics education through curriculum design and educational practice. Comparative studies inform practice, encourage debate, give rise to self-reflection and form a basis for future research.

Postlethwaite (1988) defined comparative education in The Encyclopedia of Comparative Education and National Systems of Education by saying '*to "compare" means to examine two or more entities by putting them side by side and looking for similarities and differences between or among them. In the field of education, this can apply both to comparisons between and comparisons within systems of education'* (Postlethwaite, 1988: xvii).

Postlethwaite (1988) determines the four aims of comparative education as follows:

1. '*Identifying what is happening elsewhere that might help improve our own system of education'* (p.xix),

2. 'Describing similarities and differences in educational phenomena between systems of education and interpreting why these exist' (p. xix),

3. 'Estimating the relative effects of variables (thought to be determinants) on outcomes (both within and between systems of education)' (p. xx), and

4. 'Identifying general principles concerning educational effects' (p.xx).

Kaiser (1999) summarises Postlethwaite (1988) by distinguishing two types of comparative studies: country studies; and studies on themes within and between countries. Kaiser defines country studies as studies on a particular country's education system, while studies within and between countries are concerned with major educational themes such as the economics of education, education planning and policy, primary and secondary schooling, teacher and teacher education, curriculum etc.

In an Irish context 'Country Studies' can be conducted in an analysis of the Irish educational system, and the mathematics curriculum, syllabus and application of same within the country. 'Country Studies' can also entail the consideration of Irish styles of teaching and learning, and attitudes to the teaching of mathematics in Ireland. Societal and cultural factors that affect the Irish student are also of value. A 'Between Countries Analysis' in the Irish instance can be completed in a comparison of the Irish curriculum to another curriculum. An analysis of mathematics assessment methods in other countries is also a valuable comparative tool as there is evidence to suggest that the assessment style implemented can have a powerful influence on the teaching and learning conducted. The levels of abstraction and mathematisation in various syllabi are also interesting to note and may affect the success of students studying these syllabi.

3.3 Problems Associated with International Comparative Studies

Noah (1988) (as discussed in Kaiser, Luna and Huntley, 1999) outlines some of the problems involved with comparative studies as follows: the cost and difficulty associated with compiling data from foreign countries; difficulty in comparing data collected; the possibility that the data collected by national sources may contain some bias and may affect the validity of a cross-national comparison; problems with constructing valid scales for data; and the possibility that there may be an ethnocentric bias.

Kaiser (1999) recognises that problems may arise from implementing educational methods used in first world countries to third world curricula and voices concern regarding the implementation of '*reforms modeled on the experience of the industrialised nations as a means of maintaining their pre-eminence'* (Kaiser, 1999:7).

Possible problems that the author foresees in compiling information for an international comparative study relate to difficulties in interpreting and translating other languages, the financial costs associated with collecting information abroad, the comparison of data collected from students studying different curricula, and the difficulties associated with obtaining approval for collecting data within other countries. The author collected data solely from English speaking countries (Ireland and the U.S state of Massachusetts) as it was not possible, due to financial and practical constraints, to translate and interpret data from non-English speaking countries. It is acknowledged that this gives a westernised, industrialised slant to the research collected. The author also found that English terminology, and the comprehension of same, varies from Irish-English to American-English, with some language changes necessitated for ease of comprehension by request from the school involved in the United States.

3.4 International Commission on Mathematics Instruction (ICMI)

Early international comparative work in mathematics was carried out by the International Commission on Mathematics Instruction (ICMI). Howson, in Kaiser et al (1999), discusses how the ICMI ran a study in the early 1970s which asked authors from various countries to describe the changes in curriculum and teaching methods that had taken place in their countries during the 1960s, the aims of these changes, and the outcomes that resulted from these changes. This study resulted in the publication of a paper, published in 1978: ‘Educational Studies in Mathematics’. Howson determines that while this paper held much interesting information regarding mathematics education on an international level, it held little in the way of comparative assessment as there was little analysis or synthesisation.

3.5 The IEA Assessments: FIMS, SIMS and the TIMSS Series

The IEA (the International Association for the Evaluation of Education Achievement) is responsible for possibly the most established series of mathematical assessments at an international, comparative level. The following section considers the organisation itself, the IEA; the initial international mathematical assessments, FIMS and SIMS; and its well-established series of TIMSS assessments.

The International Association for the Evaluation of Educational Achievement (IEA) is the organisation responsible for a large number of international assessments in mathematics such as FIMS, SIMS and the TIMSS series. The IEA originated from a meeting of educational enthusiasts at the UNESCO Institution for Education in Hamburg in 1958. At this meeting problems in the evaluation of students and schools were discussed and ways forward in educational assessment were suggested. The IEA were of the opinion that all educational systems hold a common end goal: optimal results and educational success. Their interest lay in examining the different methods utilised by different education systems in order to arrive at this end point. The IEA decided that by assessing a broad range of education systems simultaneously there would be enough variability to identify common relationships between different systems. The IEA originally consisted of twelve educational institutes but currently has members from 68 educational institutions, from universities to ministries of education (www.iea.nl).

The IEA hopes to meet the following aims through its comparative assessment and research projects:

- '*Provide international benchmarks that may assist policy-makers in identifying the comparative strengths and weaknesses of their educational systems;*
- *Provide high-quality data that will increase policy-makers' understanding of key school- and non-school-based factors that influence teaching and learning;*
- *Provide high-quality data which will serve as a resource for identifying areas of concern and action, and for preparing and evaluating educational reforms;*
- *Develop and improve educational systems' capacity to engage in national strategies for educational monitoring and improvement;*
- *Contribute to development of the world-wide community of researchers in educational evaluation'.* (www.iea.nl/misson_statement.html)

The First International Mathematics Study (FIMS) was carried out in the 1960's by the IEA. FIMS was a comparative study of mathematical achievement. The fact that it concerned mathematics was primarily due to the notion that mathematics was considered easier to compare between countries than any other subject. This was possibly due to the fact that mathematical notation is common to many countries and education systems. There were two research populations studied by FIMS: the first group were students of thirteen years old, the second group consisted of students in their last year of secondary school. Problems arose in comparing the second group between the countries involved in the study, as mathematics was not a compulsory component of education for all students, in all countries, in their final year of education. The FIMS was the first major international comparative study (Travers, K.J. & Weinzweig, A. in Kaiser, Luna & Huntley, 1999).

The Second International Mathematics Study (SIMS) was undertaken in 1976. SIMS was a much more ambitious undertaking than the earlier TIMS and the primary goal was to create an international picture of mathematics education. The structure consisted of three sections:

1. The Intended Curriculum: what is mandated at national level;
2. The Implemented Curriculum: what is taught in the mathematics classroom, and
3. The Attained Curriculum: the mathematics that students actually learn.

The research population involved in SIMS consisted of two groups: the first group consisted of thirteen-year olds which was in keeping with the first population group in FIMS. The second group consisted of students in their final year of compulsory education (as per FIMS), but also those who were studying mathematics as a substantial component of their academic curriculum (Travers & Weinzweig, in Kaiser et al, 1999). By changing the second population to mandate that the students are in their last year of compulsory education this became a population that was significantly easier to compare that that in FIMS.

3.5.1. The Third International Mathematics and Science Study (TIMSS) 1995

The Third International Mathematics and Science Study (TIMSS) was first carried out in over 40 countries in 1995 by the IEA (the International Association for the Evaluation of Educational Achievement). This was the first assessment of what was to become a series of assessments that continue to be completed in a four-year cycle. The following assessments were completed in 1999, 2003, 2007 with the next assessment planned for 2011. The acronym TIMSS changed from the 'Third Mathematics and Science Study' to 'Trends in Mathematics and Science Study' after TIMSS 1995. The TIMSS 1995 assessment was a more comprehensive study of mathematics education at an international level than either of the two earlier IEA sponsored studies: FIMS or SIMS. Mathematics achievement was assessed at five grade levels in three population groups, in comparison to the two population groups assessed in the earlier studies. (Beaton & Robitaille in Kaiser et al, 1999).

TIMSS works continuously to *'ensure the reliability, validity, and comparability of the data through careful planning and documentation, cooperation among participating countries, standardised procedures, and rigorous attention to quality control throughout'* (Mullis et al, 2009). This study has been used extensively to compare learning and teaching practices, along with student performance, in the international community.

'*TIMSS followed in the wake of other reports and documents (National Commission on Excellence in Education, 1983; National Council of Teachers of Mathematics, 1989; 1995; American Association for the Advancement of science, 1989, 1993; Executive Office of the President, 1990) that have focused* *attention on the importance, conditions, and goals of science and mathematics education*' (Greene et al, 2000:1).

TIMSS was an IEA, multicultural study, with a significant role played by the United States, Germany, Canada, the Netherlands and Australia with regard to the administrative, developmental and analytical aspects of mathematics education in particular (Andrews as in Gates, 2001). More than 50 countries participated in the first TIMSS assessment in 1995. Forty-five countries took part in the achievement testing.

Argentina	Australia	Austria
Belgium	Bulgaria	Canada
Colombia	Cyprus	Czech Republic
Denmark	England	France
Germany	Greece	Hong Kong
Hungary	Iceland	Indonesia
Iran (Islamic Rep. Of)	Ireland	Israel
Italy	Japan	Korea
Kuwait	Latvia	Lithuania
Mexico	New Zealand	Netherlands
Norway	Philippines	Portugal
Romani	Russian Federation	Scotland
Singapore	Slovak Republic	Slovenia
South Africa	Spain	Sweden
Switzerland	Thailand	United States

Table 2: Countries that participated in TIMSS 1995 achievement testing.

The TIMSS headquarters is in Boston College, Massachusetts in the United States. TIMSS 1995 defined three internationally desired population groups for its assessment:

1. All students in the two adjacent grades that contain the most 9-year olds at the time of assessment;
2. All students in the two grades that contain the most 13-year olds, and;
3. Students in their final year of secondary schooling.

Sampling standards were high and required at least 85% of the selected schools to participate. The mathematical content areas assessed by TIMSS were

- number;
- measurement;
- geometry;
- proportionality;
- functions;
- relations and equations;
- data analysis,
- probability and statistics, and;
- analysis.

Contextual information was also considered important in the TIMSS assessment series, in comparison to FIMS and SIMS where mathematical achievement was the sole component of assessment. Students were asked to complete questionnaires regarding aspects of their personal life including family backgrounds and extra-curricular interests. Teachers and school principals were also asked to fill out questionnaires regarding teaching practices, school characteristics etc. (Beaton & Robitaille in Kaiser et al, 1999).

3.5.1.1. TIMSS 1995 Results

In TIMSS 1995 the Population One cohort (all students in the two adjacent grades that contain the most 9-year olds at the time of assessment) were assessed with regard to the following content areas:

- whole numbers;
- fractions and proportionality;
- measurement;
- estimation and number sense;
- data representation, analysis and probability;
- geometry;
- patterns.

The top ranking countries in the Population One TIMSS assessment were Singapore and Korea, with Japan, Hong Kong, the Netherlands, the Czech Republic and Austria also performing well (Beaton & Robitaille in Kaiser et al, 1999).

The Population Two (students in the two grades that contain the most 13-year olds at the time of implementation) assessment tested six content areas:

- fractions and number sense;
- measurement;
- proportionality;
- data representation, analysis and probability;
- geometry, and;
- algebra.

Singapore was the top performing country with respect to the Population Two results, with Korea, Japan and Hong Kong, Flemish-speaking Belgium and the Czech Republic performing well. Gender differences with respect to all population results showed little gender differentiation between male and female students, but those that did exist tended to favour boys over girls. A clear and positive relationship is noted between students verbalising a stronger liking of mathematics and higher scores in the TIMSS mathematics assessment (Beaton & Robitaille in Kaiser et al, 1999).

3.5.1.2. Ireland and TIMSS 1995

Lyons et al (2003) consider Ireland's involvement in TIMSS 1995 and make the observation that mathematics teachers in Ireland attribute more importance to the memorisation of formulae and procedures than teachers elsewhere. Seventy-four per cent of Irish teachers rated this as an important teaching and learning method compared with an importance rating of 40% in other participating countries. The ability to think creatively, and understand the underlying mathematical concepts, was rated highly in the majority of countries, but teachers in Ireland attributed a relatively low ranking to these skills. The memorisation of formulae and procedures was rated much higher than the ability to think creatively. This would suggest that Irish education is more traditional in style, led by more traditional teachers, who are teaching in a more traditional way. This is reinforced by the examination system which is based, as discussed previously, on a terminal assessment system. As a result, examinable skills are valued and the closed-ended questions asked in the Junior and Leaving Certificate examinations test memorisation and reproduction skills, therefore these are the skills that Irish teachers tend to place value on.

3.5.2. Trends in Mathematics and Science Study (TIMSS) 1999.

TIMSS 1999 was the second TIMSS study in the series and occurred four years after the first. Just seven countries participated in the mathematics part of this study: Japan, the Netherlands, Switzerland, Hong Kong SAR, the United States, Australia and the Czech Republic. Five countries participated in the science component of the study. The study focused solely on mathematics at the eighth grade. It was a videotape study that also used worksheets and textbook content used in class to supplement the information collected. Teacher questionnaires were also used. The study followed mathematics lessons in more than one thousand classrooms over a period of one academic year. All aspects of mathematics teaching and learning were considered (www.timss.bc.edu/timss1999.html).

All seven countries involved in TIMSS 1999 shared some features in their mathematics lessons:

1. The majority of the time (80%) spent in the mathematics lessons at eighth grade is spent on solving mathematics problems;
2. In all seven countries a combination of whole class work and private individual work is used;
3. The majority of mathematics lessons, in all the participating countries, considered previous mathematical work covered, and introduced new mathematical concepts also;
4. At least 90% of mathematics lessons observed involved the utilisation of a mathematics textbook or worksheets; and
5. Teachers spent considerable more time talking in mathematics lessons than students in a ratio of 8:1.

It is noted in the results of TIMSS 1999 that the high scoring countries tended to employ a variety of teaching methods in their mathematics classrooms, as opposed to a single, shared teaching method in lower-achieving countries.

'Distinctions included the length of time spent introducing new content, the coherence across mathematical problems and within their presentation, the topics covered and the procedural complexity of the mathematical problems, and classroom practices regarding individual student work and homework in class' (www.timss.bc.edu/timss1999.html).

TIMSS 1999 was a cross-national study which observed teaching and learning practices in mathematics, and science, in seven countries. In each of the participating countries approximately 100 schools were randomly selected. Eighth grade mathematics is the area studied, with the videotaping distributed evenly throughout the year in order to gain insight into the full syllabus as covered in the eighth grade in each of the countries involved. Bilingual coders were involved in the coding and analysing of data in non-English speaking countries. Data collected was weighted in order to obtain reliable information among participating countries (www.timss.bc.edu/timss1999.html).

3.5.3. Trends in Mathematics and Science Study (TIMSS) 2003

The third TIMSS (Trends in International Mathematics and Science Study) was carried out in 2003, and considered mathematics and science learning at two grades, fourth and eight, in forty-nine countries (Mullis et al, 2005). The aim was for students to have eight years of formal schooling at the time of assessment in the eighth grade, and for students to have had four years of formal schooling at the time of testing in the fourth grade. TIMSS 2003 resulted in Singapore performing the best in mathematics at both the fourth and eighth grades (Muliis et al, 2004). Singapore's mathematics performance is significantly higher than the rest of the performing countries. Other countries that performed well are the Republic of Korea, Hong Kong SAR and Chinese Taipei at the eighth grade, while Hong Kong SAR, Japan and Chinese Taipei performed well in mathematics at fourth grade (Mullis et al, 2004).

The five mathematics content areas in TIMSS 2003 are:

- Number;
- Algebra;
- Measurement
- Geometry; and
- Data.

(Mullis et al, 2004)

The participating countries, and regions, in TIMSS 2003 are Australia, Belgium (Flemish), Bulgaria, Cyprus, England, Hong Kong SAR, Hungary, Iran (Islamic Republic of), Israel, Italy, Japan, Korea (Rep. of), Latvia, Lithuania, Netherlands, New Zealand, Romania, Russian Federation, Singapore, Slovak Republic, Slovenia, South Africa, United States, Ontario Province (Canada), Quebec Province (Canada), Argentina, Chile, Chinese Taipei, Indonesia, Jordan, Macedonia (Rep. of), Malaysia, Moldova (Rep. of), Morocco, Philippines, Tunisia, Indiana State (U.S.), Norway, Scotland, Sweden, Armenia, Bahrain, Botswana, Egypt, Estonia, Ghana, Lebanon, Palestinian National Authority, Saudi Arabia, Serbia, Syrian Arab Republic, Yemen, Basque Country (Spain) (Mullis et al, 2004).

The mean score for eighth grade students in mathematics is 467, across all forty-six countries at this level. The mean score for students at the fourth grade is 495 and is an average of the twenty-five participating countries. The benchmarking participants are not utilised in calculating either average. TIMSS 2003 saw a significant difference in the mean scores in mathematics achievement, at eighth grade, for the highest and lowest scoring countries with an average of 605 for Singapore and an average of 264 for South Africa. At eighth grade twenty-six countries, and the four benchmarking participants (the Basque Country, Spain; Indiana State, U.S.; Ontario Province, Canada; and Quebec Province, Canada), scored significantly above the international average, and eighteen countries scored significantly lower. Assessment in mathematics at the fourth grade saw a high of 594 in Singapore to a low of 339 in Tunisia (Mullis et al, 2004).

The vast majority of the countries assessed in TIMSS 2003 had a defined national curriculum, the exceptions being the United States and Australia. The emphasis at the eighth grade was on understanding mathematical concepts, followed by competence in demonstrating basic skills; at fourth grade the emphasis is on the improvement, and mastering of, basic skills, followed by mathematical understanding (Mullis et al, 2004). The five content domains in mathematics in TIMSS 2003 were:

1. Number;
2. Algebra;
3. Measurement;
4. Geometry; and
5. Data.

Algebra was called ‘patterns and relationships’ at the fourth grade level. There were four acknowledged cognitive domains:

1. Knowing Facts and Procedures;
2. Using Concepts;
3. Solving Routine Problems; and
4. Reasoning (Mullis et al, 2005).

The table in Appendix II shows the results of the TIMSS 2003 achievement testing in mathematics for the eighth grade population.

The content areas in TIMSS 2003 consider number, algebra, measurement, geometry, and data. Topic areas were decided on with objectives specified to the two relevant populations: fourth and eighth grades. An NRC (National Research Coordinator) was selected by each of the participating countries and the NRCs were involved throughout the process. An item-writing task for the NRCs from each of the participating countries commenced the process of selecting test items for TIMSS 2003. Countries participating in the assessment were then invited to submit topics to be included in the test. At this stage subject-matter specialists considered all the ideas. The mathematical items were field-tested in each of the participating countries with representative sample groups of students. The NRCs were also involved in the review of test items and in the review of the scoring criteria (Mullis et al, 2004).

The student samples were picked carefully in accordance with the TIMSS selection data. TIMSS spent a considerable amount of time constructing procedures and guidelines to ensure that the national samples were of the highest possible quality and were valid and efficient. The NRCs were responsible for the implementation of the assessment in each of the participating countries (Mullis et al, 2004).

At the eighth grade 194 items were tested, while at the fourth grade 161 items were tested. For the eighth grade assessment approximately one-third of the test items were constructed-response items, and involved students generating and writing their answers. Some of these constructed-response questions required extended answers.

3.5.4. Trends in Mathematics and Science Study (TIMSS) 2007

TIMSS 2007 was the fourth study carried out in this cycle of international mathematics and science assessments, and involved 425,000 students from fifty-nine participating countries (Mullis et al, 2008). At the fourth grade the top performing countries were Hong Kong SAR and Singapore, followed by Chinese Taipei and Japan in that order. Other countries that performed well were Kazakhstan, the Russian Federation, England, Latvia, and the Netherlands. The U.S. state of Massachusetts performed extremely well at fourth grade at a level comparable with Chinese Taipei. At the eighth grade the top performing countries were Chinese Taipei, Korea, and Singapore, followed by Hong Kong SAR and Japan. There was a significant gap in average mathematical achievement between these five, top-performing Asian countries and the next group of countries in terms of achievement. The next group of countries consists of Hungary, England, the Russian Federation, and the United States. The U.S. state of Massachusetts performed at a higher mathematical level than this group of four countries, but was outperformed by the five Asian countries at the top of the league performance wise (Martin et al, 2008). The author is particularly interested in the mathematical performance of students in the U.S. state of Massachusetts as the U.S. research collected for this study is obtained from this state. This facilitates a comparison of Irish students with the highest achieving U.S. state in TIMSS assessments.

Countries participating in TIMSS 2007 are: Algeria, Armenia, Australia, Austria, Bahrain, Bosnia and Herzegovina, Botswana, Bulgaria, Chinese Taipei, Colombia, Cyprus, Czech Republic, Denmark, Egypt, El Salvador, England, Georgia, Germany, Ghana, Hong Kong SAR, Hungary, Indonesia, Iran (Islamic Republic of), Israel, Italy, Japan, Jordan, Kazakhstan, Korea (Republic of), Kuwait, Latvia, Lebanon, Lithuania, Malaysia, Malta, Mongolia, Morocco, Netherlands, New Zealand, Norway, Oman, Palestinian Nat'l Auth., Qatar, Romania, Russian Federation, Saudi Arabia, Scotland, Serbia, Singapore, Slovak Republic, Slovenia, Sweden, Syrian Arab Republic, Thailand, Tunisia, Turkey, Ukraine, United States, Yemen, with benchmarking participants Alberta, Canada; Basque Country, Spain; British Columbia, Canada; Dubai, UAE; Massachusetts, US; Minnesota, US; Ontario, Canada; and Quebec, Canada (Mullis et al., 2008). The table in Appendix 3 shows the results of the TIMSS 2007 achievement testing for the eighth grade population.

Students were assessed at both the fourth and eighth grades as part of TIMSS 2007. Fourth grade students were assessed in three content areas: number; geometric shapes and measures; and data display. At the eighth grade students were assessed in four content areas: number; algebra; geometry; and data and chance. Students were given a test that resulted from extensive test development. At the fourth grade the test included 179 items and a total score of 192 points, while at the eighth grade the test contained 215 items, and a total of 238 points. At both fourth and eighth grades roughly half the test items are constructed-response, and half are multiple-choice. Representatives from each of the participating countries were involved in the test design. Each of the countries involved received training and support at each stage from the TIMSS and PIRLS International Study Centre, in Boston College (Mullis et al, 2008).

The first step in test design was an item-writing workshop for the National Research Coordinators from each of the participating countries. Countries involved in TIMSS 2007 were then encouraged to submit possible test items which were reviewed by the selection committee. The test items were tested with representative samples of students in each of the participating countries. There was a constant review of the test items and the scoring criteria to ensure the items would effectively, and fairly, meet the test objectives for all students in the participating countries. TIMSS ensured that stringent procedures were implemented with regard to student sampling. Particular care was given to ensuring that the sample offered an accurate, representative estimate of the student population. Staff from 'Statistics Canada' were involved in ensuring that high sampling standards were implemented throughout (Mullis et al, 2008).

TIMSS again considered three aspects when designing the test:

1. The intended curriculum: what each participating country intends to teach;
2. The actual curriculum: the aspects of the curriculum that are actually taught; and
3. The achieved curriculum: what students actually learn.

In considering the intended curriculum the IEA considered not only the curriculum, as defined by the participating country, but also the supports provided for curriculum implementation: teacher qualification levels, formal assessments etc. All the participating countries were invited, and expected, to write a chapter for the TIMSS 2007 Encyclopedia on their intended curriculum. The major components of the mathematics and science curriculum had to be reiterated in this chapter. Each participating country also answered a questionnaire regarding their mathematics curricula, and its implementation. The implementation of the curriculum was the focus of data collected in questionnaires completed by the principals and teachers of assessed students and the students themselves. Teachers had to supply information on each of the TIMSS content areas. Students were asked to complete questionnaires that considered social influences such as their home experiences, classroom events, extra-curricular factors etc. Teachers and school principals answered questionnaires to provide information about the socio-economic status of their students, school ethos, resources etc. (Mullis et al, 2008).

The main methods of reporting the results from the TIMSS 2007 assessments were based on item response theory (IRT) scaling methods. As TIMSS 2007 is part of the TIMSS series of assessments, it is essential that the scaling methods used to score responses are comparable with the data collected in the preceding TIMSS assessments: 1995, 1999 and 2003. The TIMSS and PIRLS International Study Centre reviewed achievements item statistics for every participating country. Each participating country appointed a National Research Coordinator (NRC). The NRC was responsible for implementing the TIMSS 2007 test in their country, and ensuring that this was done in accordance with the TIMSS guidelines (Mullis et al, 2007).

3.5.5. Trends in International Mathematics Achievement (TIMSS 1995, 1999, 2003 and 2007).

The table in Appendix IV considers international trends in mathematics achievement at the eighth grade as noted in the various TIMSS assessments

(1995, 1999, 2003 and 2007). It is important to note that some of the differences in mathematics performance within particular countries can be anticipated due to the impact of major reforms. The TIMSS Encyclopedia 2007 predicted the possibility of an improvement in mathematics scores in both the Russian Federation and Slovenia due to the addition of an extra year of compulsory schooling at primary level in addition to other positive educational reforms (Mullis et al, 2007). Mullis et al (2008) consider the trend in mathematics achievement by summarising that, at the eighth grade, ten countries had higher average achievement in the 2007 TIMSS assessment than in their initial testing, fifteen countries had a significantly lower mathematics score in 2007, and eleven countries showed no significant change. In contrast, at fourth grade, the authors noted that ten countries had higher achievement in 2007 than their earlier results, five scored lower in 2007, and eight countries showed no significant change.

As TIMSS works on a four-year cycle assessing students at the fourth grade, and again four years later when they are in the eighth grade, it necessitates particular grades performance as they move through the educational system in their country. In comparing results from 2003 with 2007, Mullis et al (2008), note that nine countries performed above the mean average in 2003 and again in 2007. These countries were Singapore, Hong Kong SAR, Japan, Chinese Taipei, Lithuania, the Russian Federation, England, Hungary, and the United States. These also held true for the benchmarking provinces of Ontario and Quebec, both in Canada. Australia, Scotland, Norway, Iran and Tunisia performed close to the scale mean score in both years 2003 and 2007. Performance in Italy deteriorated, from a similar score to the scale average in 2003 to below it in 2007. Mathematical performance improved during this time period in both Slovenia and Armenia. Both countries were below the scale average in 2003 and move closer to the average in 2007 (Mullis et al, 2008).

3.5.6. TIMSS in an International Context

Andrews (in Gates, 2001) suggests that where mathematics is deemed to be taught successfully there are often common teaching and learning practices. He focuses on research undertaken in Japan, France and Hungary in particular. The following are considered effective teaching and learning practices of mathematics based on his findings:

1. Learners are taught in mixed-ability classes for the most part;
2. Each individual class is taught as a unit;
3. The majority of the class is dominated by the teacher presenting information, or managing the talk of others;
4. Learners operate in a public domain;
5. There is a constant review of what is being done by teachers;
6. There is little time spent working from textbooks, and;
7. Homework is used to provide a coherent link between mathematics lessons.

Andrews (in Gates, 2001) also acknowledges common major issues in mathematics teaching in Japan, France and Hungary. These include the following:

1. Mathematics is acknowledged as being difficult;
2. Mathematics is considered a problem-solving activity;
3. Mathematics problems tend to be chosen to exemplify generality;
4. Development of mathematical ideas is considered important;
5. Mathematical vocabulary is emphasised;
6. Proof and justification are an important part of mathematics lessons;
7. Mathematical ideas are constantly revisited, and
8. Routine procedural work is not considered to be particularly important to mathematics learning, and relatively little time is spent on this aspect of mathematics.

It is interesting to compare these core mathematical beliefs from countries with an established, successful mathematical teaching model to the mathematical teaching principles in Ireland, which differ significantly. As discovered in the 'Inside Classrooms' study Irish mathematics teachers value procedural learning above all else and the vast majority of class time is spent on this (Lyons et al, 2003). This is in stark contrast to the afore-mentioned countries: Japan, France and Hungary. In Ireland proof and justification are considered an important aspect of examination success but outside of this they are not deemed an essential mathematical tool.

The TIMSS cognitive domains are

1. Knowing facts and procedures;
2. Using concepts;
3. Solving routine problems; and
4. Reasoning.

The difficulty arises with an international comparative study when it comes to developing reliable, valid achievement scales for the cognitive domains. This is due to the fact that the differences between students, especially across countries, can make it difficult to recognise which cognitive abilities students are utilising when problem solving in mathematics. The TIMSS content domains, (number, algebra, measurement, geometry, and data), are, for the most part, consistent with the curricula of the participating countries. It is interesting to note the different curricula areas where students perform at different levels and competencies, within and across countries. The results from TIMSS 1995, 1999 and 2003 suggest that eighth grade students in the United States perform relatively poorly, in an international context, in geometry, and relatively well on data items (Mullis et al, 2005).

3.5.7. Criticisms of TIMSS

The use of English as the official language in TIMSS is seen to favour some countries, to the detriment of others argue Keitel and Kilpatrick (in Gates, 2001). It is understandable that an assessment that is designed using the English language may naturally benefit English speakers in the style and language used. Others, including Dylan William, argue that the process of 'horse-trading' whereby TIMSS representatives will invariably favour the curriculum and syllabus style of their own individual country will also influence the equity of the assessment (Andrews in Gates, 2001).

3.6 The Programme for International Student Assessment (PISA)

PISA, the Programme for International Student Assessment, is an international assessment that assesses mathematical ability and achievement together with other subjects and skills, including literacy skills. It is a project of the OECD, the Organisation for Economic Co-Operation and Development, and its participants include both OECD and partner countries. The PISA testing is carried out every three years and Ireland participates in each cycle of PISA assessments.

In this section the author considers the PISA assessment process, and Irish performance in PISA. This section is primarily focused on PISA 2003 as it is the PISA assessment that focused on mathematical literacy. The initial PISA assessment was implemented in 2000 and every three years thereafter. The following table provides details of the PISA cycles implemented to date, and the forthcoming PISA 2012 cycle.

Cycle	Major Domain	Minor Domain
PISA 2000	Reading	Mathematics, Science
PISA 2003	Mathematics	Reading, Science and Cross-curricular problem-solving
PISA 2006	Science	Reading, Mathematics
PISA 2009	Reading	Mathematics, Science
PISA 2012	Mathematics	Reading, Science and Cross-curricular problem-solving

Table 3: PISA cycles (www.erc.ie)

The skills and knowledge of 15-year olds are assessed. In 2003 the major assessment focus was on literacy within mathematics. Reading literacy, scientific literacy and cross-curricular assessment solving were also assessed but to a lesser extent. The OECD defines mathematical literacy as

'an individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgments and to use and engage with mathematics in ways that meet the needs of that individual's life as a constructive, concerned and reflective citizen' (OECD, 2003:24 as quoted in Cosgrove et al, 2005).

3.6.1. PISA 2000

In order to assess mathematical literacy, PISA 2000 identified three broad dimensions:

- Processes: the ability to analyse, reason and communicate mathematical ideas.

  Processes are divided into three sub-divisions:

  (i) Reproduction, definitions and computations;

(ii) Connections and integration for problem-solving; and

(iii) Mathematisation, mathematical thinking and generalisation.

- Content: Themes such as change & growth, space & shape, chance, quantitative reasoning, and uncertainty & dependent relations;
- Context: Doing and using mathematics in a variety of different situations including in one's personal life, school life, work and society (OECD, 2000).

The PISA processes used include modelling and problem-solving. OECD, 2000, suggests that the modelling process requires the student to engage in the following:

- Structuring the situation to be modeled;
- Mathematising (reality to mathematics);
- De-mathematising (mathematics to reality);
- Reflecting, analysing and offering a personal critique of models and their results;
- Validating the model; and
- Communicating about the model and its results.

In PISA 2000 the mathematics section covered two topics: 'change and growth'; and 'shape and space'. These mathematics sections covered aspects of the Junior Certificate curriculum including measurement, algebra, functions, geometry and statistics. Ireland performed on a par with the OECD average of 500.0 with a score of 502.9. This performance ranked Ireland in 15th place out of 27 countries. The five highest scoring countries were:

1. Japan: 556.6

2. Korea, Rep. of: 546.9
3. New Zealand: 536.9
4. Finland: 536.2
5. Australia: 533.3.

Ireland demonstrated a strong performance by mathematically weaker students and at the 10th percentile Ireland ranked 14th out of 27 countries, with a score of 394.4 versus 366.6. In contrast to this Irish students struggled to perform at the highest level in mathematics in PISA 2000, and at the 90th percentile Ireland ranked 20th out of 27 countries with a mean score of 606.2 versus an OECD average of 624.8 (Shiel et al, 2001).

3.6.2. PISA 2003

The emphasis of PISA is on education for citizenship and preparedess for adult life. PISA is very much influenced by the Realistic Mathematics Education (RME) movement which is concerned with solving mathematical problems which relate to authentic, real-life problems. The PISA assessments are carried out at three-year intervals, with the primary assessment focus changing within the three main assessment areas: mathematics, reading and science. Cosgrove et al (2005) explain that PISA 2003 was an innovative assessment of mathematics as it did not strive to assess mathematics in terms of the national curricula of participating countries, rather it focused on assessing how well prepared 15-year old students are for participation in society and for meeting real-life challenges currently and in future work and life situations. In this regard PISA 2003 strongly emphasised realistic mathematics. PISA 2003 assessed students in 41 countries. The table in Appendix V displays the participating countries.

PISA mathematics is assessed with regard to three dimensions:

- Context;

- Content; and
- Competency.

There are four recognised context areas:

- Personal;
- Social/occupational;
- Public; and
- Scientific.

The four content areas assessed are:

- Shape and space;
- Change and relationships;
- Quantity; and
- Uncertainty.

The areas of competency for consideration are:

- Reproduction;
- Connections; and
- Reflection.

Reproduction concerns the performance of routine mathematics skills: performing basic calculations, recalling memorised facts, solving basic problems following learned routines etc. Connections is the process of making connections between, within and across mathematical domains. Reflection is the ability to recognise the necessary mathematics in realistic problems, the ability to analyse mathematical procedures, and the ability to develop arguments and generalisations.

3.6.2.1. Ireland and PISA 2003

As 15-year old students are assessed by the Programme for International Student Assessment (PISA), the students examined at this age in Ireland tend to be studying the Junior Certificate Curriculum. The Junior Certificate mathematics syllabus that is currently studied by students was revised and implemented in 2000 and examined for the first time in 2003. This revised syllabus is examined at the end of a three-year period and this is the sole means of recognised assessment. The mathematics course can be followed, and subsequently examined, at three levels: higher, ordinary and foundation level. The terminal examination consists of two papers for both higher and ordinary level, each paper containing six questions with three parts in each question. The foundation level examination consists of one paper which again contains six questions. There is no element of choice and each student must answer all the questions on the papers at their level.

The syllabus has two aims:

1. to contribute to the personal development of students; and
2. to provide them with the necessary mathematical skills needed to further their mathematical education, and furthermore to provide them with adequate mathematical skills for life and work.

The terminal examination assesses the following skills:

- recall;
- relational understanding;
- instrumental understanding; and
- application.

Each of the six questions asked per paper in the established Junior Certificate mathematics examination consists of three parts, labeled (a), (b) and (c). Close and Oldham (2005) believe that the three parts: (a), (b) and (c) assess the skills of

- reproduction;
- slightly harder reproduction; or
- basic connections and connections respectively.

The revised, established Junior Certificate curriculum emphasised a need for the development of relational understanding, and for students to establish the communication skills necessary to debate and defend their reasoning and results. The PISA movement is heavily influenced by the need for realistic mathematics education. PISA assesses the skills of 15-year olds and their mathematical problem-solving abilities in solving real-world problems using mathematical methods. The main aim of PISA is to assess real-world knowledge and preparedness for adult life. The Junior Certificate mathematics syllabus and PISA both have common aims. It is therefore important to establish the Irish outcomes in the PISA examination as a means of assessing the success of the Junior Certificate in meeting its aims.

The PISA assessment is a pen and paper test. This is a familiar method of testing for Irish students as all mathematics examinations in Ireland involve pen and paper. There are five different types of mathematics questions that the student may be asked:

- traditional multiple-choice items;
- complex multiple-choice items;
- closed-constructed response items;
- short-response items; and
- open-constructed response items.

These types of questions are largely unfamiliar to Irish students with no multiple-choice items on the Junior Certificate syllabus. Both the short-response items and the open-constructed response items pose difficulty for an

Irish 15-year old student as there may be a range of possible correct answers for both. This is not a concept that is addressed in the Junior Certificate mathematics curriculum. The closed-constructed response items are most familiar as the answer tends to be required in numerical form. The mathematical content areas focused on in the PISA assessment are space and shape, change and relationships, quantity and uncertainty. The area of uncertainty includes probability which poses a problem for Irish Junior Certificate students as it is not on the curriculum until the Leaving Certificate.
The Irish performance in the PISA assessment in 2003 is distinctly average, ranking 17th of the 29 OECD countries and 20th of 40 participating countries. The three highest achieving countries are Hong Kong- China, Finland and Korea. The top-ranking European Union country is the Netherlands/Belgium. Cosgrove et al (2005) believe that the Irish mathematics performance in PISA 2003 is perhaps better than expected due to the amount of mathematising required. Mathematisation is the process of utilising mathematical skills to solve real life problems. There is little emphasis on realistic mathematics in the Irish syllabus, with the style of question used focusing on abstraction.

The gap between the best and poorest performing students is also considered by PISA in the observation of the gap between the 10th and 90th percentile ranks within the country. The gap between the highest achievers and the lowest achievers in the PISA assessment in Ireland in 2003 was relatively low, indicating a narrow spread of achievement in comparison to the OECD average. In the PISA assessment an item response theory scaling is used so that the difficulty of questions and student scores can be placed on the same scale. Students can score results from Level 1 up to Level 6 with Level 1 being the lowest and 6 the highest. The lowest achieving students in Ireland were ranked higher than their OECD colleagues, with 17% of Irish students scoring at the lowest level (level 1) in comparison to an OECD average of 21%. The highest achievers in Ireland were ranked lower than those in other countries with 11% of Irish students scoring at the highest proficiency level (levels 5 and 6 combined) in comparison to an OECD average of 15%. This indicates that while we are providing adequate mathematics education for the weakest students within our system we are not sufficiently nurturing and challenging those that are at the top of the group. This results in a situation where while we have fewer low-achievers in mathematics in Ireland than other OECD countries we also have fewer high-achievers. Irish mathematics students tend to perform very much within the average parameters with very few students at either extreme. In 2003, 72% of Irish students scored at Levels 2, 3 and 4 in comparison to 64% scoring at these levels in other OECD countries. This illustrates the mediocrity of our mathematics scores in comparison to other countries. This is a fundamental issue which must be addressed if we want a successful, knowledge economy in Ireland.

In 2003, Irish students performed at a comparative level to the OECD average with the average Irish student achieving a mean of 503 compared to the OECD average of 500. In the PISA assessment Irish students performed similarly to those in other OECD countries in the area of Quantity with an average score of 501.7 compared to the OECD average of 500.7. In the areas of Change and Relationships, and Uncertainty Irish students performed higher than the OECD average with a score of 506 in Change and Relationships compared with the OECD average of 498.8, and 517.2 in Uncertainty compared with 502. In the content area of Space and Shape Irish students had difficulty, significantly under performing in comparison to their OECD counter-parts with a score of 476.2 compared with 496.3. The 2000 PISA assessment tested two of the areas in mathematics that were tested in 2003: Space and Shape, and Change and Relationships. There was no significant change in Irish performance between 2000 and 2003 despite the introduction of the revised Junior Certificate curriculum during this period. This is of some concern as there was a significant increase in the OECD average score in the content area of Change and Relationships during this time (Shiel et al, 2007).

Shiel et al (2007) explain that while many of the objectives of the Junior Certificate curriculum compare favourably with those of the PISA assessments not all of the Junior Certificate objectives are examined in the final examination. It is the author’s opinion that this may lead to a situation where non-examinable objectives in the Irish curriculum may be neglected in favour of those that are needed to perform well in the Junior Certificate examination. An objective of the Junior Certificate mathematics curriculum is the development of relational understanding and the ability to apply one's mathematical knowledge to solve real-life mathematical problems. This is the concept known as mathematisation. This is consistent with the primary aim of PISA: to assess students' ability to use their mathematical knowledge to solve real-world problems. However, while the aims of both the Junior Certificate mathematics curriculum and PISA with respect to relational understanding appear comparable this skill is not actually assessed in the Junior Certificate examination which leads to this concept being given less time, if any, in the Irish mathematics classroom. The Junior Certificate curriculum also aims to foster an appreciate of mathematics, which is also a PISA aim, yet as this is not an examinable skill it again may be neglected within the classroom due to the examination driven nature of the Irish system. The Junior Certificate requires the skill of reproducing for many of the content items which is not a skill that is encouraged by PISA. For these reasons teachers in the Irish mathematics classroom will probably tend to focus on examinable skills such as the reproduction of examination style questions, as opposed to higher-order skills involving reflection, discussion and explanation.

Cosgrove et al (2005) suggest that there is a divergence between what is learned and what is assessed when one compares the aims and objectives of the Junior Certificate curriculum and PISA. The Junior Certificate syllabus necessitates vertical learning, where increasingly difficult mathematics are presented, usually in abstract contexts. The PISA assessment uses horizontal mathematising, where mathematical skills are used to solve real-life problems. Freudenthal (1991) described horizontal mathematisations as leading 'from the world of life to the world of symbols'. Cosgrove et al (2005) recognise that the Junior Certificate assessment is more generous in recognising effort, by the award of attempt marks, than the PISA assessment. Conway and Sloane (2005) discuss the importance of the Junior Certificate providing students with an opportunity to experience fully the dual nature of mathematics in terms of both horizontal and vertical mathematising. The authors discuss how one danger of reform of the Irish mathematics syllabus is the possibility that only one type of learning may be deemed appropriate for each of the levels higher, ordinary and foundation based on an assessment of student ability.

In 'PISA Mathematics: A Teacher's Guide' the authors asked curriculum experts in Ireland to rate the familiarity for Irish 15-year olds of each of the PISA mathematics items. Two-thirds of items were rated as being somewhat or very familiar to higher and ordinary level Junior Certificate students, with a half of items rated similarly for foundation level students. The context in which items are presented, usually in terms of real-life situations, and the format for answering these items were rated to be largely unfamiliar to all Irish Junior Certificate students. It was determined that PISA items could not always be found on the Junior Certificate syllabus. Of the PISA items, 29% could not be found on the higher level mathematics course, 33% were not on the ordinary level curriculum and 49% were deemed to be missing on the foundation level course. The topics that could not be located on the Junior Certificate syllabus included the space and shape items which could not be found in the Junior Certificate at any of the three levels, despite geometry being part of the curriculum. The curriculum experts determined that this was due to the fact that geometry in the Irish curriculum focuses on traditional Euclidian geometry, while PISA geometry emphasises visualisation skills. Therefore, Irish students are not given the opportunity to demonstrate the geometry skills they have obtained by the age of fifteen. The experts also expressed their opinion that there are discrepancies between Junior Certificate algebra, which is a significant part of the syllabus, and PISA algebra. This creates a situation where Irish students did not get sufficient opportunity to demonstrate their algebraic skills and ability. Despite the differences in the assessments there was a strong correlation between student performance in the Junior Certificate examination and the PISA assessment 2003, with a correlation between the two of 0.75, as discussed in Shiel et al (2007).

Close and Oldham (2005) examine the link between the 2003 Junior Certificate Examination and the PISA assessment. Their findings indicate that Junior Certificate students have not developed the necessary analytical and reflective skills, therefore leaving them unprepared for the high proportion of PISA items requiring reflection. The authors find that Irish teaching methods, concentrating on exposition and practice, provide little time for in-class reflection or discussion. As reflection, discussion and analysis are not key objectives of the Leaving Certificate syllabus, and are not easily examinable skills, it is unlikely that older Irish mathematics students would perform well in these areas either.

3.6.3. PISA 2006

Ireland's mean score in PISA 2006 is 501.5 points which does not differ significantly from the OECD mean of 497.7. Ireland scored 16th out of the 30 OECD countries and 22nd out of the 57 participating countries. The five top-scoring countries are:

1. Chinese Taipei: 549.4
2. Finland: 548.4
3. Hong Kong-China: 547.5
4. Korea: 547.5
5. Netherlands: 530.7 (Eivers et al, 2007).

Ireland had fewer students performing at the highest proficiency level, Level 6, than the OECD average (1.6% versus 3.3%). At the lowest proficiency level, Level 1, Ireland performed better than the OECD average (16% versus approximately 21%), but poorer than high-scoring countries such as Korea and Hong Kong-China (approximately 9%) and Finland (6%) (Eivers et al, 2007). The following table, provided by Eivers et al (2007) demonstrates Irish performance at each mathematical level identified by PISA, in comparison to the OECD average.

Level cut-off point	At this level most students can:	Ireland %	OECD %
Level 6 (<669.3)	Evaluate, generalise and use information from the mathematical modelling of complex probems.	1.6	3.3
Level 5 (607.0-669.3)	Develop and work with the mathematical modelling of complex situations.	8.6	10.0
Level 4 (544.7-607.0)	Work with mathematical models of complex, concrete situations	20.6	19.1
Level 3 (482.4-544.7)	Work in familiar contexts, usually requiring multiple steps for solution.	28.6	24.3
Level 2 (420.1-482.4)	Work in simple contexts that require no more than direct inference.	24.1	21.9
Level 1 (357.8-420.1)	Work on clearly defined tasks with familiar contexts where all the relevant information is present and inference is not required.	12.3	13.6
Below Level 1 <357.8	Do not respond correctly to more than 50% of Level 1 questions.	4.1	7.7

Table 4: Irish test performance in PISA 2006 compared to OECD average (Eivers et al, 2007)

In PISA 2006 Irish boys outperformed girls in mathematical literacy (507.3 versus 495.8). Only 0.9% of Irish females reached Level 6, versus 2.4% of Irish males. The OECD average was 2.5% of females and 4.2% of males reaching Level 6. This is similar to the OECD average (503.2 versus 492.0 in favour of males). There was a gender difference, favouring males, in 22 of the 30 OECD countries (the largest gap was 23 points in Austria). Qatar was the only country where there was a significant gender difference favouring females (Eivers et al, 2007).

3.6.4. PISA 2009

Ireland obtained a mean score of 487.1 in mathematics in PISA 2009. This is significantly below the OECD average of 495.7. Ireland ranked 26th out of 34

OECD countries and 32nd out of 65 participating countries. The six top-scoring countries are:

1. Shanghai-China (600.1)
2. Singapore (562.0)
3. Hong Kong – China (554.5)
4. Korea (546.2)
5. Chinese-Taipei (543.2)
6. Finland (540.5) (Shiel et al, 2010).

Again, Ireland has significantly fewer students scoring at the higher levels (Level 5 and 6). At the top two levels 6.7% of Irish students performed sufficiently versus the OECD percentage of 12.7%. The United Kingdom and Poland both scored similarly in terms of the overall mean score in mathematical performance, but outscored Ireland significantly at Level 5 and 6 with percentages of 9.8% and 10.4% respectively. Interestingly, despite Northern Ireland obtaining a mean score of 492.2, which is similar to Ireland, they significantly outperformed the Republic in performance at the top levels with 10.3% of students performing at levels 5 and 6. Finland had an amazing 21.6% of students obtaining the necessary points at levels 5 and 6. At the lower end of the scale Ireland had 20.8% scoring at or below level 1, which is slightly fewer than the OECD average (Shiel et al, 2010).

In PISA 2009 Irish males achieved a higher mean score (490.9) than females (483.3) in mathematics, but the difference was not significant. Both male and female scores are significantly lower than the OECD averages of 501.4 and 489.9 respectively. Twenty-one OECD countries had a significant gender difference favouring males. The remaining 13 OECD countries had no significant gender difference. The proportion of Irish males, 20.6%, and females, 21%, at or below level 1 is comparable to the OECD averages of 20.9% and 23.1% respectively. At Levels 5 and 6 fewer Irish male (8.1%) and female (5.1%) students scored sufficiently compared to OECD figures of 14.8% and 10.6% respectively (Shiel et al, 2010).

3.6.5. Changes in Irish PISA Performance from 2003 to 2009.

Irish mathematical performance in PISA declined from 502.8 in PISA 2003 to 487.1 in PISA 2009. This is a drop of 16 points. Only one other country experienced a greater decline, the Czech Republic, with 24 points. Following from this decline, Ireland's rank dropped from 20th to 26th among the participating OECD countries. Ireland's position also altered from being at the OECD average in 2003 to significantly below it in 2009 (Shiel et al, 2010). This is a worrying trend that shows a downward slide from results that are already mediocre at best.

In 2003 the proportion of Irish students at or below Level 1 was 16.8%. By 2009 this figure had increased to 20.8%. The number of students at or above Level 5 decreased from 11.3% in 2003 to 6.7% in 2009. Performance for both male and female students dropped significantly also (down 19 points for male students and 12 points for female students) (Shiel et al, 2010).

The following table illustrates the key changes in Irish mathematical test performance from PISA 2003 to PISA 2009:

Mathematics (Irish test performance)	***2003***	***2009***	***Change from 2009 to 2003***
Mean Score	503	487	-16
Mean Score (Males)	510	491	-19
Mean Score (Females)	495	483	-12
Gender Difference	15.0	8.0	-7.0
% at or below Level 1	16.8	20.8	4.0

Mathematics (Irish test performance)	*2003*	*2009*	*Change from 2009 to 2003*
% at or above Level 5	11.3	6.7	-4.7

Table 5: Changes in Irish performance in mathematics, PISA 2003-2009 (Shiel et al, 2010)

3.7 International Assessment of Educational Progress (IAEP)

In 1988 the Educational Testing Services (ITS) carried out the First International Assessment of Educational Progress (IAEP) (Howson in Kaiser et al, 1999). The primary purpose of IAEP was to collect data on what students already know and their ability to apply this information, but cultural and societal factors plus students attitudes were also considered. IAEP-1 was carried out in 1988 and assessed the achievement of 13-year old students. The students were assessed in mathematics and science in six countries, the United States and five others.

IAEP-1 is of particular interest to the author as Ireland was one of the six countries to participate in the study. The study considered five countries and four Canadian provinces (as Canada does not have a federal system of education). These included:

- British Colombia, Canada;
- New Brunswick (French and English), Canada;
- Ontario (French and English), Canada;
- Quebec, Canada;
- Ireland;
- Korea;
- Spain;
- The United Kingdom (with students represented from Scotland, Wales and England); and

- The United States.

(Board of International Comparative Studies in Education, National Research Council, 1995:49).

IAEP-2 was a similar assessment, carried out in 1991, but on a much larger scale as it involved twenty countries. In IAEP-2 the mathematical and scientific skills of both 9-year olds and 13-year olds were considered. Twenty countries assessed 13-year olds and fourteen countries assessed the science and mathematical skills of 9-year olds. The samples collected in IAEP-2 varied significantly from country to country, with some countries assessing all age-eligible students but others selectively assessing for various reasons (including geographical and language restrictions). There were also a significant number of age-eligible children omitted in some countries due to the fact that they were not attending school. The variation in assessment levels within countries makes it difficult to accurately compare the data collected and leads to a bias (www.nap.edu).

Countries involved in IAEP-2, and their level of participation are considered in the following table (all students refers to all age-eligible students):

Brazil	São Paulo and Fortaleza, restricted grades, in-school population
Canada	Four provinces at age 9 and nine out of 10 provinces at age 13
China	20 out of 29 provinces and independent cities, restricted grades, in-school population
England	All students, low participation at ages 9 and 13
France	All students
Hungary	All students
Ireland	All students
Israel	Hebrew-speaking schools only
Italy	Province of Emilia-Romagna, low participation at age 9

Jordan	All students
Korea	All students
Mozambique	Cities of Maputo and Beira, in-school population, low participation (Mozambique did not assess Science.)
Portugal	Restricted grades, in-school population at age 13
Scotland	All students, low participation at age 9
Slovenia	All students
Soviet Union	14 out of 15 republics, Russian-speaking schools
Spain	All regions except Cataluña
Switzerland	15 out of 26 cantons
Taiwan	All students
United States	All students

Table 6: Participating countries in IAEP-2 (Board on International Comparative Studies in Education, National Research Council, 1995:50)

3.8 The Learner's Perspective Study

Clarke et al (2006) believe that it is *'an essential thesis of the Learner's Perspective Study (LPS) that international comparative research offers unique opportunities to interrogate established practice, existing theories and entrenched assumptions'* (Clarke et al, 2006:1). The Learner's Perspective Study examines mathematics practice in eighth-grade classrooms in twelve countries. The authors believe that their study achieves this in a more integrated and comprehensive fashion than previous international studies. Clarke et al (2006) designed a series of research questions to assist data collection:

1. '*Within the classrooms studied in each country, is there evidence of a coherent body of student practice(s) (and to what extent might these practices be culturally specific)?'*;

2. *'What are the antecedent and consequent conditions and actions (particularly learner actions) associated with teacher practices identified in earlier studies as culturally specific and nationally characteristic?';*
3. *'To what extent does an individual teacher employ a variety of pedagogical approaches (and/or lesson scripts) in the course of teaching a lesson sequence?';*
4. *'What degree of similarity or difference (both locally and internationally) can be found in the learner (and teacher) practices occurring in classrooms identified by the local education community as constituting sites of competent teaching practice?';*
5. *'To what extent are teacher and learner practices in a mutually supportive relationship?';*
6. *'To what extent are particular documented teacher and learner practices associated with student construction of valued social and mathematical meanings?';* and
7. '*What are the implications for teacher education and the organisation of schools of the identification of those teacher and learner practices that appear to be consistent with the realisation of local goals (and those which are not)?*' (Clarke et al, 2006:7-9).

The research design of the Learner's Perspective Study attempts to complement the survey style approach by giving more time to the perspective of the learner. Clarke et al (2006) recognise three key requirements:

1. The recording of interpersonal conversations between students during mathematics lessons;

2. The documentation of sequences of mathematics lessons, ideally covering a complete mathematics topic; and
3. The identification of the intentions and interpretations underlying the actions and statements of the students during the lesson.

Clarke et al (2006) seek to challenge the traditional approach of a videotape study:

'*single-camera and single-microphone approaches, with a focus on the teacher, embody a view of the passive, silent student, which is at odds with contemporary learning theory and classroom experience'* (Clarke et al, 2006:16).

This single-camera approach does not consider the perspective of the learner. Important factors that affect any classroom video study are considered in the process often known as 'data reduction' or 'data construction'. Clarke et al consider the following as being of the utmost importance: the choice of classroom; the number of cameras used in the classroom; and who the cameras focus on, who is kept in view constantly and who is only picked up on camera in certain circumstances (Clarke et al, 2006). Data construction in the LPS (Learner's Perspective study) utilised a three-camera approach: a camera focused on the teacher, another on the collective body of students, and a third focusing on the class as a whole. The use of these images (generally with a picture-in-picture image that showed the teacher in the top right-hand corner of the screen) was then used in post-lesson interviews to stimulate participants accounts of what had occurred during the mathematics lesson in question. This method was used for a sequence of at least ten consecutive lessons. The same research design was used in each of the twelve participating countries in order to enable the comparison of data. Three mathematics teachers were selected in each country to participate in the study. All teachers involved in the study were selected based on their acknowledged competence at a local level. Teacher selection was made by a local research group, in accordance with defined criteria (Clarke et al, 2006).

Students were given control of the video player, and the ability to replay certain events they considered important, in the post-lesson interviews, and were asked to comment on the classroom events. Post-lesson interviews were conducted on an individual basis in all countries except Germany, Israel and South Africa, where students strongly requested group interviews. Each of the three teachers in each country were also interviewed, at least three times, in a similar manner. 'The validity of students' and teachers' verbal reconstructions of their motivations, feelings and thoughts was given significant thought' due to the interviews used in the study (Clarke et al, 2006:21).

In analysing the data for the Australian and American participants in the LPS the following:

1. There is evidence that the structure of a single U.S. lesson could not capture the essence of a typical lesson structure for all observed U.S. classrooms;
2. Four distinct classroom activities occurred in the U.S. mathematics lessons observed: reviewing previous material; demonstrating how to solve problems for the day; practicing; and correcting seatwork and assigning homework;
3. The lesson pattern reported in the TIMSS Classroom Videotape study (that a classroom begins with reviewing previous material followed; then with a demonstration of how to solve problems for the day; followed by practicing; and ending with correcting seatwork and assigning homework) did not appear as the complete lesson structure in any of the 80 U.S. lessons observed;
4. Almost all U.S. lessons began with students correcting their homework from a transparency displayed by the teacher;
5. Large portions of U.S. time were devoted to student practice and little time was given to teacher demonstration;
6. The US lesson pattern did not match any of the observed Australian lessons;

7. There appeared to be significant structural differences between the Australian and the US lessons analysed;
8. Australian mathematics lessons demonstrated a rapid alternation of activity types (Mesiti and Clarke, 2003).

In relation to the comparison of LPS data between the U.S. and Australia Mesiti and Clarke (2003) make the important point that sometimes the similarities in lessons in one country are only evident when they are compared to lessons in another country. They also address the fact that there is a huge variation in lesson structure within countries and that it is important not to characterize an entire nations mathematics lessons with a single pattern. This highlights the value of comparative studies within and between countries.

3.9 Mathematics in South East Asia

No analysis of international mathematics education is complete without a reflection on mathematics achievement in East Asian countries. When the TIMSS 1995 results were first published in 1996 the mathematics education community was surprised at the sheer excellence of the achievement scores in East Asian countries. The four countries in particular that scored in the first four positions in mathematics achievement in TIMSS 1995 were Hong Kong, Japan, Korea and Singapore. Taiwan joined TIMSS in 1999 and since them these five East Asian countries are consistently in the top five positions in the TIMSS cycle of assessments: TIMSS 1999, 2004 and 2007. These five countries perform equally well in the PISA assessments in mathematics. Mainland China has not participated in either TIMSS or PISA but students from Mainland China have performed at an equally impressive level in mathematics in assessments such as IAEP (Leung and Li, 2010).

3.9.1. Mathematics Education in Mainland China

There has been significant reform in Mathematics Education in Mainland China over the past decade. Until ten years ago Chinese students performed relatively well in mathematics competitions and large-scale mathematical studies, but there was an over-emphasis on acquisition of knowledge and skills. Mathematics instruction tended to focus on lecturing and memorisation to the detriment of other mathematical skills. Chinese students also spent more time studying mathematics, both in and out of school (Liu and Li in Leung & Li, 2010).

Liu and Li (in Leung and Li, 2010) acknowledge the following steps towards mathematics curriculum reform that have led the process of change over the last ten years:

1. Curriculum Objectives: A move away from an over-emphasis on knowledge acquisition;
2. Curriculum Structure: The school curriculum was deemed to have a need to be more balanced, with a move away from over-emphasising content-based subjects;
3. Curriculum Content: Emphasises mathematical connections with every-day life and knowledge, developed in conjunction with science and technology.
4. Careful selection and encouragement of the skills needed for life-long learning;
5. Curriculum Implementation: A move away from an over-emphasis on students' acceptance, memorisation, drill and practice. Encourage students to learn through active participation, analysing and solving problems, communication and collaboration;
6. Curriculum Program Evaluation: Less differentiation and selection by students' ability;
7. Curriculum Administration: Adapt the curriculum so that it is accessible to all students, regardless of their region and/or school. Administer the curriculum at three levels: national, regional and school.

(Liu and Li, in Leung and Li, 2010).

Textbooks in Mainland China underwent reform as part of the over-haul of mathematics education. The new experimental textbooks have been found advantageous for the following reasons as they:

- Provide opportunities for students to explore mathematics independently;
- Place value on connections in mathematics with students' own personal experience;
- Provide opportunities for 'exploration-orientated' teaching; and
- Provide teaching options for teachers.

(Liu and Li, in Leung and Li, 2010).

The curriculum changes implemented in China changed the focus of mathematics education away from knowledge acquisition and towards preparing students for future life experience. A new value was also placed on students having a positive school and mathematical experience. Openness and collaboration were also emphasised for the first time, and as outlined above the emphasis has moved from rote-learning towards exploratory learning and developing creativity in mathematics.

3.9.2. Mathematics Education in Japan

Andrews (in Gates, 2001) describes a typical Japanese lesson as one which moves at a slow pace because thinking about a problem, discussing it and focusing on the problem at hand are deemed more important than getting the correct answer. There is little time spent on textbook exercises. Students tend to work on mathematical problems in either small groups or individually, before a teacher-led discussion is held, with students ideas presented on the board. The discussion element of the lesson is significant with the teacher also leading a discussion about the previous lesson's problem before moving on to the current lesson's topic (Andrews in Gates, 2001).

Conway and Sloane (2005) note that students in Japan are encouraged to solve problems by working with each-other, which lends to Brown's (1997) notion of Fostering a Community of Learning (FCL) (see Chapter 2 for further explanation). Conway and Sloane note that in an analysis of 8th grade classrooms in Japan the lesson often commenced with the teacher posing a mathematics problem on the board, telling students to consider the problem at hand and consult each-other, and then encouraging students to share their solutions and opinions with the whole class. This also encourages mathematical language skills. Conway and Sloane note that this type of FCL teaching is in marked contrast to observed lessons, at the same level, in the United States and Germany where the mathematics teaching was didactic and teacher-led, with students practicing their skills on almost identical problems. The author believes that the didactic, autocratic, teacher-led classroom style followed in these countries is very similar to what is currently happening in the Irish classroom.

Stigler and Hiebert (1999) considered mathematics classrooms, of eighth grade students, in Japan, Germany and the USA. Their research has focused on the essentially culturally nature of teaching. They observe that Japanese lessons are never interrupted from the outside. *'The lesson as a unit is the central element in the culture of the Japanese school, and each lesson must tell a coherent story'* (Clarke et al, 2006:10). In comparison mathematics lessons in the USA are combinations of smaller units. For this reason mathematics lessons in Japan are not disrupted in the same way by external factors (such as public address system, lunch monitors etc.) as they may be in the USA where interruptions are more common.

In the Irish education system, in contrast to Japan and more in line with the education system in the USA, interruptions do tend to happen. Not only are public address systems common-place but also visits by year-heads, student-teachers etc. occur. It is also a common occurrence in the Irish education system for students to miss mathematics classes (along with classes in other subjects) as a result of being involved in extra-curriculum activities such as sports. Due to the fact that it is extremely unlikely that all students in a mathematics class will play the same sport for the school it is most likely that a minority of students will miss school for a given sport. As a result the mathematics lesson tends to continue as normal and these students miss out on the topic covered on that particular day. This is particularly detrimental for students that are involved in more than one extra-curricular activity in the school.

3.10 Conclusion

The author uses the insight gained from the literature reviewed in this chapter to design the research questions to effectively test the hypothesis: ‘That Irish students have the ability to transfer the mathematics learned in school to solve real-life problems’. The author plans to introduce an international component to her research by testing students not only from Ireland, but also from the U.S. state of Massachusetts. The comparative nature of the implemented tests gives an international component which raises interesting questions about the Irish curriculum and assessment style.

4.0 Chapter 4: Modelling and mathematics education

4.1 Introduction

This chapter considers modelling as a means of linking classroom mathematics with real-life scenarios. The following is a summary of the literature surrounding the possibilities available if one is to consider introducing modelling as a means of incorporating the realistic into the classroom.

4.2 Traditional Mathematics

Yanagimoto (2005) discusses how traditionally taught mathematics often consists of unnatural problems whose sole purpose is to teach a particular concept. The author suggests an example of what she considers to be a traditional, meaningless type of mathematics question:

> '*I bought ten writing instruments, both pencil and ball-point pens. I paid 960 yen. The pencils cost 60 yen each; the ball-point pens cost 120 yen each. How many pencils did I buy?*' (Yanagimoto, 2005:12).

Modelling problems differ from problems posed in traditional mathematics education in that the problems introduced are ones which are faced in the society in which one lives, not unnatural problems that have been constructed solely to teach a mathematical principle. In traditional mathematics the value of the solution is judged by whether it is important to the system of pure mathematics, or the regularity with which the problem comes up in examinations; in modelling the value of the solution is judged by the results of its application.

Lege (2005) explains that imitation and practice are cornerstones of traditional mathematics education, especially in relation to problem-solving. Keast (1999) highlights the following features of traditional mathematics. A traditional mathematics lesson:

- Has an authoritative figure who gives out information in a non-contextual way. This information does not appear to have any relevance to the life of the student;
- Promotes learning that is based on remembering and applying rules;
- Consists of contrived exercises, examples and problems bearing little resemblance to real-life;
- Encourages students to work individually; and
- Promotes the idea that answers in mathematics are always known and pre-determined, and that an answer is either right or wrong. This leaves little room for discovery learning and/or creativity.

Reusser and Stebler (1997) explain that students solve stereotypical, traditional mathematics problems with little relevance to real life (even if they are unsolvable). This outlook is based on the following assumptions by the student:

- That every problem presented by a teacher or in a textbook makes sense;
- That every question in mathematics is inherently correct and complete;
- That there is only one 'correct' answer to every problem;
- That one is obliged to give an answer to every question presented;
- That all numbers provided in the question must be used in order to arrive at the correct solution;
- If a chosen mathematical operation results in an answer that is a whole number (with no remainder) then one is probably on the correct track;
- If one does not understand a problem then look for key words and at previously solved problems and repeat the same steps. Reproduction and pattern detection are key mathematical skills in solving traditional mathematics problems.

Reusser and Stebler (1997) assert that these assumptions inhibit correct mathematisation and the ability of students to deal with realistic, not always tidy, variables. If students are to develop an ability to transfer their mathematical skills to real-life situations than school experience should promote their ability to do so and not impede their mathematical development by dressing mathematics up with rules, procedures and formulae that must always be followed if the one, correct answer is to be arrived at.

4.3 Mathematical beliefs

Maab (2005) proposes that mathematical beliefs may be the main barriers to the integration of modelling in the traditional school setting. The mathematical belief system is formed by all beliefs regarding mathematics, mathematics instruction and the studying of mathematics. Maab (2005) raises the following questions:

- How do students' mathematical beliefs change over a course of mathematics classes during which modelling exercises are introduced and integrated?
- How far do such lessons enable students to carry out modelling processes independently?
- What connections exist between mathematical beliefs and modelling competencies?

When modelling is introduced in the classroom setting Maab found that the single factor that affects the process the most is the mathematical belief system of the students and their teachers. Students' attitudes to mathematics and mathematical modelling are closely related to their mathematical beliefs.

Grigutsch (1996) identified students' mathematical beliefs as falling into four categories. Mathematics is referred to by Grigutsch as a field of science. The four main categories for mathematical beliefs are mathematics as:

1. A science which mainly consists of problem solving processes.
2. A science which is relevant for society and life.
3. An exact, formal and logical science.
4. A collection of rules and formulae.

4.4 Attitudes towards mathematics

In designing an introductory course in mathematics for pre-university students in a university, UNEG, in Buenos Aires, Falsetti and Rodriguez (2005) considered the introduction of mathematical modelling as a means of combating negative attitudes towards mathematics. Their aim was to use modelling to improve mathematical confidence and hence performance in mathematics. A previous assessment determined the following characteristics among the students in the university towards mathematics:

- A feeling of inhibition towards mathematics;
- Intimidation when faced when variations in problems from the ones they have previously practiced and learned;
- No reflection period when solving a problem;
- A feeling that mathematics is very formal and regimental;
- No problem solving strategies; and
- Difficulty in making simple deductions and reasonings.

Falsetti and Rodriguez (2005) aimed to instil in the students an image of mathematics as something they could contribute to and be a part of.

'If the boundary between mathematical and non-mathematical contexts can be seen as permeable, mathematics can be seen as a science that gives approximate, not categorical answers to the problems' (Falsetti and Rodriguez, 2005:15).

Maab (2005) identified four main types of learners with regard to the mathematical modelling process:

- Type 1: The reality-distant modeller: is opposed to the modelling process and embraces context rich mathematics. This results in a lack of competency in dealing with modelling problems. This type of learner has problems with the construction of the real model, the validation and the interpretation.
- Type 2: The mathematics-distant modeller: gives preference to real-world problems. This type of learner has a negative attitude to

traditional mathematics and generally performs poorly in mathematics tests. The mathematics-distant modeller is good at constructing the real model but runs into difficulty when it comes to mathematising and constructing the mathematical model.

- Type 3: The reflecting modeller: has a positive attitude towards both mathematics and mathematical modelling.
- Type 4: The uninterested modeller: has a negative attitude towards both mathematics and mathematical modelling. This type of learner struggles with the modelling process and also lacks basic mathematics competency.

(Maab, 2005: 70).

4.5 Anti-modelling environment

Lin and Yang (2005) describe a non-friendly modelling environment as coming from three primary sources:

- The background of mathematics teachers and students;
- The examination process;
- An education system that relies heavily on textbooks.

The specific situation that Lin and Yang consider is that of Taiwanese students. Mathematics students in Taiwan perform very well in international assessments such as TIMSS but the authors believe that this does not demonstrate relational understanding. In Taiwan the traditional mathematics environment in the classroom leads to a situation where mathematical applications and modelling are unfamiliar and as a result students and teachers may feel excluded. The study carried out by Lin and Yang found that not all teachers are content with their students using their own approaches without being instructed by the teacher on how to go about solving the problem. The students in turn are used to being told what is correct and when their solution is the appropriate one. Students are also familiar with the teacher demonstrating procedures which they then follow in solving mathematical problems.

Lin and Yang (2005) also describe the over emphasis of examination preparation as having a negative impact on modelling in the classroom. In Taiwan students and teachers are very focused on preparing for the Joint University Entrance Examination (JUEE). This is of primary concern with regard to teaching and learning. Not only does this examination determine the future academic success of the students involved but the authors also explain that the teachers' performance is also determined by the success of their students. This leads to an emphasis on memorisation of formulae and procedures, recall, drills and repetition of algorithms and constant practice of typical and traditional examination problems. This situation is very similar to the Irish examination system's impact on the teaching and learning of mathematics in Irish schools. Perhaps the sole discrepancy is the reputation of the teacher being at risk. Despite this many Irish teachers and educators would possibly argue that a teacher's reputation is determined within schools and communities by the class groups they teach and the examination success of those students. In this regard perhaps the Taiwanese situation is more similar than the author initially believed to be the case.

The role of textbooks in the Irish education system and the over-reliance on textbooks by Irish students and teachers may have a negative impact on any attempt to introduce modelling to the traditional mathematics classroom. The primary purpose of the textbook is to support the mathematics teacher and provide a useful resource to assist one's teaching. Yet many Irish mathematics teachers are led through the curriculum, with the textbook leading the way. A situation arises where teachers are slavishly following the textbook with little variation or adaption introduced by themselves. Textbooks emphasise drill and practice through the process of solving numerous mathematical questions of similar format and content. This in turn dominates these skills in Irish mathematics lessons and reproduction of familiar solutions becomes the key skill.

'"*Real" situations in textbooks were like ornaments that do not motivate learning and like an artificial reality that do require modelling with solving problems'* (Lin and Yang, 2005).

A further issue which arises due to an over emphasis on the use of a textbook is the dependency on the quality of textbook itself. The quality of some of the popular textbooks used and the inherent pedagogical value may be questionable. This is an issue that is not given adequate consideration in Irish schools where brand loyalty may be an over-riding factor when textbook selection is under way.

4.6 The modelling process:

Yanagimoto (2005) proposes that it is necessary to review existing mathematics teaching methods that have nothing to do with real life and the functioning of society. Yanagimoto believes that the future in mathematics education relies on school mathematics focusing on undeveloped mathematical problems that have no proper answers or solutions. The most important aspect of mathematics teaching and learning should be the development of a creative, scientific mind and this is stifled by traditional mathematics education methods. An over emphasis on question recognition and reproduction is in direct opposition to the development of creative or flexible mathematical thought processes. The predictability of the Junior and Leaving Certificate mathematics examination questions are also negating creative thought. Students in Ireland are currently taught that mathematics is about recognising a pattern and solving the question as you normally would. The idea that mathematics is creative and unpredictable is not a concept that the majority of Irish mathematics students would agree with on completing their second level education. This is in contrast to what Yanagimoto suggests is the future in mathematics education.

'The aim is to show students a zigzag thought process in the making, as well as models of refined perfection, in order to remind them of the importance of flexibility in ideas' (Yanagimoto, 2005:2).

Yanagimoto (2005) suggests that the benefits of mathematical modelling in education include:

- Helping students to solve complicated problems involving actual phenomena using mathematics;
- Modelling enables students to demonstrate their creativity in their approach to solving the problem;
- Students are given the opportunity to approach problems they have not solved before, using methods they have not encountered before.

(Yanagimoto, 2005).

The following steps occur as part of the modelling process according to Maab (2005: 62):

1. A real situation is simplified, idealised and structured;
2. This leads to a model of the original, realistic situation;
3. The real model is then mathematised (i.e. restructured in terms of the mathematical information deemed to be important). This leads to the formation of a mathematical problem;
4. The mathematical problem is solved in order to obtain a mathematical solution;
5. This solution must them be interpreted with respect to real-life;
6. The procedure and solution have to be validated with respect to suitable reference values; and
7. If the solution does not correspond to reality, then aspects of the modelling process must be adapted and then the process repeated.

Modelling competencies as outlined by Maab (2005) include:

- Competencies to understand the real problem and to set up a model based on reality;

- Competencies to set up a mathematical model from the real model and competencies to solve mathematical questions with this mathematical model.
- Competencies to interpret mathematical results in a real situation.
- Competencies to validate the solution.

Some features of applied mathematics include, according to Yanagimoto (2005):

- The use of concrete problems.
- The value of the solution is judged by the results of its application.
- Importance is attached to ideas rather than to the rigid application of theories.
- There can be more than one solution.
- Students must decide which model they feel is the most suitable.
- Students must think independently about the process before coming to a conclusion.
- Students' comments after a modelling lesson included: '*After this lesson, however, I came to realize that mathematics could be quite helpful to us in our real lives'* (Yanagimoto, 2005:11).

Humble (2005) proposes that students should complement their decision making process by always questioning the answer to a given solution. Humble proposes that by asking why from the answer the students will generally find out more about the question. The beauty of mathematics for Humble is the puzzlement and confusion that must be faced before the solution is complete. In traditional mathematics students rarely embrace this puzzlement in their quest to find a solution as quickly as possible. By embracing the mathematical process, and enjoying this confusion, students will rely on their own mathematical instincts rather than those of their teacher. By questioning their solution students are not merely performing by rote but delving into the mathematical process that has just occurred.

Geraniou et al (2009) are working on a research project in the hope of developing a method of mathematics learning to improve mathematical generalisation. What students need to develop their mathematical thinking is the opportunity to '*design situations that are rich in the construction and analysis of patterns, and provide both a rationale and computational support for expressing generality'* (Geraniou et al, 2009: 75). Modelling can be a useful tool to develop the mathematical generalisation skills of students with a constructivist pedagogical approach supporting students with their modelling activities. The preliminary data results from Geraniou et al suggest that while it is difficult to move from specific mathematical examples towards the general, the constructivist approach to learning allows the students to develop their skills by following a number of understandable steps.

4.7 Positives and negatives attributed to mathematical modelling

Like all concepts there are both positives and negatives attributed to the modelling process. Some of these are highlighted in the following section. Positives include the facilitation of in-class differentiation, the engagement of previously dis-engaged students, and the promotions of sense-making and reasoning skills. Negatives can include behaviour problems in students when faced with the modelling process and a perception that modelling is not 'real' mathematics.

Maab (2005) introduced mathematical modelling to a group of students in the German lower-secondary school system. All the students involved in her programme developed mathematical modelling competencies over this period, regardless of their mathematical ability. The author determined that modelling questions have self-differentiating properties in that they can be as challenging as the student needs them to be, or as basic as required for the less-able student. Solutions can be developed and extended or reduced depending on students' capabilities. The individual needs of all students can therefore be realised.

Lege (2005) introduced a mathematical modelling scheme to students that were deemed to be 'at-risk' in two schools in the United States. 'At-risk' is determined to be the probability that the students are at-risk of not graduating from high school. The students involved in Lege's study were deemed to be at least one grade behind the grade they were actually in in terms of mathematical ability. The study involved approximately one quarter of the total 'at-risk' population of the two schools involved. Positives of introducing modelling into these two schools, as suggested by the author, include:

- A positive change in the environment of the classroom;
- A repositioning of the teacher as a guide rather than an expert;
- The engagement of students in both mathematics and meaningful discussion;
- The introduction of group work in a traditional classroom setting;
- The elevation of critical-thinking skills;
- Challenging students with a complex, problem-solving environment;
- Developing sense-making;
- The provision of a realistic problem-solving experience; and
- The injection of a social component into mathematics education.

Lin and Yang (2005) suggest that introducing modelling to the mathematics classroom will benefit situational reasoning, mathematisation skills and students' ability to interpret and communicate in mathematical terms.

Lege (2005) suggests that there is evidence that some students may become disengaged and demonstrate some behavioural problems when introduced to the mathematical modelling process. Reasons for this may include a perception that modelling is not really mathematics and the suspension of normal classroom roles.

In order to effectively introduce modelling in the Irish mathematics classroom it is essential that teachers have an awareness of the negative possibilities and are mindful of introducing mathematical modelling as 'real' mathematics. Examples of where mathematical modelling could be used in real life situations may be helpful in this situation. Through effective practice mathematical modelling will become a 'normal' classroom activity and appreciated by the students as such.

4.8 Modelling and gender

Palm and Nystrom (2010) examined the modelling of real-world problems with respect to gender. They considered the idea that there may be gender differences in the way students approach the modelling of real-world situations. The authors were also interested in task authenticity, and the appropriate use of real-world knowledge, affecting male and female students differently. Authentic school tasks are determined to be those that successfully emulate real-life tasks. Palm and Nystrom investigated their hypothesis with 161, eleven year-old students. The students were from eight fifth grade classes from a selection of schools in Sweden. The study consisted of teacher interviews, student testing and post-testing interviews with the students involved. There were two different types of test: the first consisted of traditional style word questions while the second test consisted of authentic, realistic questions. No gender differences were found. While this is a small sample, it is indicative of what may be expected from other fifth-grade Swedish students.

Keast (1999) discovered marked differences in mathematical learning styles that are not necessarily gender specific but they are gender related. In this study the researcher was involved in assessing the affect of single-sex mathematics classes in a small, rural, secondary school in Australia. It was proposed that the introduction of single-sex mathematics classes for students in years 7 and 8 may result in an increase in self-confidence for female students which led to a higher uptake of mathematics courses at senior level in the school. Keast identified two learning styles: separate and connected knowing; and two associated teaching styles: separate and connected teaching. Separate teaching is thought to be traditional teaching of the 'chalk-and-talk' form. The following features of male and female learning styles were noted in the appropriate single-sex mathematics class in Keast's study.

Male students in the single-sex boy's mathematics class:

- Preferred to work individually;
- Did not want to share their ideas;
- Disliked group work;
- Were more content as 'separate knowers';
- Responded well to competition; and
- Preferred learning from the board with the teacher instructing the students on what the important aspects to learn were.

Female students in the single-sex girl's mathematics class:

- Formed small groups;
- Liked working with others and sharing ideas;
- Worked well through discussion and developing ideas in a connected way;
- Sought help from each other;
- Enjoyed the opportunity to investigate problems;
- Were more content as 'connected knowers'; and
- Preferred discovery learning to traditional book work.

Keast (1999) highlighted the fact that while all boys were content to learn in this individual, separate manner not all girls were content to be taught in a connected way. A minority of very able female mathematics students demonstrated a preference for learning individually and not as part of a group. Some girls developed from separate knowers to connected learners over the course of the year. Keast also noted that it was found to be ineffective to teach female mathematics students in the traditional way but that it was very difficult to involve boys in discussion regarding their mathematical understanding and they worked better as individuals.

Nathan and McMurchy-Pilkington (1997) investigated empowering the Maori community in New Zealand through mathematical power. They found that the preferred learning style of female Maori students was active-learning with a hands-on element. These women performed well in mathematical activities that were visual and engaging, and described them as interesting and fun. Students who enjoy mathematics and find it engaging will have a more positive attitude towards learning mathematics and making it part of their every day life.

4.9 Modelling specific methodology

Ikeda and Stephens (2010) carried out an experimental teaching program for 9th grade students in Japan. The authors hoped to establish a method of assessing the effectiveness of their modelling intervention. It was decided to pre and post test the students involved in the study using both a PISA problem and a general question, which involved determining what were important aspects of the modelling process. Responses to both the PISA problem and the general question were given a coding system and were analysed based on the students responses before and after the intervention. The intervention phase involved an experimental teaching programme in which the researchers attempted to improve students' conceptual knowledge through modelling. This phase involved students in a 9th grade Japanese high school and was carried out over a period of nine weeks. Three main teaching practices were emphasised in relation to improving the modelling skills of the students involved:

- Conflicting situations: where the teacher presents the key conflicting situations that arise from particular modelling problems and the students then drive key ideas that arise from these situations;
- Repeated connections: where the teacher constantly makes connections between students' thinking which promote modelling; and

- Spiral reflections: where the teacher encourages the students to reflect on the modelling process and all which that entails through the series of nine lessons.

Ikeda and Stephens (2010) study had a distinct research plan involving the following three steps:

1. The three underlying teaching principles as outlined above;
2. A planned program of experimental teaching; and
3. A set of assessments to evaluate the effectiveness of the program.

The planned program of experimental teaching included the constant emphasis of the three teaching principles, the phased introduction of a range of modelling tasks and a clear focus on the key ideas in order to promote modelling. Modelling was introduced to these students in order to assist them in solving real-world questions mathematically. Reflection was key to the experimental teaching phase and was used to encourage and promote understanding of the purpose of modelling and how it may used in real life, outside of the classroom environment.

Ikeda and Stephens (2010) assessed the success of introducing modelling strategies to the above group by using a PISA 2006 assessment question and a general question on modelling as pre and post-test questions. The post-test showed significant improvement in both question types which suggests that the three teaching principles that were emphasised (conflicting situations, repeated connections and spiral reflections) were effective in improving student mathematical performance through modelling techniques.

4.10 Conclusion

In order for Irish mathematics teaching and learning to evolve effectively, and for the introduction of the ‘Project Maths’ curriculum to be a success, it is important that we consider opportunities in the Irish curriculum for mathematical modelling. The author is of the opinion that modelling is an effective strategy for promoting connections between the mathematical theory traditionally learned in the classroom and realistic, authentic mathematics situations. However, she is fully aware that a willingness to embrace modelling on the part of the teacher is not sufficient when it comes to effectively introducing modelling in the mathematics classroom – students and teachers must fully embrace the modelling process and practice the requisite techniques for the introduction of modelling to be effective.

It is important to note that the new 'Project Maths' curriculum does not engage in the modelling process, despite references to real-life mathematics and authentic problem-solving. The author believes that this is a design flaw in the 'Project Maths' curriculum as mathematical modelling is the perfect opportunity to link traditional mathematics with innovative and effective problem solving techniques. Mathematical modelling affords the opportunity to allow students to develop as creative mathematicians while providing parameters within to do so. The techniques learned through modelling allow mathematical knowledge a valuable place in society and the work place.

5.0 Chapter 5: Methodology

5.1 Introduction

The research process and collection of data is discussed in this chapter. Both the methodology and the research methods selected by the author are examined, and the chronology of the research process is outlined. It is important to appreciate that while the term ‘methods’ refers to the procedures and instruments used, the word ‘methodology’ refers to the analysis of these procedures (Cohen and Mannion, 1992). Teddlie and Tashakkori (2009) describe research methods as specific strategies and procedures for implementing research design, while they define methodology as a broad approach to enquiry which specifies how research questions should be asked and answered. Gray (2009) explains that methodology is the analysis of a particular method used in research, and the broad philosophical and theoretical justification behind those methods.

The purpose of this chapter is to introduce the reader to the theoretical framework supporting the research decisions, explain the background to deciding on one research instrument over another, and engage in an analysis of these research procedures. The author documents the move from the general research problem (Irish students performing poorly in international mathematics assessments such as PISA and TIMSS) towards the specific research question (student difficulty in transferring mathematical knowledge learned in the classroom to the problem-solving process in a different setting). Cohen et al (2000) refer to this process of focusing a research question as ‘operationalisation’, and describe it as a move from the general towards a specific question for which the researcher seeks actual answers.

‘Research is a systematic process of collecting analysing, and interpreting information (data) in order to increase our understanding of a

phenomenon about which we are interested or concerned' (Leedy and Ormrod, 2010:2).

Leedy and Ormrod (2010) suggest that there are eight distinct characteristics of research projects. The research:

1. Originates with a question or problem;
2. Requires clear articulation of a goal;
3. Requires a specific plan for proceeding;
4. Usually divides the principle problem into more manageable sub-problems;
5. Is guided by the specific research problem, question or hypothesis;
6. Accepts certain critical assumptions;
7. Requires the collection and interpretation of data in an attempt to resolve the problem that initiated the research; and
8. Is cyclical by nature.

The author illustrates the progression of her research with respect to these characteristics throughout the methodology chapter. The outcome of this research will lead to a greater understanding of knowledge acquisition versus understanding in Irish mathematics education, and will contribute towards a growing understanding of mathematics teaching and learning in Irish society.
McNiff (2002) explains how research consists of three main components:

- Ontology: the way we view ourselves;
- Epistemology: how we understand and acquire knowledge; and
- Methodology: how we do things.

The author endeavours to maintain an awareness of these three components in the process of this research project.

5.2 The Research Question

Can students utilise the mathematics they learn in school in unfamiliar situations? This is the primary research question. In order to answer this question the author did a significant amount of desk research on the topic. By considering the significant body of literature available the research question was refined. Three methods of data collection are used to collect the field research:

- a structured observation to analyse the classroom situation in relation to mathematical learning theories;
- two tests: the first a traditional pen and paper mathematics test and the second a realistic, problem-solving question involving justification of the answers and reflection.;
- A semi-structured interview.

5.3 The Research Sample

The research sample consists of mathematics students in their second year of second level schooling. The author decided on this year group for several reasons including the following:

- The students are not in their first year in Irish second-level education, therefore they have had time to make the necessary social adjustments and are familiar with the curriculum. The students also have had time to adjust to the teaching styles that pertain in mathematics teaching at second level;
- It is a non-state examination year and therefore both the students and their teachers are under less time pressure;
- The students generally have a mean age of fourteen years. This enables some comparison with the research subjects in PISA, who were fifteen, and TIMSS, where students in the 8th grade were assessed. The 8th grade is very comparable age wise with Ireland's 2nd year. It would not be feasible to assess Irish fifteen years olds as the majority of them are

sitting the Junior Certificate examination at the end of that school year. As a result both students and teachers are under particular pressure in that academic year which would make participation in external research projects difficult.

The research sample consists of students in six mathematics class groups in five Irish second-level, co-educational schools. The schools were selected at random from a geographical region encompassing three counties. The location was selected for convenience reasons for the author in order to make visiting the schools feasible within the school week. It was decided not to consider students in single-sex schools as gender may be introduced as a variable affecting the research question.

5.4 Research Aims

The primary research aim of this study is to test the hypothesis as set out in the introduction: 'That students have difficulty transferring mathematical knowledge learned in the classroom to unfamiliar, realistic situations'. The author sets out to consider the hypothesis in relation to the theoretical framework as set out in section 5.9. The effects of different mathematical learning theories are considered through the data collection methods implemented: the structured observation, testing and semi-structured interview, and the subsequent data testing (both quantitative and qualitative). The ability of students to transfer mathematical knowledge is primarily assessed through the two tests:

1. The traditional style test with questions and mathematical topics familiar to the student; and
2. The realistic, problem-solving test with familiar mathematical topics posed in an unfamiliar format.

Through assessing students' mathematical knowledge by implementing two distinct tests the author hopes to resolve the primary research question. The structured observation gives some insight into the learning styles that are involved in the teaching and learning of mathematics in the classroom situation. The effect of learning styles on mathematical transfer can be noted through considering the students involved in this research situation in light of their classroom experience.

5.5 The Purpose of the Research

The purpose of the author's research is to identify any disparities that may exist between students' ability to solve traditional versus realistic mathematical problems. The author is of the opinion that lack of transfer ability (from learned mathematical knowledge to problem-solving skills) can be demonstrated if students have difficulty in solving realistic questions that involve mathematising but can solve traditional questions that require reproduction relatively easily. The purpose of the structured observation is to incorporate the different learning theories/styles that are implemented in the classrooms that are involved in the study. The ability of the student to utilise the mathematical knowledge learned in the classroom to address realistic, unfamiliar, untidy situations can therefore be examined in terms of mathematical learning styles.

5.6 Research Design

Creswell (2009) describes a research design as a plan and procedure for research that incorporates everything from broad assumptions to detailed methods of data collection and analysis. Creswell (2009:5) suggested the following framework for design, as illustrated in the following diagram:

5.7 The Research Process

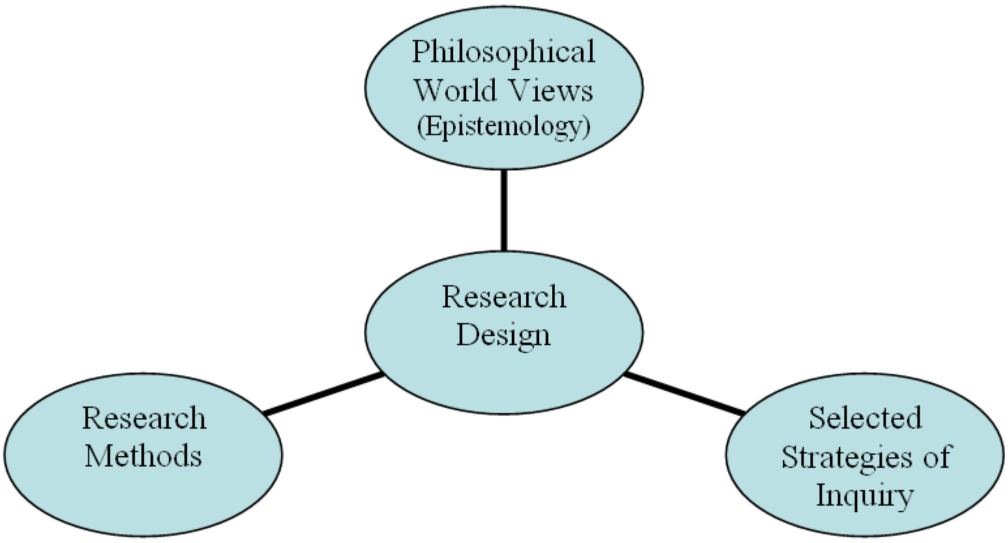


Figure 1: The research process

The author commenced her investigation into the research question by considering the considerable body of literature available. The focus in the literature review was initially on Irish students' poor performance in international mathematics assessments. Through immersion in the vast body of research on the subject the author moved towards a more specific question: the ability of a student to transfer mathematics from the classroom to realistic situations.

The literature introduced the author to the concept of different learning styles and the influence they can have on the ability of students to process knowledge correctly. The research question considers the concepts of reproducing knowledge versus the demonstration of understanding. If knowledge is not completely processed a difficulty can arise when students are expected to transfer this mathematical knowledge to an unfamiliar situation. The concept of different learning theories and styles (absolutist versus relativist philosophies) developed into a theoretical framework on which much of this study is based.
The research process is illustrated in the following schematic:

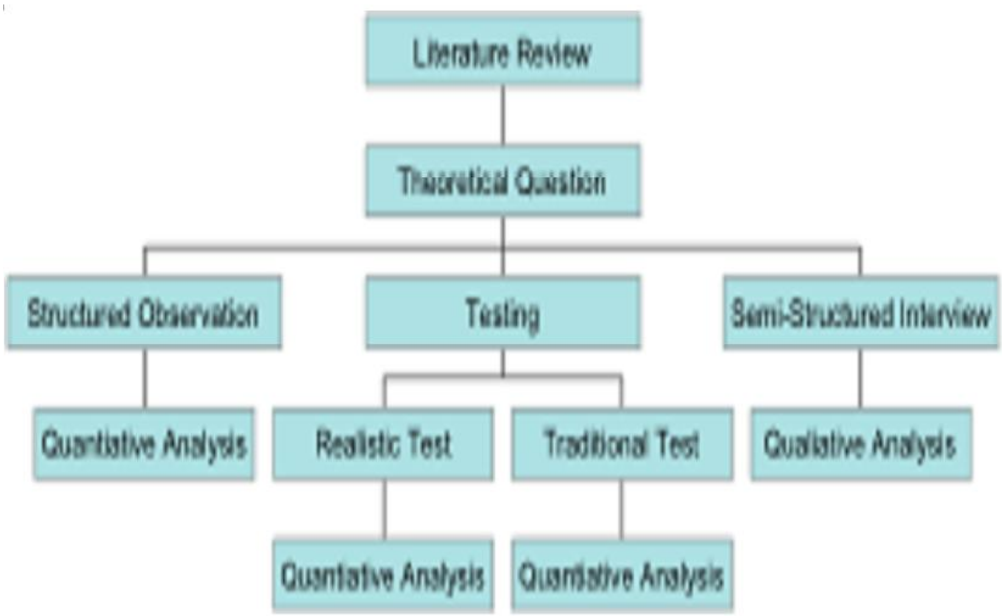


Figure 2: An illustration of the research process carried out by the author

5.8 The Chronology of the Research

The author completed the research over a three-year period. A chart documenting the chronological life of the research project is shown below:

Year	Jan	Feb	Mar	Apr	May	June	July	Aug	Sep	Oct	Nov	Dec
2008						Literature Review						
2009	Literature Review											
				Decide on Research Question								
						Write up of literature review to date				Analysis of research methods		
2010	Design of real-world test and pilot of same. Design of traditional test and pilot of same.			Analyse results from pilot tests. Research and write research methods and methodology chapter.		Research and write chapter on maths ed and modelling. Continue to research and write research methods and methodology chapter.						
						Design structured observation			Pilot of structured observation			
									Approach schools for involvement in research and access to mathematics classes	Visit all schools and classes involved in research and implement tests, observation and hold interviews.		
2011												
	Analysis of data collected (structured observations and test data). Validity and reliability testing.			Write up of thesis to include proof-reading all chapters to date, completing chapters etc.								

Table 7: Chronological life of the research project

5.9 The Theoretical Framework

The theoretical framework underpinning the research is examined in this section. The author is using a framework based on the major learning theories in mathematics. The various learning theories are considered under one of two philosophical/epistemological categories (as outlined by Lyons et al (2003)):

- Absolutist, and
- Relativist.

These general labels for learning theories provide a basic division between behaviourist style learning theories (absolutist) and cognitive theories (relativist). The author decided to use the two contrasting epistemological traditions of the absolutist and relativist traditions as a means of sorting the various learning theories into categories. 'Inside Classrooms' (Lynch et al, 2003) is a significant body of literature based on research on Irish mathematics education at second-level and the author wished to acknowledge this work by using the same terminology used. The work carried out by Lynch et al (2003) draws on the work of Leone Burton for its discussion on epistemological traditions. As a result the author considers the work of Burton (1994, 1995, 1999) in discussing the absolutist and relativist epistemological stances.

Behaviourist, cognitive and constructivist learning theory are the primary learning styles considered by the author and are located within the contrasting epistemological traditions: the absolutist tradition and the relativist tradition. The author attempts to place the current Irish mathematics curriculum, and the teaching and learning happening in Irish mathematics classrooms, within this framework. The performance of the students participating in this research is considered in terms of the teaching and learning methods they are exposed to, and this is then considered in terms of the theoretical framework.

The following table illustrates the categorisation of various learning theories into the categories: absolutist and relativist:

Absolutist Learning Theories	**Constructivist Learning Theories**
Behaviourism	Cognitive Learning Theories
Objectivism	Constructivism
Separate Knowing	Connected Knowing

Table 8: Absolutist and relativist learning theories

Gergen (1997) claimed that objectivism/behaviourism and constructivism represent opposite extremes on an epistemological continuum. The objectivist learning theory is very similar to the behaviourist learning theory and the author will use these two terms interchangeably. Keast (1999) identifies two learning styles: separate and connected knowing, and two associated teaching styles: separate and connected teaching. Separate knowing and teaching is typical of traditional mathematics teaching. Keast believes that traditional mathematics teaching alienates the connected knower who benefits from mathematics that relates to one's own life experience.

In practice the conflicting epistemological view-points of absolutism and relativism and the learning theories of behaviourism and cognitive knowing are not mutually exclusive. Indeed many combinations exist between the various categories. In practice, many teachers speak of embedding a mixture of directed instruction, as per behaviourism, and constructivist learning theories, such as student-direct learning, in their teaching. It is also possible that a teacher may hold the belief that mathematics is objective, logical and consistent (as per the absolutist philosophy) while embracing constructivist practices, such as co-operative learning, in his/her classroom as a means of imparting this mathematical knowledge. The author acknowledges the various combinations of learning theories and philosophical stand-points that mathematics teachers may hold with relation to teaching and learning. In the course of the data collection and analysis of this research, the author considers the Irish mathematics education system within this notion of separate knowing and teaching versus connected knowing and teaching while acknowledging that these are not always mutually exclusive.

5.9.1. Behaviourism versus Constructivism

Despite the reality of teaching practices frequently consisting of a mixture of behaviourist and cognitive learning theories, the literature often speaks of the opposing nature of the two. Handal (2003) demonstrated the polarity of the terms used to describe the behaviourist perspective and the constructivist perspective. The following table shows some of those terms used to represent the two learning theories:

Behaviourist Perspective	Constructivist Perspective
Behaviourism	Constructivism
Traditional	Progressive
Mimetic	Transformational
Basic skills	Higher order thinking
Content	Process
Positivist	Relativist
Subject-centred	Child-centred
Transmission of factual and procedural knowledge	Emphasis on qualitative transformation in outlook of the learner
Euclidean	Quasi-empirical
Absolutist	Fallibilist
Technical-positivism	Constructivism

Table 9: Terms associated with the behaviourist and constructivist learning theories.

Burton (1994) explains that a dichotomy appears to exist between the absolutist and relativist philosophies. She explains that this apparent dichotomy is in fact false as there are many positive and negative examples that can fit under each philosophical category. Burton (1994:209) provides the following explanation of aspects of both philosophies:

An absolutist philosophy	A relativist philosophy
Mathematics is seen as information	Mathematics is a result from know-how
Transmission of knowledge from teacher to students	Enquiry-based learning
Learners are dependent on the teacher	Autonomous learners
Competitive and individualised	Collaborative and group-based
Interaction through questions and answers	Discussion based learning
Assessment based on unseen written tests	Assessment through integral tasks and by both the self and with peers.

Table 10: Characteristics of the absolutist and relativist perspectives.

From an Irish point of view the author is interested in considering which of the labels, seen in the table above, relate to mathematics education as it exists in Irish mathematics classrooms. The author intends to discuss the above table in the data analysis chapter with relation to the data collection methods: tests, structured observation and semi-structured interviews. Through analysing the data provided by these mixed method techniques the author will seek to link the findings to the absolutist and/or relative philosophies of learning.

5.9.2. The Absolutist Philosophy:

The absolutist tradition is that which is most typically associated with 'traditional' mathematics and is very familiar to those involved in mathematics education in Ireland. The absolutist philosophy considers mathematics as a logical and value-free process. It often underpins the philosophical stand-point held by teachers who use behaviourist teaching methods such as the use of didactic teaching and the idea of the teacher as the transmitter of knowledge with the student as a receiver of this information. However, as mentioned earlier an absolutist philosophy does not guarantee the sole use of behaviourist teaching and learning methods in the classroom: it is possible to consider mathematics as logical and value-free while utilising child-centred teaching methods in practice.

Burton (1994: 207) describes behaviourist teaching methods as follows: *'A hierarchy is thus defined for those engaged in teaching and learning, with lesser mortals, learners, attempting to obtain knowledge and skills from higher mortals, teachers'.* Burton expands further when she simplifies her definition of the absolutist tradition to one in which the primary focus of mathematics education and the aim of educators regarding mathematical facts is to *'convey them into the heads of the learners'* (Burton, 1995:520).

Burton (1994) discusses how mathematics is considered to be a subject that is value-free, logical and knowledge based, and thus not a subject that is perceived to demand a demonstration of creativity and/or imagination. In the absolutist tradition of learning the knowledge, not the learner, is at the centre of all teaching and learning activity. Mathematics, in the absolutist tradition, has a learning style that 'assumes transmission of unchallengeable content' (Burton, 1994:207). Burton (1995) explains that despite the fact that absolute objectivity in mathematics is a myth, nevertheless it is a powerful myth which continues to exercise considerable power in mathematics education – both in curricular and methodological terms.

5.9.3. Behaviourist Learning Theory

Behaviourist learning theory has had a strong influence on mathematics education in the 20th century, and as an educational movement it possibly portrays 'traditional' mathematics education in the minds of most. Behaviourism promotes drill learning, repetition of procedures, the idea of the teacher as the expert and the centre of knowledge, memorisation of formulae and the notion of mathematics being a subject where there is only ever one correct answer.

'*Learning and teaching in behaviourist terms is a matter of optimising and manipulating the instructional environment towards the fulfilment of rigidly and specifically designed educational objectives'* (Handal, 2003, 5).

Behaviourism is defined by Sloane and Conway (2005) as direct teaching followed by controlled practice, with a distinct focus on learning hierarchies and vertical transfer. Behaviourism focuses on the teaching and learning of mathematics in a formulaic way, following these steps:

- Tasks are broken down into small, manageable pieces;
- The basics are taught first;
- Mathematics learning is incrementally reinforced and observable behaviour is rewarded; and
- Time is not spent reasoning, reflecting or problem solving (Conway and Sloane, 2005:83).

5.9.4. The Relativist Philosophy

The relativist philosophy considers all knowledge to be culturally and politically situated. This philosophy is often associated with cognitive learning theories. Cognitive theory has been a popular movement in mathematics education over the last thirty years. Cognitive theory introduced a move towards active learning and advancement from the tradition of 'chalk-and-talk' teaching, allowing room for the cultural value of mathematics. Conway and Sloane (2005) describe cognitive learning theory as a move away from just recording information, as is common in behaviourist learning theory, towards interpreting information. The basic behaviourist stance is that the information already exists; cognitive theory believes that knowledge is constructed by the learners experience and actions. Important educational insights offered by cognitive learning theory include:

- Learning is active;
- Learning is about the construction of meaning;
- Learning is helped and hindered by our prior knowledge and experience;
- Learning re-organises our minds;
- The mind develops in stage; and
- Learning is often unsettling (Conway and Sloane, 2005:87).

5.9.5. Constructivist Learning Theory

Constructivism, as developed by Piaget, views mathematical learning as the construction of meaning and understanding based on the modelling of realistic situations, analysis of patterns and the acquisition of a mathematical outlook (Gales and Wefan, 2001:4). Constructivist learning theory, like the relativist epistemology, is based on the belief that all mathematical learning is linked to current and past knowledge and individual real-life experience. Active and discovery learning are the basis of constructivist learning, with creative thinking and a questioning mind highly valued.

Holt (2001) explains that a social constructivist approach to the teaching and learning of mathematics will allow and encourage problem solving, articulate communication, active learning, participation and social interaction. The constructivist method of learning encourages communication, and active participation by each student as required. Oral communication is valued and viewed as a necessary mathematical tool. Improvisation co-action is described as being *'a collaborative practice of acting, interacting and reacting, of making and creating, in the moment, without script or prescription, and in response to the stimulus of one's context and environment'* (Martin and Towers, 2009:3).

Mathematical understanding is a key component of constructivist learning theory. The growth of collective mathematical understanding through group work, and shared mathematical actions and thinking, is the basis of collective mathematical understanding. Mathematical understanding is seen as the interplay between actions and general conceptualisations. Martin and Towers (2009) use eight layers of action for mathematical understanding to label the learner's progress and their mathematical actions as their understanding grows.

The theoretical framework used by Martin and Towers focused on the learners working in small groups and their collective mathematical activity in these groups. Collective understanding is seen as a dynamic process. The *'improvisational perspective focuses on the way in which collective mathematical understanding is constantly changing and growing (a process) as a group of learners work together in the moment, rather than on the establishment of collective classroom norms'* (Martin and Towers, 2009:2)

5.10 A Mixed-Methods Study

The researcher decided to use a mixed-methods design. The initial idea for the research focused on a quantitative study as the main research body, in which mathematical proficiency was assessed in two different types of mathematical test. This was followed by a statistical analysis of these results. As test design was undertaken, and preparation for implementation of these tests evolved, the researcher became aware that tests alone would not provide a clear picture as to what is happening in Irish mathematics classrooms. For this reason a qualitative aspect was introduced to the Irish data collection, this involved teacher/principal interviews. A further mixed methods aspect was also introduced in the form of systematic, structured classroom observations. The objective of the inclusion of the qualitative aspect was to situate Irish mathematical activity, provide a meaningful content for the information collected and link teaching and learning in the classroom to the learning theories and hence the guiding theoretical framework.

Creswell and Clark (2011) suggest the following mixed methods evaluation criteria. The researcher:

- Collects quantitative and qualitative data;
- Uses rigorous procedures in collecting and analysing data;

- Integrates the quantitative and qualitative data so that their combined use provides a better understanding of the research question;
- Includes the use of a mixed-methods research question design and integrates all features consistent with the design;
- Frames the study within philosophical assumptions, and;
- Conveys the research using terms currently used in mixed methods research.

5.10.1. Convergent, parallel mixed-methods design

The author decided to use a convergent, parallel design as the basis of the mixed-methods research. Creswell and Clark (2011) describe a convergent study as one in which the researcher collects and analyses quantitative and qualitative data at roughly the same time, within the same research phase, and merges both sets of results (quantitative and qualitative) for an overall interpretation of the research question. Quantitative results can be compared and contrasted with qualitative results for corroboration and validation of data. This can lead to a greater understanding of not only the research question but also the surrounding issues. Creswell and Clark (2011) suggest that a convergent mixed-methods study is also efficient as both quantitative and qualitative data can be collected at roughly the same time, and in some cases in one visit.

Teddlie and Tashakkori (2009) believe that a major advantage of mixed methods research is the way in which it enables researchers to ask confirmatory and exploratory questions at the same time. A parallel mixed-methods study uses both qualitative and quantitative methods in independent strands, in the same research phase, to answer both exploratory and confirmatory questions. It is typical that exploratory questions are usually qualitative (such as the semi-structured interview) and confirmatory questions are quantitative (as with the closed-item questions in the tests). However, Teddlie and Tashakkori (2009) emphasise the importance of remaining aware of the challenges that can arise in parallel research methods; primarily the complexity of implementing multiple research strands simultaneously.

While the data collected by the author required three visits for each class group involved, the data was still collected in a short period of time, and in several instances in one school week. The order in which the research was collected was not deemed to be of any significance, and the author was led by the teacher involved as to when they felt each step was most appropriate, for whatever reason, for they themselves and their class. The author implemented one sole research phase, with all data (quantitative, qualitative and qualitative into quantitative) collected and analysed in roughly the same time period. For each individual class group there was a very short data collection time period. For the overall research groups (excluding the pilot study) data was collected and analysed over a period of five months. There was no distinguishing break or feature in moving from data collection for one group to the next, and in terms of a research phase all data collection and analysis was conducted in a similar manner.

By using a convergent mixed-methods design data, can be collected and analysed separately and independently, using data collection and analysis methods that are appropriately suited to each type (Creswell and Clark, 2011). Therefore, quantitative data can be analysed quantitatively and qualitative data qualitatively. By using a convergent mixed-methods design the author is of the opinion that the strengths of each research method (quantitative and qualitative) can be played to, the weaknesses accounted for and a greater sense can be achieved regarding the research question. By using a convergent research design both quantitative and qualitative data collection methods are equally valid and equally vocal in the voice they give to the research question.

5.11 Data Collection Methods

The author gave significant consideration to the data collection methods selected as these methods would have a significant impact on how effectively the research question could be examined. Creswell and Clark (2011) recommend that in designing a mixed-methods study the researcher utilises a qualitative strand that incorporates ‘persuasive’ qualitative data collection procedures and a quantitative strand that included ‘rigorous’ quantitative data collection procedures. The author was conscious of effectively answering the research question through effective data collection and analysis.

5.11.1. The Structured Observation

The author selected classroom observation as a research method based on the fact that testing alone would not allow for an insight into the context in which the teaching and learning of mathematics takes place. Six mathematics classes were selected from five, Irish co-educational schools. The five schools were selected at random from a geographical region encompassing three counties. This geographical region facilitated the collection of data by the author. One mathematics second-year class group was nominated by four of the schools with the fifth school providing two second-year, mathematics class groups. As only one mathematics lesson was observed for each of the class groups involved in the research, the researcher is very aware that it only provides a snap-shot into how mathematics is taught and learned in a typical Irish, second-year mathematics classroom. The schools involved in the research are co-educational to allow for comparison between schools without the possible influencing factors that may occur by introducing single-sex schools to the mix. All five schools are non-fee paying schools. The activities involved in teaching and learning in a ‘typical’ Irish mathematics classroom can be considered within the context of all the classrooms observed, and typical behaviours identified. The author believes the mixture of school types (community school, community college, secondary school from both rural and urban areas) provides a combination of factors that allows an insight into what a ‘typical’ Irish mathematics classroom looks like. Teaching and learning behaviours can be considered in terms of mathematics learning theories (absolutist and relativist), and thus Irish mathematics education placed within this spectrum of varying mathematics learning methods.

In considering classroom observation as a data collection method the author first considered how this would relate to the research question she wished to explore. As outlined above the researcher hoped to gain some insight into the learning theories implemented in the Irish mathematics classrooms observed, thus linking the observed behaviour to the theoretical framework of the research project. It was also hoped that by considering the teaching and learning behaviour in Irish classrooms the author could consider these activities in terms of the test performance by Irish students, and gain an indication as to how students perform based on classroom activity.

Classroom observation as a data collection technique can take one of two forms: unstructured or structured observation. Teddlie and Tashakkori (2009: 218) define the differing observational techniques as follows:

- *Unstructured/open-ended observations:* These observations take the form of a running narrative. The recorder generally takes extensive field notes and records as many interactions as possible. Unstructured observations generally result in qualitative data.
- *Structured/closed-ended observations:* Structured observations use data recording instruments or pre-determined protocols to record the observed situation in a structured format. These instruments/protocols tend to involve numeric scales. Structured observations result in quantitative data, which be statistically analysed.

The author decided to use a structured observation data collection technique in order to facilitate the comparison of data collected in different mathematics classrooms. Numeric, quantitative data facilitated this comparison easily compared to the extent of the difficulties that may arise in attempting to compare qualitative data. The structured observation is a *'technique in which the researcher employs explicitly formulated rules for the observation and recording of behaviour. The rules inform observers about what they should look for and how they should record behaviour'* (Bryman, 2001:508).

Cohen et al (2005) highlight some features of the structured observation:

- It is time consuming to prepare but if correctly designed it takes little time to analyse the data;
- It is systematic and enables the researcher to generate numerical data;
- Numerical data facilitates comparisons, frequencies, patterns and trends to be noted;
- The observer adopts a passive, non-intrusive role where they are merely noting the incidence of factors;
- The categories for observation are discrete with no overlap;
- It is essential that a pilot is developed, tested and re-tested for the structured observation to be effective;
- A pre-designed observation schedule should be designed with appropriate space for noting incidence, presence and frequency;
- Each column should represent a certain time interval with movement from left to right representing the chronology of events;
- The researcher must practice completing the research schedule until proficient in entering data; and
- The researcher must decide on notation to be used for coding purposes.

The second decision the author made in relation to observation as a data collection method involved the debate regarding the participant-observer continuum. In the complete observer role the researcher is at an extreme observer level and does not participate in the research at all, to the extent that the researcher involved would not enter the research setting (i.e. the classroom) at any stage when there are people present. At the opposite end of the continuum the researcher becomes fully engaged in the research setting and a full member of the group they wish to observe (Teddlie and Tashakkori, 2009:222). The labels given to the various levels of observer within the participant-observer continuum are:

- Complete participant;
- Participant as observer;
- Observer as participant; and
- Complete observer (Teddlie and Tashakkori, 2009:222).

The author identifies herself as being closer to the 'complete observer' end of the continuum, but as she enters the classroom and meets with the students prior to, and during, the observed mathematics lesson the author identifies herself as an 'observer as participant'.

Coding is an important aspect of the structured observation. It is important to have devised an effective coding system in order to effectively analysis one's findings. Bryman (2001) defines codes as tags that are attributed to data about people or other units of analysis. The aim of the coding system is to assign data relating to each variable to groups which are a category of the variable in question. A number can then be assigned to each category making the data quantifiable for quantitative research. In qualitative research coding is the process of data being broken down into its component parts.

Concerns regarding the collection of numerical data include:

- The method is behaviourist and excludes any mention of intentions of what/who is being observed;
- Individual's subjectivity is lost;
- There is an assumption on the part of the researcher that certain observed behaviour provides evidence of underlying feelings and motivations (Cohen, 2000: 309).

The author is fully aware of difficulties that may arise in the observation process and made every effort to ensure that all groups observed were treated in the same manner and data was recorded in a similar fashion.

5.11.2. Testing (As a mixed-methods technique)

Tests are designed in order to assess knowledge, intelligence or ability (Teddlie and Tashakkori, 2009). The author was interested in assessing both knowledge and ability; the mathematical knowledge to provide the information required and the ability to utilise this information to solve realistic problems. Tests as a mixed-methods technique incorporate both quantitative and qualitative techniques by posing questions in such a way as to demand/necessitate a particular style of answer. For example closed-ended items result in a quantitative response, whereas open-ended questions typically necessitate a qualitative answer. The author utilised both types of questions; closed-ended questions in the traditional test and open-ended questions in the realistic test. Teddlie and Tashakkori (2009) note that qualitative data collected by testing is often quantitised as researchers using testing as a data collection method typically want numeric data which can be analysed using statistical methods. This is true of the data collected by the author in the realistic test; in order to quantitse the data, the responses provided to the open-ended questions were given numerical scores for ease of analysis.

Rubrics are scales developed by researchers in order to rate responses generated by testing. Rubrics provide guidelines for assessing responses to open-ended questions, performances on tasks and products related to the topic of interest (Teddlie and Tashakkori, 2009). By providing criteria for assessing written responses to questions asked in tests, these numeric scales enable researchers to summarise results across all research participants. This enables the quantitising of both qualitative and quantitative data which facilitates a direct comparison between the research participants (Teddlie and Tashakkori, 2009).

When constructing a test Cohen et al (2007:321) suggest that the researcher considers the following:

- The purpose of the test (ensuring that it tests what it is supposed to be testing);
- The type of test (e.g. diagnostic, achievement, aptitude, criterion-referenced, norm-referenced etc.);
- The objectives of the test should be set out in very specific terms so that the content of the test can be seen to relate to the specific objectives of a curriculum;
- The content of the test;

- The construction of the test should incorporate item analysis in order to clarify the discriminability and the item difficulty of the specific test;
- The format of the test including its layout, instructions, methods of working and its completion;
- The validity and reliability of the test; and
- The provision of a manual of instructions for the administration, marking and data treatment of the test.

The author remained aware of the above considerations when designing, administering and analysing the test. The effort to design, implement and assess the tests in a fair manner is essential to the effectiveness of the research.

5.11.3. The Semi-structured Interview

An interview involves the researcher (the interviewer) asking a person involved in the research questions relating to the research (the interviewer). The interview is a popular data-collection technique and allows for direct interaction between the interviewer and the interviewee. In some cases it facilitates the expansion of relevant topics as appropriate and it allows the interviewee to ask for clarification if there is any aspect of the interview that they do not fully understand. Research interviews take one of three particular formats in a qualitative/quantitative sense:

- Interviews involving open-ended questions (generally qualitative);
- Interviews based on closed-ended questions (usually quantitative); and
- Interviews that include both closed-ended and open-ended questions (quantitative AND qualitative – mixed methods).

Teddlie and Tashakkori (2009) present four types of interviews:

1. *Informal conversation interview:* There are no pre-determined question topics. The interview takes the form of a conversation and questions emerge from the conversation as it progresses. This is a very fluid, organic type of interviewing technique. This interview style provides qualitative data.

2. *General interview guide approach (semi-structured interview):* Topics and issues are specified in advance. The interviewer decides on the order in which to ask the questions and on the wording of the questions during the course of the interview. There is the possibility of expanding on topics as the need arises. This interview style provides qualitative data.
3. *Standardised open-ended interview:* The exact wording of the questions is determined by the interviewer in advance. So also is the order in which the questions will be asked. All interviewees are asked the same questions, in the same order. Questions are worded in a completely open-ended format. This interview style provides qualitative data.
4. *Closed fixed-response interview:* Questions, the order of questions, and the range of responses are decided on in advance of the interview. Responses are fixed and the interviewee chooses from among these responses. Questions are worded in a closed-ended format. This interview provides quantitative data (Teddlie and Tashakkori, 2009:229).

Gray (2009) describes the informal, conversational interview as one which relies on the spontaneous generation of questions as the interview progresses. While this method may be informative it was decided by the author that in order to obtain answers to the questions deemed pertinent to the study, it was essential to follow a more structured type of interview technique. Despite the need for more structure than in an informal, conversational interview the structured interview was deemed to be too restrictive as it does not provide the opportunity to expand on topics as they arise. Gray (2009) describes a structured interview as one where pre-prepared questionnaires and standardised questions are used, and as a result all the respondents are answering identical questions that facilitate the recording of all responses in a standardised schedule. The author decided on a semi-structured interview as the optimum method of obtaining qualitative data of a high standard that would yield valuable information for the study in question. Gray (2009) explains that in a semi-structured interview the researcher has a list of issues and questions to be covered but they may not all necessarily be dealt with in every interview.

The semi-structured interview will provide the author with qualitative data to support the research question: Do students have the mathematical understanding to transfer the knowledge they learn in Irish schools to unfamiliar, problem-solving situations? By asking the teachers and/or principals involved in the study pertinent questions surrounding the teaching and learning of mathematics in their school the researcher hopes to place the classroom observations in context. This provides a basis to consider what is happening in Irish, and indeed Massachusetts', schools. The author decided on a semi-structured interviewing technique as she is of the opinion that it allows the major topics to be asked of all the interviewees but allows the interview participants the opportunity to ask questions themselves and expand on topics that they feel are particularly relevant.

5.12 The Data Collection Process

While the selection of the data collection methods is critical in ensuring that the research question is effectively addressed, the data collection process is also important if the methods selected are to be utilised to their full effect.

5.12.1. The Structured Observation

As discussed earlier the author decided to use a structured observation data collection technique in order to facilitate the comparison of data collected in different mathematics classrooms. Numeric, quantitative data facilitated this comparison easily compared to the extent of the difficulties that may arise in attempting to compare qualitative data.

The second decision the author made in relation to observation as a data collection method involved the debate regarding the participant-observer continuum. As discussed earlier, the labels given to the various levels of observer within the participant-observer continuum are:

- Complete participant;

- Participant as observer;
- Observer as participant; and
- Complete observer (Teddlie and Tashakkori, 2009:222).

The author identifies herself as being an 'observer as participant' but closer to the 'complete observer' end of the scale. The author was aware of maintaining her position as 'observer' and not engaging or participating in the teaching and learning process at any stage. The sole verbal interaction the author had with the research participants during the observation process was at the beginning of the class when she thanked the students and teacher involved for facilitating the observation and referred very briefly to what this would entail. The teacher and students in each group had at this stage already been fully briefed as to what they were engaging in and what this information would be used for. The teachers involved were also given access to a copy of the observation template. It should be noted that all teachers involved refused a copy of the template and preferred to rely solely on the author's information regarding the proposed observation.

Observation was selected as a means of gaining insight into the learning theories that are implemented and facilitated in Irish mathematics lesson. The structured observation was decided on as the type of observational tool to be used in the data collection process as it can translate time spent in the classroom into numerical data which can be analysed in a quantitative sense using statistical analysis. This facilitates a direct comparison of teaching and learning activity in the mathematical classrooms observed. A precise time-structure can be decided on and different aspects of teaching and learning can be considered within this timeframe. Time spent on different elements of teaching and learning in a mathematics class can give some insight into the learning theories promoted in that particular classroom and this can then be considered in terms of the students' ability to transfer mathematical knowledge.

Gray (2009) explains that one of the major problems with the process of observation is that of actually gaining access to the research setting, in this case the classroom. The author encountered significant resistance in this regard, in the most part due to resistance from the mathematics teachers involved but also, on occasion, from school principals. Despite repeated reassurance that the teachers themselves were not being examined, many teachers felt that the presence of a researcher in their mathematics class would be an overly intrusive, critical presence and chose not to be involved in the study. In one instance, despite the facilitation of the project by the school principal and board of management, all five second-year mathematics teachers in a particular school chose not to be involved in the study which resulted in the school in question not participating in the research. As a result of this unexpected resistance the researcher was delighted when two teachers from the same school volunteered their involvement in the research study. Prior to this it was a major achievement if one teacher in a school agreed to participate. Due to the resistance encountered by the researcher in gaining access to mathematics classes, a situation occurs in which the teachers who self-nominated to participate in the study are possibly confident being observed. This may indicate that the teachers involved in the research consider themselves to be proficient in their field and confident in their skills, therefore resulting in a particular type of teacher that is observed teaching in the course of the research.

The majority of schools (three out of the five visited) volunteered their higher-level class (and for those schools with more than one higher-level class the class involved was the highest of those studying the higher-level course). As a result the researcher specifically approached the remaining two schools, and the final two schools to come onboard the research, with a request that ordinary or foundation level students be involved in the study. This resulted in one school (which volunteered two class groups) volunteering their two mathematics class groups that involved students requiring the most assistance. These students were following the ordinary level mathematics course but it was anticipated that a minority of students in these class groups may decide to sit the foundation level mathematics paper in the Junior Certificate examination.

The fifth school allowed access to a mixed-ability, ordinary level class group. As a result all student abilities were involved, to some extent, in the research.

Despite the resistance met by the researcher in obtaining access to mathematics classrooms, those teachers who did participate in the study were 100% committed to their involvement in the study and were comfortable with being observed. Out of the six class groups observed for the study, plus the initial pilot group, five of the teachers observed were female and two of the teachers (including the teacher involved in the pilot were male). All except two of the teachers had twenty years plus teaching experience, one had five years teaching experience and the teacher involved in the pilot observation was a trainee teacher.

The author utilises the classroom observation as a means of considering the qualitative data observed in a quantitative manner. This is known as 'quantitising data' and is the process of converting qualitative data into data that can be analysed in a statistical manner (Teddlie & Tashakkori, 2009). This was achieved by identifying a number of activities that frequently occur, or have the capacity to occur, in a typical mathematics lesson. Tasks were assessed at thirty-second intervals for the duration of the mathematics period. Tasks were not mutually exclusive, and it was possible for more than one identified activity to occur simultaneously (e.g. board-work and teacher explanation). Any of the identifiable tasks that occurred in the thirty-second period were marked – the duration of the individual identifying activities within the thirty-second period was not recorded. The observable qualitative data was in this manner converted into numerical data and as a result it was possible to analyse same in a quantitative manner. Teddlie & Tashakkori (2009) call this process of converting qualitative data into numerical data 'data conversion/transformation'.

The objective of the classroom observation was to gain an insight into what occurs in a typical Irish mathematics lesson. By observing a mathematics lesson involving the class group that were involved in the test component of the research it was hoped that the author would gain knowledge regarding the teaching and learning methods used. As a result of considering the data collected through the observation, in conjunction with the data collected through testing, the effect of the teaching and learning styles on examination results can be considered.

The pilot of the structured observation schedule is of paramount importance to the structured observation being effective. Cohen et al (2000:305) describe the decisions the pilot researcher must make:

- The foci of the observation (both people and events);
- The frequency of the observations (i.e. the time interval attributed);
- The length of the observation period (i.e. a 40 minute class); and
- The nature of the entry (i.e. an appropriate coding system).

The author gave careful thought to the purpose and focus of the observation. The observation schedule was carefully designed to include all teaching and learning activities that the author expected to occur.

5.12.2. Testing

Through considering the research question in relation to the Irish situation, and the body of literature surrounding the area of testing, the author decided on testing as a research tool for collecting data. It was decided to use a process of testing that would consider the ability of Irish students' to solve traditional, familiar questions, and also to test the same students with unfamiliar, realistic questions that utilise the same mathematical skills. In order to consider the performance of Irish students in relation to international performance it was decided to also implement both tests in the state of Massachusetts in the United States. The decision to include students from another country was carefully considered by the author. The state of Massachusetts in the United States was decided on for several reasons, among which were:

- The commonality of a shared language: English;

- The results that the state of Massachusetts have achieved in international mathematics testing. In TIMSS 2007 eighth and fourth grade students in Massachusetts outperformed all other states in the USA in mathematics, and also ranked among the highest achievers mathematically in the world. 4th grade students from Massachusetts tied for third place in mathematics with Chinese Taipei and Japan (behind Hong Kong SAR (1st) and Singapore (2nd)). At eighth grade Massachusetts' students ranked in 6th place behind Chinese Taipei, Republic of Korea, Singapore, Hong Kong SAR and Japan (www.doe.mass.edu).

The students from Massachusetts that participated in the study were from one particular school district initially, and due to access constraints this was narrowed to students from one middle-school. The school involved was a public middle-school of roughly 1,000 students in a town on the outskirts of Boston.

The author gave much thought to the style of test to be implemented and the order in which the testing would occur. After much consideration it was decided to implement two thirty minute tests. The length of the test was decided on based on the length of a typical Irish mathematics class of forty minutes. The author felt it was important that the test would not necessitate the full class time to be utilised in order to allow time to re-introduce the researcher, the test and to settle the students.

5.12.2.1. The Realistic Test

The author made the decision to draft the realistic test first. The reasoning for this was that the unfamiliar style of the realistic would be more difficult and time-consuming to draft, especially if it was to fully embrace the aspects deemed important by the author:

- Questions in a format not familiar to Irish students;
- Engagement by the student in the reflection process;

- Open-ended questions with more than one correct solution in some parts of the test;
- The provision of surplus information that may not necessarily be required in finding a solution to the given problem;
- A necessity for students to justify some part of their answer; and
- Authentically realistic questions.

The author explored the literature surrounding modelling (as seen in the literature review) and the concept of authentic questions. After much consideration the given realistic questions were decided on. It was also important to the author that the students were not overly constrained by time as it would be an added pressure; however the author felt it was important that the students be fully engaged by the questions posed for duration of the time available.

5.12.2.2. The Traditional Test

The traditional test consists of traditional mathematics questions posed in a familiar format. Irish students are particularly focused on preparation for the state examinations and therefore are familiar with a recognisable form of question. Some characteristics of traditional mathematics questions in the Irish Junior Certificate curriculum include the following:

- Closed-ended questions;
- There is one correct answer to each question;
- Justification of the problem-solving methods using words is unusual;
- Evidence of reflection is not typical;
- All the numerical information provided in the question is necessary in finding the solution. No redundant information is provided. (Therefore the students believe that you must use all the given information, and that there will never be a situation when you have to possibly leave something irrelevant out);
- Keywords act as prompts for students as to what to do;
- Reproduction and procedural skills are required; and
- Real-life experience is rarely called on.

In order to successfully replicate the assessment format that Irish students are familiar with, the author made the decision to utilise test questions from previously implemented Junior Certificate assessment papers. The author decided to select arithmetic and algebraic questions from the Junior Certificate examinations in order to achieve consistency with the skills necessary to successfully solve the mathematical problems in the realistic test. The author is of the opinion that this facilitates a comparison of results.

5.12.2.3. The Semi-structured Interview

The author highlights the following questions as being pertinent to her interviewing technique:

- The level of the mathematics course followed by the class group in question;
- The number of mathematics students in the class;
- The number of class periods per week for mathematics in the time-table for the year group in question (second years);
- Any behavioural issues that affect teaching and learning of mathematics with the class group in question;
- The availability of extra assistance for students that may require it;
- The number of mathematics class groups in the year;
- The level of mathematics followed by the year group (is foundation level mathematics available for the students?);
- The number of mathematics teachers in the school;
- The value placed on a mathematical ethos in the school;
- The use of information technology in mathematics lessons;
- The implementation of ‘Project Maths’ teaching methods in preparation for the introduction of ‘Project Maths’ for all year groups;
- The predicted Junior Certificate results for the class group in question.

The author implemented the semi-structured interview as a means of obtaining qualitative data that provides an insight into what is happening in Irish schools as regards the scheduling of mathematics classes, distribution of mathematics teachers and the provision of special help in mathematics for students who require extra assistance. The provision of smaller class groups for students who are less mathematically able was also investigated and noted. This concept of streaming students for mathematics is interesting as it is a relatively common concept in Ireland while research suggests that it is not a positive. The research relating to streaming mathematics students suggests that the 'elite' students in the top classes benefit from this system but all other students suffer from the glass-ceiling that streaming provides (Boaler, Wiliam & Brown, 2010). In Ireland the availability of different mathematics courses of varying degrees of difficulty (higher, ordinary, foundation) affords an imposed streaming of sorts. While it is difficult to argue with the provision of small classes with extra help for students who more require it, the debate regarding streaming is interesting and provides interesting food for thought.

The topics selected for discussion in the semi-structured interview with the participants from the United States school varied slightly. The questions asked included (but were not restricted to):

- The selection process for students involved in the study;
- The mean age of the students involved;
- Average class size;
- The utilisation of Algebra 2 students in the study and the possibility that these students are more mathematically able than other students in the year group;
- The number of students participating in the study as a percentage of the year group as a whole;
- The length of a mathematics lesson;
- The number of mathematics periods per week;
- The levels of mathematics studies;
- Topics covered in Algebra 2;
- Other mathematics topics studied in addition to, or in place of, Algebra 2.

5.13 Validity and Reliability

It is essential that both validity and reliability are considered when conducting research of a high standard. The validity and reliability of the data collected will have a profoundly positive or negative effect on the end result of the research. Invalid and unreliable results completely undermine a research project. Creswell and Clark (2011) explain that validity serves the purpose of checking the quality of the data, the results, and the interpretation. Validity of inferences made during the interpretation by the researcher, from the assessment results, can be adversely affected if items, tasks and conditions in the research instrument fail to match the construct that the researcher initially set out to assess (Chatterji, 2003:55). Chatterji explains that validity can also be lowered when the population and/or subpopulation assessed using the research instrument are different to the population for whom the research instrument was initially designed.

5.13.1. Determining Validity and Reliability

Teddlie and Tashakkori (2009) are adamant that the following two questions are pertinent when considering data quality:

1. *Measurement validity/credibility:* Is the researcher accurately measuring / recording / capturing what they intended to, rather than something else?
2. *Measurement reliability/dependability:* Assuming that the data collected is valid and credible, is the measurement and recording of data consistent and accurate, yielding little error? (Teddlie and Tashakkori, 2009:209).

These two questions form the basis of the author's analysis of her own research and data collection in terms of validity and reliability. Data collection methods (tests, classroom observation and semi-structured interview) were carefully selected and constructed keeping measurement validity and credibility in mind. In analysing the data collected the author was fully aware of the importance of consistency and accuracy when recording and analysing all data collected (both quantitatively and qualitatively). By ensuring both validity and reliability for all data collected in the research process and by ensuring the data collection process and methods used were accurate and focused on the research question at hand, the author is confident that the research involved in this project is of a high standard and is both valid and reliable.

There are different methods of assessing validity and reliability in mixed-methods research. It is important to keep in mind that for a mixed-methods project to be of a high standard the individual components of the research (both quantitative and qualitative) must be equally valid and reliable (Teddlie and Tashakkori, 2009:208). Creswell and Clark (2011:210-211) give examples of different types of validity and reliability as follows:

- *Quantitative Validity:* The validity of data collected in a quantitative manner serves two primary purposes: ensuring the quality of the scores collected, and the quality of the conclusions drawn from the quantitative analysis of the results. Quantitative validity means that the scores received from participants are meaningful indicators of the construct being measured;
- *Quantitative Reliability:* The reliability of data collected in a quantitative manner is also essential. Quantitative reliability is established by ensuring that the results obtained from research participants are consistent and stable over time. Statistical procedures can confirm internal consistency and hence reliability.
- *Qualitative Validity:* In qualitative research validity plays a more significant role than reliability. Qualitative validity focuses on whether the account provided by the researcher and the research participants is accurate, credible and can be trusted. Qualitative validity is gleaned from the analysis procedures of the researcher, and involves assessing whether the information obtained through the qualitative data collection is accurate.
- *Qualitative Reliability:* A minor role is played by reliability in qualitative studies. Reliability in qualitative research is primarily concerned with the reliability of multiple coders in a team research

problem. As a result this type of reliability plays no role in the author's research project as there is no teamwork involved, and all research involved is carried out by the author.

- *Construct Validity*: is concerned with assessing if the data collection procedures used measure what they were intended to measure;
- *Criterion-related Validity:* is concerned with the scores adhering to some external standard;
- *Content Validity*: assesses whether the items and questions used in the data collection procedures are representative of possible items;
- *Internal Validity*: considers the cause and effect relationship between variables; and
- *External Validity*: assesses the extent to which the researcher can determine that the results are applicable to a larger population.

(Creswell and Clark, 2011, 210:211)

Teddlie and Tashakkori (2009) explain that content validity is often achieved by asking others to judge if your data collection instrument actually measures what you hope to assess, and is useful when the research instrument hopes to measure a specific and well-defined attribute. Content validity is of particular interest to the author as her research question is a well-defined one: 'Have Irish mathematics students the ability to utilise the mathematics knowledge learned in school to solve unfamiliar mathematical problems that necessitate a level of understanding?'. The validity of the content in the tests implemented by the author is paramount as it is essential, for the research to be valid, that the tests effectively assess the research question. To ensure content validity the author organised a group of content and pedagogical experts consisting of experienced mathematics teachers, Junior Certificate examiners, school management with an interest in mathematics and university mathematics education personnel. This panel examined the content provided in the structured observation template, the tests and the proposed questions for the structured observation and offered their advice and comments. Further to several discussions with all those involved in this process the author is confident that content validity is reached. The implementation of a convergent, parallel mixed-methods design also insured a level of validity among data collection methods. The author also uses triangulation between the three types of data collection to ensure validity.

The reliability of the test, particularly the Realistic test, was ensured by operating a test-retest policy during the pilot study. Test-retest reliability examines the extent to which scores in the administration of a test are stable over time (Creswell, 2008). The Realistic test was implemented to the same group of students approximately eight weeks apart and the results for both tests were found to be similar establishing reliablity. The author did not operate a test-retest check on the Traditional test as it was based directly on the Junior Certificate examination and as a result the author was confident that it was already deemed reliable by the Department of Education and Skills.

5.13.2. Triangulation

Triangulation '*refers to the combinations and comparisons of multiple data sources, data collection and analysis procedures, research methods, investigators, and inferences that occur at the end of a study'* (Teddlie and Tashakkori, 2009: 27).

Triangulation is the process of using more than one data collection method, and hence more than one data analysis method. In a mixed method design it is typical for the process of triangulation to occur due to the fact that data is collected and analysed both quantitatively and qualitatively. Cohen et al (2000) describe triangulation as a process in research where two or more methods of data collection are used as a means of establishing validity. Teddlie & Tashakkori (2009) describe triangulation as a means of not only determining the quality of data but also a useful method of analysing mixed methods data. The purpose of triangulation is to seek corroboration of results using different methods, while the rationale of triangulation is to increase the validity of the constructs used by minimising the impact of irrelevant sources of variance inherent to bias (Gray, 2009).

The author uses triangulation as a means of testing the validity and quality of the data collected. Research methods and data collection procedures used in the triangulation process include:

- A systematic, structured classroom observation;
- The implementation of two tests; and
- An informal interview with the mathematics teacher and school principal.

5.13.3. Research Ethics

The ethics involved in any research project involving human participants are complicated as it is important that the rights of each individual are not sacrificed in the name of research. Mathematics is a subject that often causes anxiety. As a result the author considers it imperative that all possible efforts are made to reduce the possibility of anxiety in implementing the mathematics assessments, keeping in mind that assessments in general are also prone to creating feelings of anxiety. Burns (2000) suggests that it is difficult to conduct research without encountering some ethical issues. Burns (2000:22) suggests adhering to the following ethical code:

- Risks to the participants should be minimised and subjects not exposed to risk;
- The benefits outweigh the risks in relation to participants;
- The rights and welfare of the research subjects are protected;
- Participation is voluntary;
- The participant has the right to know the nature, purpose and duration of the research study;
- The subject is free to withdraw anytime without penalty;
- Information obtained is confidential; and
- Participants are fully debriefed after the study.

The author fully adhered to the ethical code suggested by Burns as outlined above.

5.13.4. The ethical dilemma

Cohen et al (2000) describe an ethical dilemma as a situation which can arise in response to the conflict between the researcher's quest for the truth and the subjects' rights and values. Ethical problems can arise as a result of:

- The research question;
- The methods of data collection;
- The age of the participants;
- The mental capacity of the participants;
- The procedures to be adopted; and
- What will be done with the data collected.

The author adhered to a strict ethical code in order to reduce any ethical conflict or the risk of encountering an ethical dilemma. The author was fully aware at all times during the research process of the possibility of ethical problems and made every effort to reduce and/or avoid ethical conflict.

5.13.5. Informed consent

The principle of informed consent arises from the individual's, and therefore the research subject's, right to freedom, which is a condition of living in a democratic society. Restrictions to personal freedom must be justified and consented to (Cohen et al, 2000:51). Cohen et al. identify four elements of informed consent:

- *Competence*: implies that the subject, or those legally responsible for the subject, is capable of making correct decisions when given the relevant information.
- *Voluntarism*: ensures that research participants make the decision to participate (or not) in the research knowingly and voluntarily.
- *Full Information*: implies that the subject is fully informed as to what the research entails.
- *Comprehension*: refers to the fact that the subject should fully understand what he/she is agreeing to.

The author was aware of the ethical implications of ensuring informed consent and fully adhered to the four principles of informed consent as suggested by Cohen. Students and parents/guardians were given information sheets providing information regarding the research project. Parent/guardian and student consent was required for inclusion in the study. It was made clear to all research participants that participation was voluntary and that they were entitled to leave the project at any stage (even if the data had already been collected). The author was available (by telephone and email) to parents/guardians and students at all times during the research and was willing to answer any questions that arose. As a result the author believes that she fully adhered to the following principles underlying informed consent. The researcher should provide:

1. An unbiased and understandable explanation of the nature of the research, its purpose and the procedures to be followed;
2. An understandable description of any reasonable level of discomfort that may be experienced by the subject;
3. An explanation of any benefits which may be expected as a result of participating in the study;
4. An understandable disclosure of any appropriate alternative procedures which may be advantageous to the research subject;
5. An offer to answer any queries the subject may have with regard to the research; and
6. A clear instruction that the subject is free to withdraw from the research project at any stage prior to its termination. (www.enmu.edu/services/grants/human-subjects-policy.doc).

5.14 Conclusion

The purpose of this chapter is to place the research question within the research methodology and research methods that pertain to it. The author is confident that her decision to use a mixed-methods design is an effective technique for considering the research question: ‘the ability of Irish students to transfer mathematics from the classroom to unfamiliar, real life situations’.

To summarise, the author decided on a mixed-methods study which uses the following research methods:

- Systematic, structured classroom observations;
- Semi-structured interviews; and
- Testing.

The methods selected effectively contribute to a convergent, parallel mixed-methods design. The following chapters (chapter’s 6 and 7) consider the data collection using these methods and the analysis of the data collected.

6.0 Chapter 6: The Data Collection Process

6.1 Introduction

This chapter considers the data collection process and the implementation of this process. The author outlines the process, involving the following:

- Obtaining Consent;
- Selecting a research sample;
- Gaining access to the schools involved in the research;
- Selecting the data collection methods (as discussed in Chapter 5); and
- Implementing the data collection methods.

6.2 Obtaining Consent

As this study involves young people, ethical clearance was obtained from the National University of Ireland Maynooth (NUIM) ethics board. In addition to providing the ethics board with information regarding the study and the purpose of the research, the author's application also included:

- An information sheet for mathematics teachers;
- An information sheet for parents;
- An information sheet, using appropriate language, for students;
- Consent forms for parents; and
- Consent forms for students.

Once ethical clearance was obtained from the NUIM ethics board it was then necessary to approach the schools. In the first instance the author approached the principals of the schools, selected at random, as possible participants in the study. While the schools were selected at random it is important to reiterate that no single-sex schools were involved in the study (as single-sex education could then become an influencing factor on the data collected) and all schools approached were from three bordering counties for ease of data collection (based on proximity to the author's place of work). The principals of the schools selected were provided with information about the research study and asked to participate. The principal in each school acted as the gate-keeper to that institution and had complete power to deny access to the author. Those principals that agreed to participate in the study were then asked to discuss participation with the mathematics teachers in their school. One volunteer was needed from each school involved. The school principals were also responsible for approaching the board of management in their particular school for permission to initiate the research project in each individual school.

The next step in the research process was obtaining consent from the students in the class group selected and the parents of these students. Consent forms were designed for distribution to both parents and students. It was decided by the author that each individual student would have the final say regarding their right to refuse to participate in the study, regardless of their parents' consent. However, a parent or guardian's right to refuse consent could not be invalidated by the student's interest in participating.

6.3 The Research Sample

The author tested students in six mathematics class groups from five different Irish secondary schools, after an initial pilot study. The students involved in the research were all in second year at the time of testing. As mentioned earlier, all five schools were co-educational and comprised of a variety of school types: community school, community college and secondary school. Schools were located in both rural and urban areas. None of the schools selected were DEIS (Delivering Equality of Opportunity in Schools) schools; this was by accident rather than design as all schools were randomly selected as outlined below. Each school approached was asked to nominate one second-year mathematics class for inclusion in the study. This required the mathematics teacher of this class group to be a willing participant in the study. It was at the school's discretion as to whether the nominated class group was an ordinary or a higher level class. All students within the class participating in the study had the opportunity to opt out of the study. One school volunteered two class groups for participating in the study.

6.3.1. School Selection

Five Irish schools were involved in the study. The schools were selected at random from co-educational schools in three bordering counties. While the selection was random the author initially approached a broad range of school types: secondary, comprehensive schools, community schools and vocational/community colleges. The schools approached were selected based on their proximity to the author's place of work in order to facilitate the collection of data. Seven schools were contacted and asked to participate in the study. All seven schools were initially approached via a telephone conversation with the principal of the each individual school. After some consideration and discussion with staff members all seven schools initially agreed to be involved in the study. The principal of one school initially agreed but the mathematics teachers refused to participate when requested. The author believes that this may have been due to a lack of comfort regarding the observation element of the data collection. Despite repeated reassurance that the teaching of teachers was merely being noted with reference to 'Learning Theories in Mathematics' some teachers were uncomfortable with what they felt was a critical presence in their classroom.

A second school out of the seven who initially agreed to participate did not follow through. In this instance, despite repeated attempts to make contact with the principal on the part of the author, the school never responded to these attempts and access was thus denied. One of the seven schools agreed to participate with two class groups. Therefore the author considered six class groups in five schools.

The following table considers the level of mathematics studied by each of the class groups involved:

Group number	Teacher	Level of Junior Certificate Mathematics studied
1	A	Ordinary Level
2	B	Higher Level
3	C	Higher Level
4	D	Ordinary Level
5	E	Higher Level
6	F	Ordinary Level
7 (Massachusetts)	G	n/a

Table 11: Schools and teachers involved and the level of Junior Certificate mathematics course studied

The following table illustrates the schools involved in the data collection process and provides information about each of these schools:

School	School Type	Location	Size	Class Group
1	Secondary	Large Town	1150-1250	Ordinary Level (Lowest Group on each side of the timetable)
2	Community College	Village	300-400	Higher Level (Wide range)
3	Community School	Town	700-800	Higher Level (Top group)
4	Community school	Town	600-700	Ordinary Level
5	Secondary	Large Town	700-800	Higher Level (Top group)
6 (same as 1)	Secondary	Large Town	1150-1250	Ordinary Level (Lowest Group on each side of the timetable)
7 (Massachusetts)	Middle School	Large Town	600-700	n/a

Table 12: School type, location, size and class group

6.3.2. Massachusetts as a research sample

In order to introduce an international component, as per the literature review, it was deemed important by the author to have data from a different country. TIMSS (Trends in Mathematics and Science Study) is based in Boston College, Massachusetts. For this reason the author decided to approach the surrounding school district in the hope that a school may be interested in participating in the study.

Massachusetts is a state noted for its top class mathematical achievement in international assessments. In the most recent TIMSS (Trends in International Mathematics and Science Study) assessment, TIMSS 2007, Massachusetts was the top scoring state in the United States of America. Eighth grade students from Massachusetts (the cohort that participated in the author's research) scored sixth in the world in mathematics with a score of 547. Massachusetts was only outperformed by Asian countries with the following scores:

1. Chinese Taipei (598);
2. Republic of Korea (597);
3. Singapore (593);
4. Hong Kong SAR (572); and
5. Japan (570).

(http://www.doe.mass.edu)

The author therefore was of the opinion that a comparison between Irish mathematics class groups and the same age cohort in Massachusetts would be particularly interesting due to the high performance of the state in international assessments.

There are 400 school districts in the state of Massachusetts and 316 Middle Schools (www.doe.mass.edu). Three Massachusetts' school districts were initially approached and asked to participate in the research. Each school district is responsible for designing and implementing their own curriculum. The superintendant in each school district is responsible for curriculum matters. The school district involved in the research had a superintendant who was particularly helpful and interested in participating in the study. For this reason, the author decided to utilise this school district in the research. The other two school districts approached were initially interested in participating in the study but it became difficult to get a guarantee of participation. The school district involved in the study committed to providing a sufficient number of students for the research to be conducted and for this reason the author was content to proceed with this school district.

The school district selected is located in a relatively affluent suburb approximately ten miles outside of the city of Boston. The suburb has a population of approximately 25,000 people. The school district is responsible for eight schools with a total student population of 4,428 students. There are five elementary schools; two middle schools and one high school. As the author’s research focus is on students in the eighth grade, the focus is on the middle school category which caters for students from the sixth to the eighth grade. The school district provided access to the larger of the two middle schools which had a student enrolment of approximately 600 students at the time of data collection.

The participating school had a student-teacher ratio of 15 to 1. 17% of students had an IEP (Individualized Education Program) which is a written plan for students identified as requiring special needs services. There were approximately 200 students in grade 8 and the mean age at the time of testing was 13.5 years. All students that studied ‘Algebra 1’ were asked to participate (50% of eighth grade students). The most mathematically able students did not participate as these students were studying a different subject, ‘Advanced Topics’ (8% of 8th graders). The least mathematically able students (42%) also did not participate as they were not studying algebra at the level required for test participation (as decided by their teachers).

6.4 Research methods

'There is growing evidence, then, that not only may schooling not contribute in a direct and obvious way to performance outside school, but also that knowledge acquired outside school is not always used to support in-school learning. Schooling is coming to look increasingly isolated from the rest of what we do' (Resnick, 1987:57).

The research seeks to examine the ability of Irish students to mathematise and use the mathematics learned in the classroom and school environment to solve unfamiliar, realistic mathematical problems. In order to consider this the author had to decide on a method of assessing both the skills of students and their mathematical ability as currently determined by the Irish assessment system. After much deliberation and analysis of the literature surrounding mathematisation, international assessments in mathematics and research methods, the author decided on a three-tier process involving the following:

- Structured observations;
- Semi-structured interviews; and
- Testing.

The following section considers each of the data collection methods in more detail.

6.4.1. The Structured Observation

A structured observation of a mathematics class involving the class group being assessed was carried out for each of the Irish groups. The purpose of the classroom observation is to gain some insight into the teaching and learning that is happening in a 'traditional' Irish mathematics class. The teaching and learning practices are considered in terms of mathematics learning theories (absolutist and relativist theories). The observations provide the context in which to consider the implemented tests.

6.4.2. The Semi-Structured Interview

A semi-structured interview is implemented in order to gain insight into the mathematics activities in the classroom and the structure of mathematics activities (timetabling, allocating of classes etc.) within the school. The semi-structured version of the interview was the data collection process decided on in order to facilitate the opinions of the interviewee to be expressed and allow elaboration of topics, questions and/or answers as the need arose.

6.4.3. The Tests

Two tests were selected by the author as a means of assessing mathematical knowledge, understanding and problem solving skills. The two tests decided on are:

- *A 'Traditional' mathematics test*: based on the current Junior Certificate curriculum and focused on the related assessment style. The test questions are directly related to the Junior Certificate examination and are selected from past Junior Certificate papers. The questions are from the algebra, statistics and arithmetic sections of the ordinary level examination. The ordinary level syllabus is covered by all second year students who eventually sit both the higher and ordinary level Junior Certificate mathematics examination. For this reason the author felt that it found common ground without excluding anyone. All students in the schools involved follow the Junior Certificate ordinary level syllabus at some stage - even if they eventually sit the foundation level Junior Certificate examination. All questions in this 'Traditional' mathematics test (and in the Junior Certificate examinations) are closed-ended questions and have one correct answer only. No surplus information is provided.
- *A 'Realistic' mathematics test:* which is not in a style familiar to Irish mathematics students and is connected to real-life experience and situations. The 'Realistic' test involves an authentic, problem-solving scenario. The scenario involves open-ended questions, for which there

can be more than one correct solution. Students are asked to demonstrate an understanding of how mathematics can be utilised to solve realistic problems. Students are asked to show evidence of decision-making and reflection. Surplus information is provided in some parts of some questions. The mathematics involved in the problem-solving are no more difficult than those required for the 'Traditional' test, and are of an ordinary level, Junior Certificate standard.

6.5 The pilot study

The pilot study involved a second year class of 32 students. The pilot school is a large school of over 1,200 students situated in a large town in the west of Ireland. The students involved in the pilot study were all studying the higher level Junior Certificate course at the time the tests were administered. The school places a strong emphasis on mathematics and each second year class group has five, forty-minute classes per week. Students were streamed into higher and ordinary level groups at Christmas of first year to allow students to work at a pace that suits their individual ability. Topics are covered sequentially for all class groups allowing for movement from lower to higher ability class groups and vice-versa depending on the progress of each individual student.

The school principal and board of management were fully supportive of the research project, and there were no significant issues with gaining access to the school. The mathematics teacher of the class group was also a willing participant in the research. Consent forms and information sheets were distributed to all students in the class group and to their parents/guardians. Consent was granted from all parents/guardians of the students, and just as importantly, all students in the class group were willing participants in the research. The author maintained a strict policy of giving the students the final say as regards refusing to participate regardless of their parents granting consent. Students were also informed on several occasions, and in the information sheet, that they were entitled to leave the research at any stage prior to the research project finishing. Separate information sheets were provided for the following groups involved in the research:

- Students;
- Parents/guardians; and
- Teachers.

The pilot study involved the implementation of the following data collection processes:

- Two 'Realistic' mathematics tests twice;
- One 'Traditional' mathematics test; and
- A structured observation.

The researcher administered two Realistic tests in the first instance, in two separate mathematics lessons with the one class group. The reasoning behind the implementation of two realistic tests was to identify which test accurately assessed realistic problem solving skills effectively. Each Realistic test was implemented twice to assure reliability under test-retest conditions. The test-retest process occurred approximately eight weeks apart. Consistency of grades for the repeated test were essential for the test to be deemed reliable. The Traditional test, as designed by the author, was also implemented. As this test consisted of ordinary level, Junior Certificate mathematics examination questions the author was confident that the students would be able to attempt varying amounts of this test depending on ability and therefore did not expect this test to need the same amount of adaption. The reliability of the Traditional test is also considered to be stable as it has been designed, tested and implemented by the Department of Education and Skills.

The format of the Traditional test was found to be suitable following the pilot of same. However, one Realistic test was found to be superior (see 6.6.3.3). This test met the reliability standards required during the test-retest process. The pilot of this Realistic test identified issues that were amended. These problems included the following:

- Formatting issues;

- No space provided for answers; and
- Difficultly in identifying the question asked.

The author adapted the Realistic test to take these issues into consideration; the format was adjusted, space was provided for answers and the actual question was highlighted for ease of identification.

A structured observation schedule was also designed by the author (see 6.6.1.1). The schedule was used to facilitate the structured observation of a mathematics lesson. The mathematics observed in the pilot study involved the higher level class group that participated in the testing of the pilot study. The author was satisfied that the template designed for the structured observation was effective and no changes were made after the pilot observation. The author initially used a regular watch for timing purposes in the pilot study but on reflection adapted this and used a stop-watch for the observations in the main research study.

6.6 The Data Collection Process

The following section introduces the data collection tools used in the research and discusses the implementation of these processes.

6.6.1. The Structured Observation

A structured observation is a quantitative method of observing activity within a classroom. This involves:

- Identifying target behaviour(s) prior to observation;
- Developing checklists or other schedules; and
- Applying these instruments on classroom settings to record the frequency of occurrence of the identified behaviour (Atweh et al, 1992:94).

The categories predetermined as target behaviours prior to implementation of the study were designed with specific learning theories and styles in mind. The author based the learning categories on behaviours she believes to be typical and/or desirable in a typical mathematics lesson.

To recap, different learning theories in mathematics can be considered in two categories:

- *Objectivist Theories of Learning*: The 'Objectivist' learning theories (to include behaviourism) are concerned with mathematics as being independent from the environment in which it is learned. The focus is on the information as the most important aspect of all teaching and learning. Teaching is didactic, with the teacher as the centre of all teaching and learning. Students focus on retaining knowledge, rote-learning and reproduction. Tasks are broken down by the teacher into manageable components, with the mathematics task as a whole not considered overly important. Mathematical applications are not considered to be of importance under the 'Objectivist' view.
- *Relativist Theories of Learning:*

  The 'Relativist' learning theories (to include constructivism) focus on the learner as the centre of all knowledge. The emphasis is on active learning and problem-solving. A distinct move away from abstraction is encouraged. Learning is not broken down into its component parts, rather learning is considered to be authentic and real. Mathematical applications are of paramount importance, with the student fully immersed with the problem as it appears in the real-world. Mathematics is not considered to be separate from one's real-life, but rather part of one's personal, as well as educational, experience. Student's prior knowledge is of value from a Relativist standpoint.

6.6.1.1. Analysis of the Structured Observation

The structured observation schedule consists of 30-second intervals and 22 teaching and learning activities that the author believes are characteristic of an Irish mathematics lesson. The activities are not mutually exclusive, and more than one activity can occur simultaneously (e.g. 'board-work' and 'teacher explanation'). The identified activities comprise of generic tasks such as 'role-call' (establishing who is absent) and 'discipline', and teaching and learning tasks. The author considers the mathematical activities in terms of Absolutist and Relativist theories of learning.

The author organises some of the identified tasks in the structured observation schedule in terms of the Absolutist and Relativist theories of learning. The author considers teaching and learning tasks from the schedule as follows:

Absolutist	**Relativist**
Board-work	Group-work
Book-work	Student computer work
Teacher explanation	Real-life reference
Student question	Student discussion
Teacher instruction	Active Learning
Teacher question	
Individual work	
Student answer	

Table 13: The categorisation of observable activities as either Absolutist or Relativist

By observing typical teaching and learning behaviour in Irish classrooms the author expects to gain an insight into the learning theory that underpins current Irish mathematics education. The type of activity occurring in each mathematics class observed will offer a window into the activity that occurs throughout the country. The author is fully aware that the six mathematics lessons observed are only a representation of what is typical, but nevertheless believes that the opportunity to observe the class groups involved in the testing and interview stages of the data collection process is a valuable opportunity to consider teaching and learning practices in Irish classrooms.

The following (fig.3) is an extract of the structured observation schedule and shows the schedule for the first ten minutes (20, 30 second intervals). The schedule is repeated identically for the next 30 minutes (60, 30 second intervals).

Type of Activity	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Role-call																				
Arrival/settling/packing																				
Discipline																				
Homework																				
Active learning																				
Book-work																				
Board-work (teacher working at the board)																				
Individual work																				
Group work																				
Teacher explanation																				
Student question																				
Teacher question																				
Student answer																				
Positive reinforcement																				
Overhead Projector																				
Interactive white board																				
Student computer work																				
Real-life reference																				
Non-mathematical activity																				
Teacher going around																				
Teacher instruction																				
Student discussion																				

Figure 3: The structured observation schedule

The three most frequent activities observed in each structured observation are highlighted in the analysis of the structured observation.

The author implemented this structured observation schedule with each of the six Irish mathematics class groups involved in the research. As discussed in Chapter 5, the author identified herself as a 'complete observer'. The teachers involved in the research were informed in advance of the mathematics lesson that would be observed. The mathematics lesson observed involved the second year class group who were participating in the testing component of the research. The author observed the teaching and learning as it occurred in the mathematics lesson and completed the observation template for the duration of the lesson. The author found the implementation of the structured observation to be a very straight-forward process and no difficulties occurred. The findings of the observations are discussed in Chapter 7.

6.6.2. The Semi-structured Interview

As discussed in the 'Research Methods and Methodology' chapter, the semi-structured interview was decided on by the author as a means of data collection. The primary reason for this is that the author felt that the data collected through testing and observation needed to be placed in context. The practicalities of what happens in the mathematics classroom, and indeed the position of mathematics within the school, are essential in considering the ability of students to transfer mathematical knowledge to solve authentic problems. The mathematics teacher is also the person who is most familiar with their mathematics class group, and therefore is perfectly placed to provide an insight into what typically happens in a mathematics lesson. A semi-structured interview provides the author with qualitative data that offers an insight into teaching and learning practices, and hence mathematics learning theories, that the quantitative data provided by the structured observation and the testing cannot offer.

To recap, the author highlights the following questions as being pertinent to obtaining the data she requires to validate data collected for her research question. The questions asked, and the information provided by the interviewee, varied from class group to class group, and between Ireland and the U.S. state of Massachusetts.

The following topics were covered in the interviews with each of the Irish mathematics teachers:

- The level of the mathematics course followed by the class group in question,
- The number of mathematics students in the class;
- The work ethos within the class group;
- The number of class periods per week for mathematics in the time-table for the year group in question (second years);
- Any behavioural issues that affect teaching and learning of mathematics with the class group in question;
- The availability of extra assistance for students that may require it (SNAs (special needs assistants), special needs tuition, homework club availability etc.);
- The number of mathematics class groups in the year;
- The level of mathematics followed by the year group: ordinary and higher level only or is foundation level mathematics available for the students;
- Does the teacher anticipate that any of the students involved in the research may sit the foundation level examination in the Junior Certificate;
- The number of mathematics teachers in the school;
- The value placed on a mathematical ethos in the school;
- The use of information technology in mathematics lessons;
- The implementation of ‘Project Maths’ teaching methods in preparation for the introduction of ‘Project Maths’ for all year groups (the author is aware that the new ‘Project Maths’ syllabus is not in place for the class groups involved in the research project but is interested in any teaching and learning changes that may be undertaken in preparation for its implementation); and
- The predicted Junior Certificate results for the class group in question.

The topics selected for discussion in the semi-structured interview with the participants from the school involved in the research project from the state of Massachusetts in the United States varied slightly. The questions asked included (but were not restricted to):

- The selection process for students involved in the study;
- The mean age of the students involved;
- Average class size;
- The utilisation of Algebra 1 students in the study and the possibility that these students are more mathematically able than other students in the year group;
- The number of students participating in the study as a percentage of the year group as a whole;
- The length of a mathematics lesson;
- The number of mathematics periods per week;
- The levels of mathematics studies;
- Topics covered in Algebra 1; and
- Other mathematics topics studied in addition to, or in place of, Algebra 1.

The author conducted the interviews with the teachers of the Irish class groups at a time after the classroom observation had been conducted. The duration of each interview was twenty minutes and the interview was held at a time that suited the teacher to be interviewed. All six Irish teachers involved in the research were willing interviewees and provided invaluable information for qualitative research purposes. The interviews provided a valuable insight into mathematics teaching practices and teacher opinions in Ireland. All Irish interviews were held in person and at the teacher's school.

The interview process was different when collecting data from the Massachusetts candidates. The restriction imposed by the author not visiting Massachusetts for data collection purposes necessitated the interview being conducted by telephone and through email. The Massachusetts teachers decided that it was most feasible if the head of the mathematics department in the school spoke on their behalf. After an initial telephone conversation between the author and the head of department, Teacher G, it was decided to conduct further interviews by email. This allowed Teacher G to consult with the individual mathematics teachers and collect answers to the author's questions. Six email interviews were conducted in total.

The findings to all seven interviews are discussed in Chapter 7.

6.6.3. The Testing Process

The author administered tests in order to assess the ability of Irish students to mathematise and their ability to transfer mathematics learned in the classroom to unfamiliar situations. Two assessments, a Realistic and a Traditional test, were decided on in order to compare the ability of the students involved in the study to solve mathematics problems that varied in style. Each test was thirty minutes in duration.

6.6.3.1. The Realistic Test

'If one views mathematics as a dynamic set of interconnected, humanly constructed ideas, then the assessment system must allow students to engage in rich activities that include problem-solving, reasoning, communication and making connections' (Romberg,1995:4).

The Realistic test focused on an unfamiliar, real-life scenario. The mathematical skills that were required by the test questions were skills that the students assessed should be familiar with. In accordance with the Irish mathematics curriculum these skills would not be considered difficult for a typical, second-year Irish student. The content validity of this test was assured by test analysis undertaken by an expert panel.

The PISA (Programme for International Student Assessment) provided a framework for the style of questioning used in the Realistic test compiled for this research project. PISA’s aim is to assess the extent to which participating countries have prepared students to play a constructive role in society. The focus of the PISA assessment is to consider if students have the ability and skills to use what they have learned in the classroom when faced with realistic situations they may encounter in their daily lives (http://www.oecd.org). The author is of the opinion that the PISA assessment style is similar in aim to the author’s own and for this reason the PISA assessment format formed a basis for the Realistic test used in this research. A high literacy level is required for this style of test. As a result all students were informed that the test could be read to them and the reading of the test could be repeated as often as they required. Irish students with identified high levels of learning difficulties had access to special needs assistance who were available to assist them with their literacy requirements. The realistic test is based on a question provided on the NRICH website (http://plus.maths.org/content/os/latestnews/jan-apr10/activity2/index).

The Realistic test implemented is shown in Appendix VII for the Irish version and Appendix VIII for the Massachusetts’ version.

6.6.3.2. The Traditional Test

The Traditional test administered was designed based on the framework provided by the relevant terminal examination, the Junior Certificate. The author decided to base the test questions on the Junior Certificate assessment and selected questions from past examination papers. The reasoning behind this was that the Junior Certificate assessment is the means used for determining mathematical ability at junior cycle, at second-level, in Ireland. The mathematics topics tested included Junior Certificate algebra, arithmetic and statistics.

The Traditional test implemented is available in Appendix VIII (Irish version) and Appendix X (Massachusetts version).

6.6.3.3. Implementing the Tests

The author visited each individual Irish school to administer the test in person. Two separate visits were required for each test. As discussed earlier, the test duration was thirty minutes. This was based on the fact that most Irish class periods are forty minutes in length and this allowed time for the author to introduce the test and for any housekeeping issues to be dealt with. The teachers involved in the study were not provided with the tests prior to implementation but were informed as to the basic concept of each. The test content was not divulged in advance. It was at the individual schools' discretion as to whether the class teacher remained in the classroom for the testing or not. There were twelve Irish test scenarios in total, two for each class group. The class teacher was present in nine out of the twelve tests.

As the author did not visit Massachusetts in person for data collection purposes, the implementation of the testing process varied slightly. The duration of each test remained at thirty minutes. Pdf versions of each test were emailed to Teacher G. There were two minor amendments to the Massachusetts' version of the tests:

- € symbols were changed to $; and
- An income tax question on the Traditional test was amended slightly (see Appendices VIII and X).

Teacher G took full responsibility for ensuring that the test content and format was not divulged before the testing process. The tests were sealed and sent by post to the author on the evening that the second test was implemented. Marking schemes are available for the tests to illustrate how they were scored (Appendix VII:ii; VIII:ii; IX:ii; X:ii).

6.7 Conclusion

In summary, this chapter introduces the reader to the research methods selected for data collection purposes and the implementation of these. The data collection methods included:

- Structured observations of mathematics lessons involving the Irish class groups participating in the research;
- Semi-structured interviews with the mathematics teachers of the Irish class groups and the head of mathematics, Teacher G, from the school in Massachusetts; and
- Testing of the Irish mathematics class groups in addition to a cohort of students from Massachusetts.

Chapters 7 and 8 consider the findings from the data collection process.

7.0 Chapter 7: Data Analysis (Structured Observations and Interviews)

7.1 Introduction

This chapter considers the analysis of the structured observations of the mathematics lessons of seven class groups and analysis of the interviews with seven mathematics teachers. The seven groups involved in the structured observation include the pilot study and the six Irish mathematics class groups. The interviews were administered to the teachers of the six Irish mathematics groups plus the head of mathematics in the school from Massachusetts.

7.2 The Structured Observation

The following section considers the analysis of the data collected through quantitative methods by structured observation of the mathematics lessons with the Irish class groups involved in the research.

7.2.1. The Pilot Observation

The Pilot structured observation involved the observation of a student teacher, teaching trigonometry to a second year mathematics class group. The mathematics topic taught was not considered to be of importance as long as the teacher felt that the topic show-cased a 'typical' mathematics lesson for their class group. The class observed were a higher level group and are the same class group that were involved in the pilot testing. The class lasted thirty-six minutes and during the observation the majority of time was spent on correcting homework, given to students during the previous lesson and for completion the previous night. The teacher observed, Teacher P, explained that while it is typical for the correction of homework to take a significant proportion of the lesson, the observed class was especially concerned with the correction of homework. There was no particular reason for this and a discussion with the teacher found that the length of class-time spent on homework varied from lesson to lesson. The three teaching and learning activities identified in the schedule that occurred the most frequently in the pilot observation are:

1. *Homework* (78%): The 'homework' observation involved the teacher dealing with and reviewing the previous night's homework by writing the correct solutions to the questions that were set on the whiteboard. The teacher asked the students questions to clarify that they understood the homework, and the students asked the teacher questions about elements of the homework and/or the explanation given by the teacher that they did not understand. The teacher also circulated the room in order to look at the homework the students had attempted the previous night in their copybooks. All of the homework set in the previous lesson comprised of questions from the prescribed textbook. The final answer to all questions are provided in the back of the text book – all questions were closed-ended with only one correct answer;

2. *Board work* (73.6%): The teacher, Teacher P, used the whiteboard to display the correct answers to the previous night's homework and to expand the topic (trigonometry) into new areas. As is very common in Behaviourist learning theories the teacher used the whiteboard as a resource for his 'chalk-and-talk' method of teaching. Again 'chalk-and-talk' is a teaching technique frequently used in Irish mathematics classrooms, and involves the teacher explaining mathematical topics by writing solutions on the white-board and speaking about their workings as they work through the question;

3. *Teacher explanation* (62.5%): The observed lesson was teacher-centered with the students relying on the teacher to tell them exactly what to do. The students in the observed class were very concerned with copying everything the teacher, Teacher P, wrote on the board word for word into their copybooks. The teacher explained much of

what was written on the board and also answered the student's questions.

The focus of teaching and learning in the observed pilot class was firmly rooted in the Absolutist philosophies, with little, if any, emphasis on Relativist tasks such as 'Group work', 'Real-life reference', 'Student discussion' or 'Active learning. Behaviourist tasks such as 'Board-work', 'Individual work', ' Teacher explanation', 'Student question', 'Teacher question' and 'Student answer' dominated the observed mathematics lesson. The purpose of implementing a pilot structured observation is so that any adjustments to the schedule, deemed necessary by the researcher, can be made. The author was satisfied with the observation schedule initially decided on. The author initially used a watch with a second hand to time the observation intervals but after the pilot study decided to use a stopwatch for better accuracy. The author also made the decision, after the pilot study, to use a clipboard for ease of recording. The teacher in each of the observed lessons was given the choice as to where they wished to physically locate the observer within the class group. The pilot teacher placed the observer in the back, right-hand corner of the classroom. It was a good position within the room from which to unobtrusively observe what is happening in the class. As planned, and explained to the research participants, the observer did not participate in the observed lesson at all.

The following table shows the time spent on the identified tasks in the pilot study:

Type of Activity	**Fraction of total time slots (72, 30 sec slots)**	**% of total class time (36 mins)**
Role-call	3/72	4.2%
Arrival/settling/packing	6/72	8.3%
Discipline	-	-

Type of Activity	Fraction of total time slots (72, 30 sec slots)	% of total class time (36 mins)
Home-work	**56/72**	**78%**
Active learning	-	-
Book-work	15/72	20.8%
Board-work	**53/72**	**73.6%**
Individual work	14/72	19.4%
Group work	-	-
Teacher explanation	**45/72**	**62.5%**
Student question	11/72	15.3%
Teacher question	26/72	36.1%
Student answer	26/72	36.1%
Positive reinforcement	-	-
Overhead projector	-	-
Interactive white board	-	-
Student computer work	-	-
Real-life reference	-	-
Non-maths activity	1/72	1.4%
Teacher going around	9/72	12.5%
Teacher instruction	-	-
Student discussion	7/72	9.7%

Table 14: Structured observation analysis of the pilot study

The following graph provides a visual representation of the teaching and learning activities that occurred during the structured observation of the pilot group:

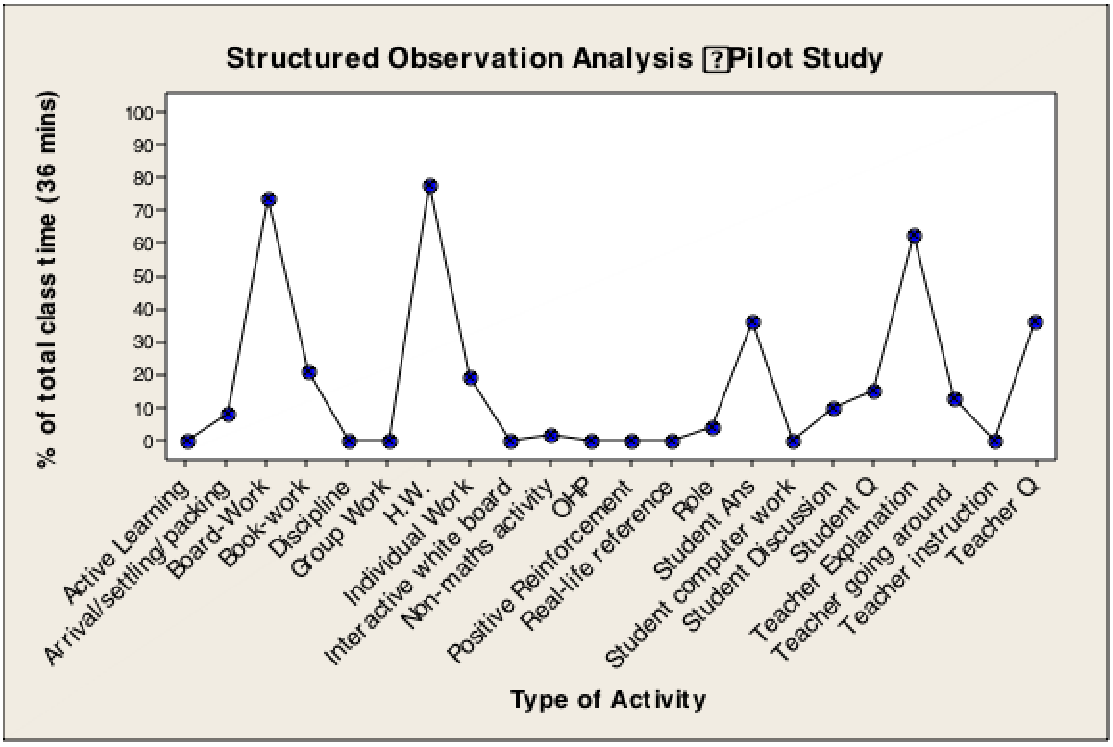


Figure 4: Graphical analysis of the observed pilot lesson

7.2.2. The Structured Observation -Group 1

The observed class group are an ordinary mathematics class in a large secondary school. The class is dedicated to providing for the students who find mathematics particularly difficult. The researcher observed the teacher teaching mathematical functions to this group of students. The author noted that the small class size enabled the teacher, Teacher A, to provide individual support and attention for each of the students. This appeared to be needed by the majority of the nine students in the class. The students were engaged and attentive, with much effort put into individual work. Teacher A explained in the interview that individual work encourages students to focus on the task at hand.

The most frequently identified teaching and learning activities in the observation schedule observed in Teacher A’s class were:

1. *Individual Work* (48%): The students worked individually to solve mathematical problems. The questions solved originated from the textbook and were closed-ended questions with one correct answer;

2. *Teacher going around* (38.75%): The teacher spent a considerable amount of time going from student to student and ensuring that each student received the individual attention necessary to solve the mathematical problems correctly; and

3. *Book-work* (30%): All the students had individual textbooks from which they worked. The teacher also selected mathematical problems from the prescribed textbook for demonstration purposes on the white board.

The author notes that the activities that dominate in the observed class (group 1) are concerned with tasks of an Absolutist nature. Tasks associated with the Relativist theory of teaching and learning include 'group-work', 'student computer work', 'student discussion', and 'active learning' were not part of the teaching and learning methods used in the observed class. However, the author notes that Teacher A made 'real-life references' more frequently than any of the other observed teachers, and made every effort to link the mathematics learned in the classroom to realistic situations. Despite this, the time spent on 'real-life references', 8.75%, was proportionately small.

The following table illustrates the time spent on the observable activities in the lesson observed for group 1:

Type of Activity	**Fraction of total time slots (80, 30 sec slots)**	**% of total class time (40 mins)**
Role-call	2/80	2.5%
Arrival/settling/packing	13/80	16.25%
Discipline	4/80	5%
Homework	-	-
Active learning	-	-

Type of Activity	Fraction of total time slots (80, 30 sec slots)	% of total class time (40 mins)
Book-work	**24/80**	**30%**
Board-work	20/80	25%
Individual work	**39/80**	**48.75%**
Group work	-	-
Teacher explanation	12/80	15%
Student question	4/80	5%
Teacher question	23/80	28.75%
Student answer	23/80	28.75%
Positive reinforcement	22/80	27.5%
Overhead projector	-	-
Interactive white board	-	-
Student computer work	-	-
Real-life reference	7/80	8.75%
Non-mathematical activity	3/80	3.75%
Teacher going around	**31/80**	**38.75%**
Teacher instruction	16/80	20%
Student discussion	1/80	1.25%

Table 15: Structured observation analysis of group 1

The following graph provides a visual representation of the teaching and learning activities that occurred during the structured observation of the group 1:

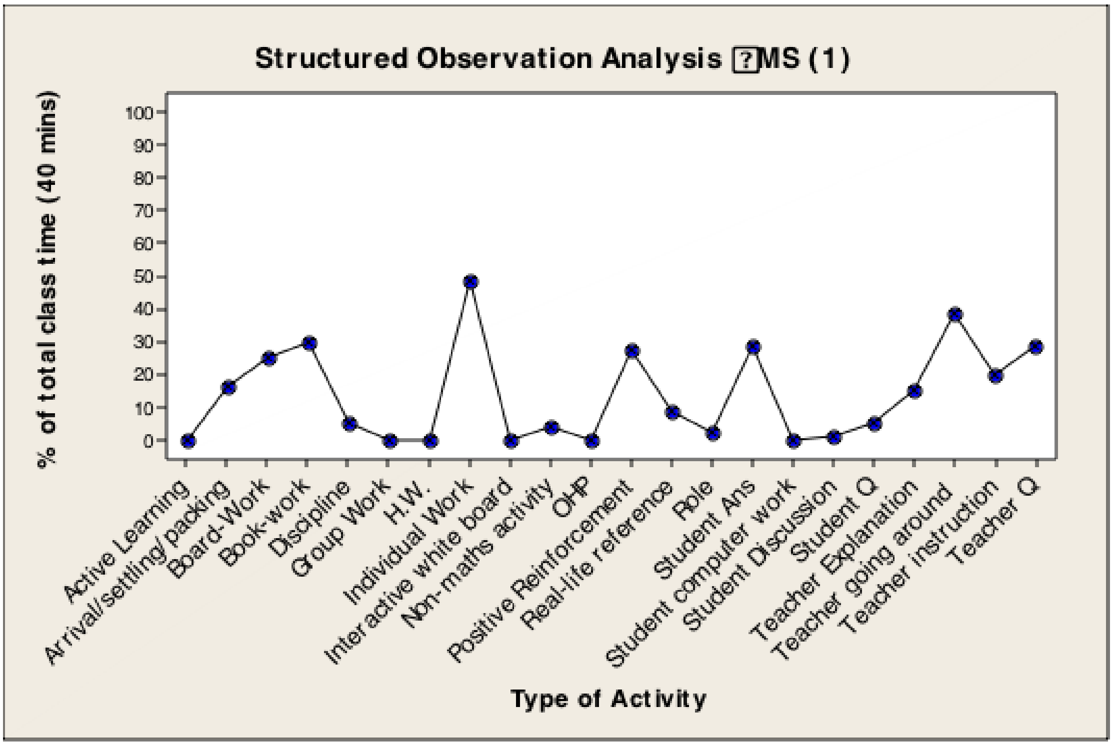


Figure 5: Graphical analysis of the observed group 1 lesson

7.2.3. The Structured Observation -Group 2

The observed class, group 2, involved a higher-level mathematics class group in a small community college. There are only two mathematics groups in second year in school 2 due to the small number of students. As a result the higher level mathematics class catered for a wide ability range. The teacher, Teacher B, involved in the research is an established teacher with more than twenty years teaching experience.

The three teaching and learning activities identified in the schedule that occurred the most frequently in the pilot observation are:

1. *Teacher explanation* (80%): Teacher B spent a significant proportion of the class explaining the mathematical procedures used to solve mathematical problems. All mathematical problems solved in the

observed lesson originated from the textbook and involved closed-ended questions with one correct answer;

2. *Board-work* (71.25%): The teacher in the observed group 2 lesson used the white-board to demonstrate problem-solving techniques. This was supported by the use of explanations and questioning to ensure the students understood what was being demonstrated; and

3. *Individual work* (46.25%): The students spent a considerable amount of time on practicing the problem-solving techniques demonstrated by the teacher. This involved solving problems that were similar on an individual basis in each student's copybook.

The following table shows the proportion of class time spent on each observed activity during the structured observation of group 2:

Type of Activity	**Fraction of total time slots (80, 30 sec slots)**	**% of total class time (40 mins)**
Role-call	1/80	1.25%
Arrival/settling/packing	6/80	7.25
Discipline	-	-
Homework	32/80	40%
Active learning	-	-
Book-work	36/80	45%
Board-work	**57/80**	**71.25%**
Individual work	**37/80**	**46.25%**
Group work	-	-
Teacher explanation	**64/80**	**80%**
Student question	15/80	18.75%
Teacher question	34/80	42.5%
Student answer	22/80	27.5%

Type of Activity	Fraction of total time slots (80, 30 sec slots)	% of total class time (40 mins)
Positive reinforcement	4/80	5%
Overhead projector	-	-
Interactive white board	-	-
Student computer work	-	-
Real-life reference	-	-
Non-maths activity	-	-
Teacher going around	6/80	7.5%
Teacher instruction	12/80	15%
Student discussion	-	-

Table 16: Structured observation analysis of group 2

The following graph provides a visual representation of the teaching and learning activities that occurred during the structured observation of group 2:

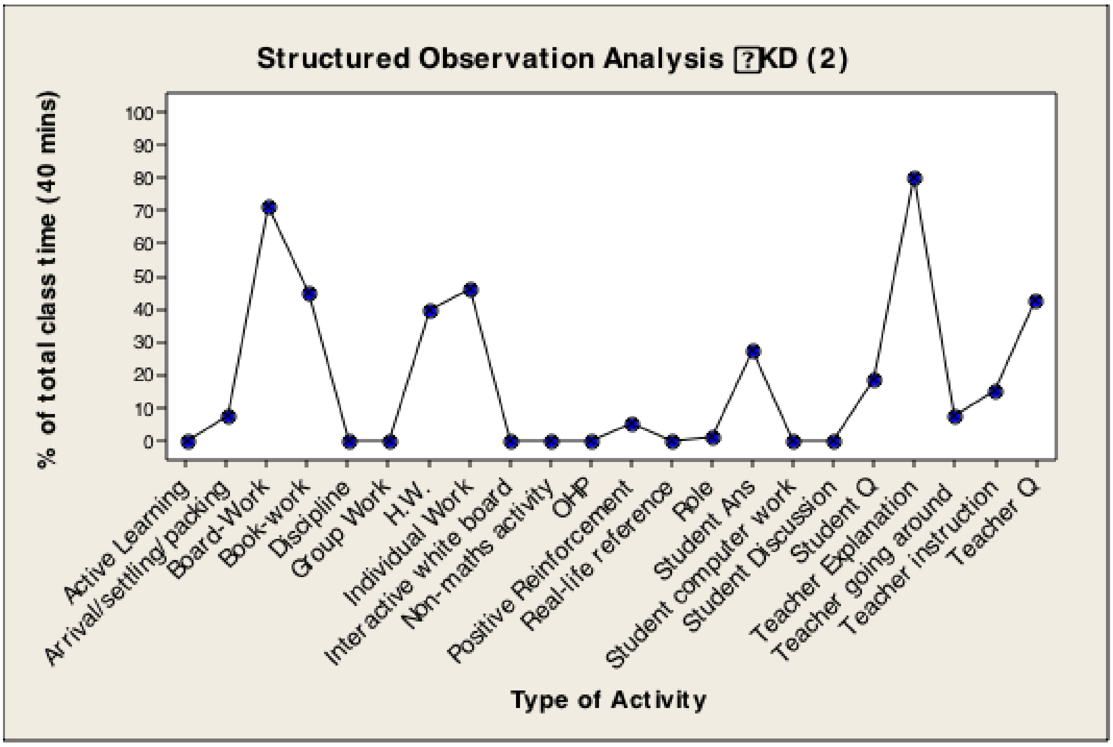


Figure 6: Graphical analysis of the observed group 2 lesson.

7.2.4. The Structured Observation -Group 3

The observed group 3 lesson involves a top stream, higher level mathematics class. The teacher involved, Teacher C, explained that the students involved are a particularly able group with the majority having a great enthusiasm and capacity for mathematics. Teacher C was an enthusiastic and nurturing teacher, and was exceptionally encouraging of the students involved in the group 3 observation. 'Positive re-enforcement' was a notable aspect of Teacher C's teaching style (32.5%), with very affectionate language used towards all students. Teacher C is an established mathematics teacher, with more than twenty years teaching experience and is head of the mathematics department in her school.

The three teaching and learning activities identified in the schedule that occurred the most frequently in the pilot observation are:

1. *Board-work* (51.25%): The teacher in the observed group 3 class spent a significant proportion of the class demonstrating procedural techniques for solving mathematical problems on the white-board. All the questions solved originated from the prescribed textbook and were closed-ended questions with just one correct answer;

2. *Individual work* (45%): the students in the observed group 3 class spent a considerable length of time on solving similar mathematical problems in their copybooks. The teacher demonstrated the technique on the board and the students then practiced this technique repeatedly by solving mathematical questions of a similar nature. The teacher regularly went from student to student, keeping a close eye on their work and providing advice where necessary; and

3. *Book-work* (38.75%) and *teacher explanation* (38.75%): Textbooks were used for much of the class and all the mathematical questions, as explained earlier, derived from the textbooks. The teacher frequently explained what she had written on the board and the students were asked questions regarding same.

The following table displays the different activities observed during the structured observation of group 3:

Type of Activity	**Fraction of total time slots (80, 30 sec slots)**	**% of total class time (40 mins)**
Role-call	2/80	2.5%
Arrival/settling/packing	6/80	7.5%
Discipline	-	-
Homework	6/80	7.5%
Active learning	-	-
Book-work	**31/80**	**38.75%**
Board-work	**41/80**	**51.25%**
Individual work	**36/80**	**45%**
Group work	-	-
Teacher explanation	**31/80**	**38.75%**
Student question	4/80	5%
Teacher question	24/80	30%
Student answer	23/80	28.75%
Positive reinforcement	26/80	32.5%
Overhead projector	-	-
Interactive white board	-	-
Student computer work	-	-
Real-life reference	-	-
Non-maths activity	2/80	2.5%
Teacher going around	25/80	31.25%
Teacher instruction	13/80	16.25%
Student discussion	-	-

Table 17: Structured observation analysis of group 3

The following graph provides a visual representation of the teaching and learning activities that occurred during the structured observation of group 3:

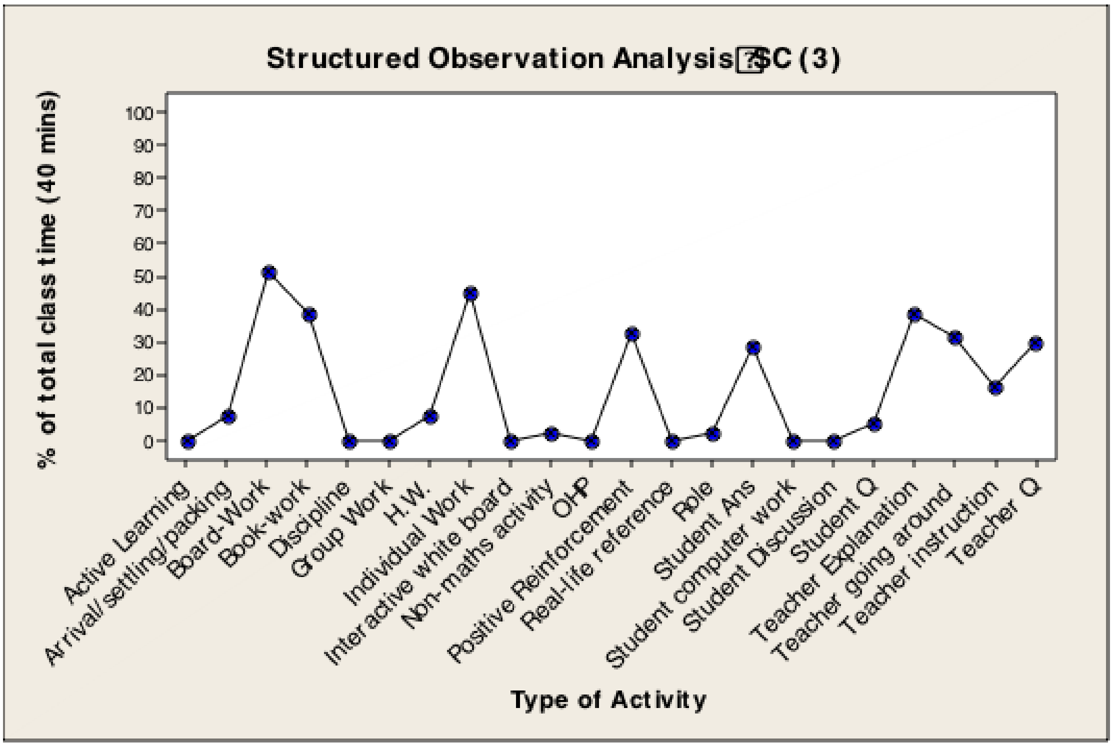


Figure 7: Graphical analysis of the observed group 3 lesson

7.2.5. The Structured Observation -Group 4

The group 4 teacher, Teacher D, involved in the research is an experienced teacher with more than twenty years teaching experience. The class observed are an ordinary level mathematics class group. Teacher D's teaching style is gentle and nurturing, disciplining is done in a most gentle and respectful manner. The students are treated as being equal to the teacher. As a result the observed class had the highest noise level but allowed for freedom of expression.

The three teaching and learning activities identified in the schedule that occurred the most frequently in the pilot observation are:

1. *Teacher explanation* (56.25%): The teacher, Teacher D, in the observed group 4 class spent a significant proportion of the lesson on explaining mathematical procedures and techniques;

2. *Board-work* (45%): Teacher explanations were supported by the demonstration of problem-solving techniques on the white board. All questions solved originated from the prescribed textbook and were closed-ended questions with one correct answer; and

3. *Home-work* (41.25%): Solutions from the previous night's homework were written on the board and the teacher explained the procedural technique used for solving each question.

The following table illustrates the time spent on each activity in the observed class for group 4:

Type of Activity	**Fraction of total time slots (80, 30 sec slots)**	**% of total class time (40 mins)**
Role-call	-	-
Arrival/settling/packing	14/80	17.5%
Discipline	8/80	10%
Homework	**33/80**	**41.25%**
Active learning	-	-
Book-work	22/80	27.5%
Board-work	**36/80**	**45%**
Individual work	22/80	27.5%
Group work	5/80	6.25%
Teacher explanation	**45/80**	**56.25%**
Student question	12/80	15%
Teacher question	17/80	21.25%
Student answer	15/80	18.75%
Positive reinforcement	7/80	8.75%
Overhead projector	-	-
Interactive white board	-	-

Type of Activity	Fraction of total time slots (80, 30 sec slots)	% of total class time (40 mins)
Student computer work	-	-
Real-life reference	-	-
Non-maths activity	1/80	1.25%
Teacher going around	20/80	25%
Teacher instruction	9/80	11.25%
Student discussion	3/80	3.75%

Table 18: Structured observation analysis of group 4

The following graph provides a visual representation of the teaching and learning activities that occurred during the structured observation of group 4:

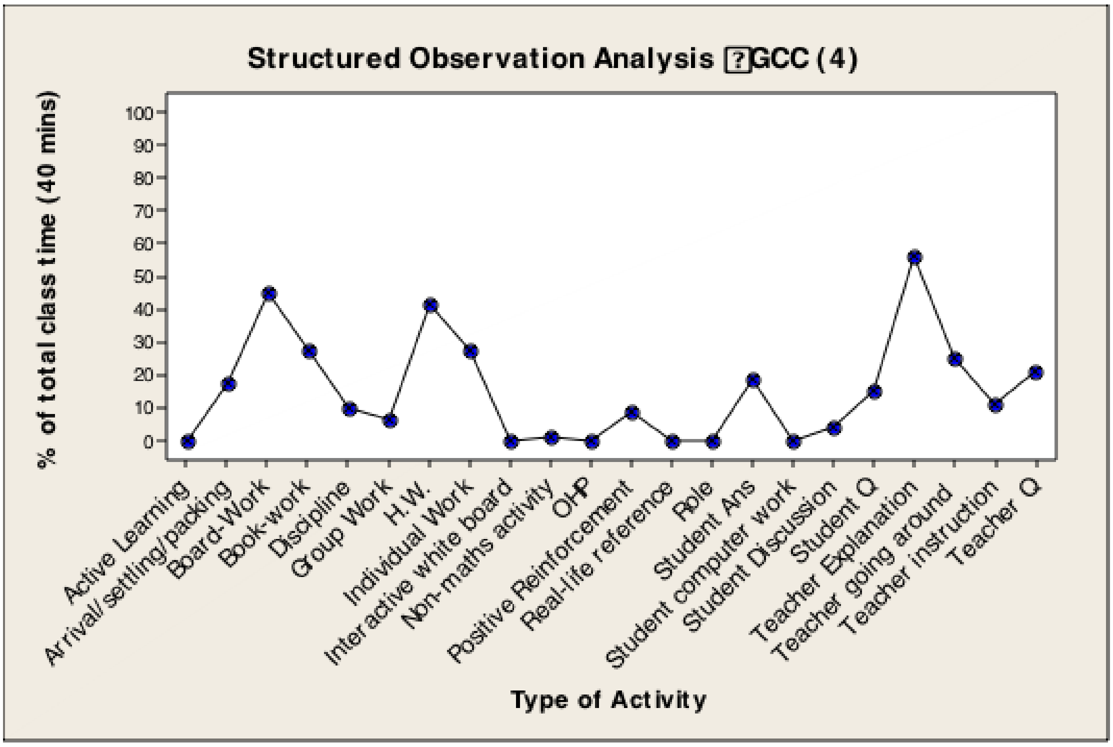


Figure 8: Graphical analysis of the observed group 4 lesson

7.2.6. The Structured Observation - Group 5

The teacher in question, Teacher E, described the observed class as reasonably typical with one or two exceptions. The first of these is the fact that 8 students (out of a possible 31 students) were absent due to involvement in school extra-curricular activities. Teacher E also referred to the fact that she was experimenting with some slight variations to her teaching style as she was preparing for the introduction of the new mathematics syllabus, Project Maths, despite the fact that the new curriculum will not affect the current second year students.

Teacher E explained that there were rarely discipline issues with this particular class and believed this was due to the fact that they were a 'Top A' class. The teacher referred to other classes that were less mathematically able (such as the 'bottom honours class') and where students were not as academically inclined as more likely to encounter discipline issues. The researcher noted that it was interesting that an experienced teacher (with over twenty years of teaching experience) was experimenting with her teaching style in anticipation of teaching changes that she will have to adapt to in the future.

The three teaching and learning activities identified in the schedule that occurred the most frequently in the pilot observation are:

1. *Teacher explanation* (57.5%): The teacher observed in the group 5 observation explained the mathematical techniques involved and the procedures for solving these mathematical problems. All the mathematical questions solved in the observed lesson were closed-ended questions, and had one correct answer only;

2. *Individual work* (55%): The students in the group 5 lesson did a significant amount of individual work. This individual work involved the students solving given mathematical questions (all of which originated from the text-book but some of which were written up on the white board). The students worked on a series of similar mathematical problems, practicing the correct procedural technique in order to arrive at the correct solution; and

3. *Board work* (52.5%): The teacher in the observed group 5 lesson used the white-board to demonstrate procedural techniques for solving the closed-ended mathematics questions from the textbook. The white-

board was accompanied by teacher explanations as shown above. Again, this shows the use of the common ‘chalk-and-talk’ technique.

The following table shows the amount of class time spent on each activity during the observed lesson for group 5:

Type of Activity	**Fraction of total time slots (80, 30 sec slots)**	**% of total class time (40 mins)**
Role-call	-	-
Arrival/settling/packing	8/80	10%
Discipline	-	-
Homework	14/80	17.5%
Active learning	-	-
Book-work	31/80	38.75%
Board-work	**42/80**	**52.5%**
Individual work	**44/80**	**55%**
Group work	-	-
Teacher explanation	**46/80**	**57.5%**
Student question	6/80	7.5%
Teacher question	-	-
Student answer	-	-
Positive reinforcement	17/80	21.25%
Overhead projector	-	-
Interactive white board	-	-
Student computer work	-	-
Real-life reference	-	-
Non-maths activity	5/80	6.25%
Teacher going around	19/80	23.75%

Type of Activity	Fraction of total time slots (80, 30 sec slots)	% of total class time (40 mins)
Teacher instruction	16/80	20%
Student discussion	-	-

Table 19: Structured observation analysis of group 5

The following graph provides a visual representation of the teaching and learning activities that occurred during the structured observation of group 5:

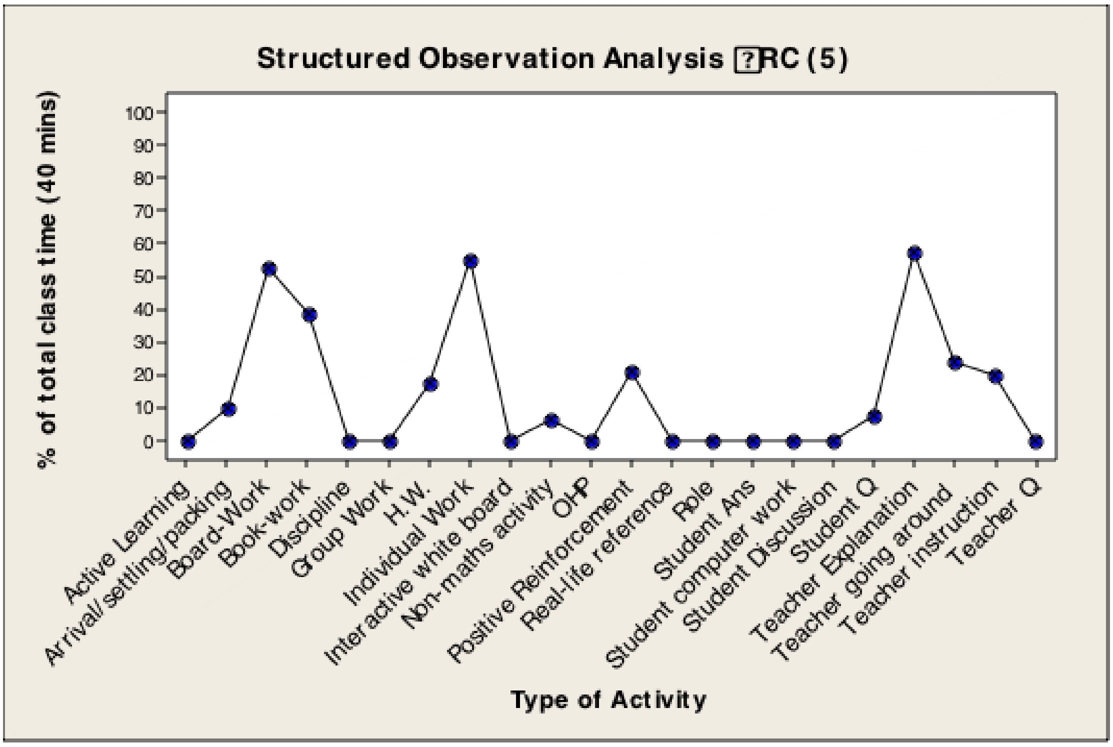


Figure 9: Graphical Analysis of the observed group 5 lesson

7.2.7. The Structured Observation - Group 6

The researcher wishes to note that the students described by Teacher F as having a particularly poor attendance history, and associated behavioural issues, were absent on the day that the class was observed. The relatively slow start to the class appeared to be due to organisational issues on the students' behalf. Once the class commenced, the students demonstrated mathematical interest and enthusiasm. There was a significant amount of mathematical discussion between the students, often initiated by the students themselves.

The three teaching and learning activities identified in the schedule that occurred the most frequently in the pilot observation are:

1. *Board work* (48.75%): The teacher observed, Teacher F, spent a significant length of the observed class period on board work. This involved the teacher showing students how to solve mathematical problems on the white board. All the questions solved were closed-ended questions and only had one correct answer;

2. *Teacher explanation* (34.29%): The teacher provided concise explanations of the mathematics demonstrated on the white board as she worked. This, again, is an example of the common 'chalk-and-talk' technique used in teaching mathematics in Ireland; and

3. *Teacher question* (34.29%): The teacher frequently asked the students questions in order to verify that they understood what was being explained.

The following table illustrates the amount of time spent on each activity during the observed lesson for group 6:

Type of Activity	**Fraction of total time slots (80, 30 sec slots)**	**% of total class time (40 mins)**
Role-call	2/70	2.86%
Arrival/settling/packing	15/70	21.43%
Discipline	2/70	2.86%
Homework	2/70	2.86%
Active learning	-	-
Book-work	1/70	1.42%
Board-work	**34/70**	**48.57%**
Individual work	15/70	21.43%
Group work	-	-

Type of Activity	Fraction of total time slots (80, 30 sec slots)	% of total class time (40 mins)
Teacher explanation	**24/70**	**34.29%**
Student question	8/70	11.43%
Teacher question	**24/70**	**34.29%**
Student answer	23/70	32.86%
Positive reinforcement	13/70	18.57%
Overhead projector	-	-
Interactive white board	-	-
Student computer work	-	-
Real-life reference	-	-
Non-maths activity	11/70	15.71%
Teacher going around	10/70	14.29%
Teacher instruction	6/70	8.57%
Student discussion	11/70	15.71%

Table 20: Structured observation analysis for group 6.

The following graph provides a visual representation of the teaching and learning activities that occurred during the structured observation of group 6:

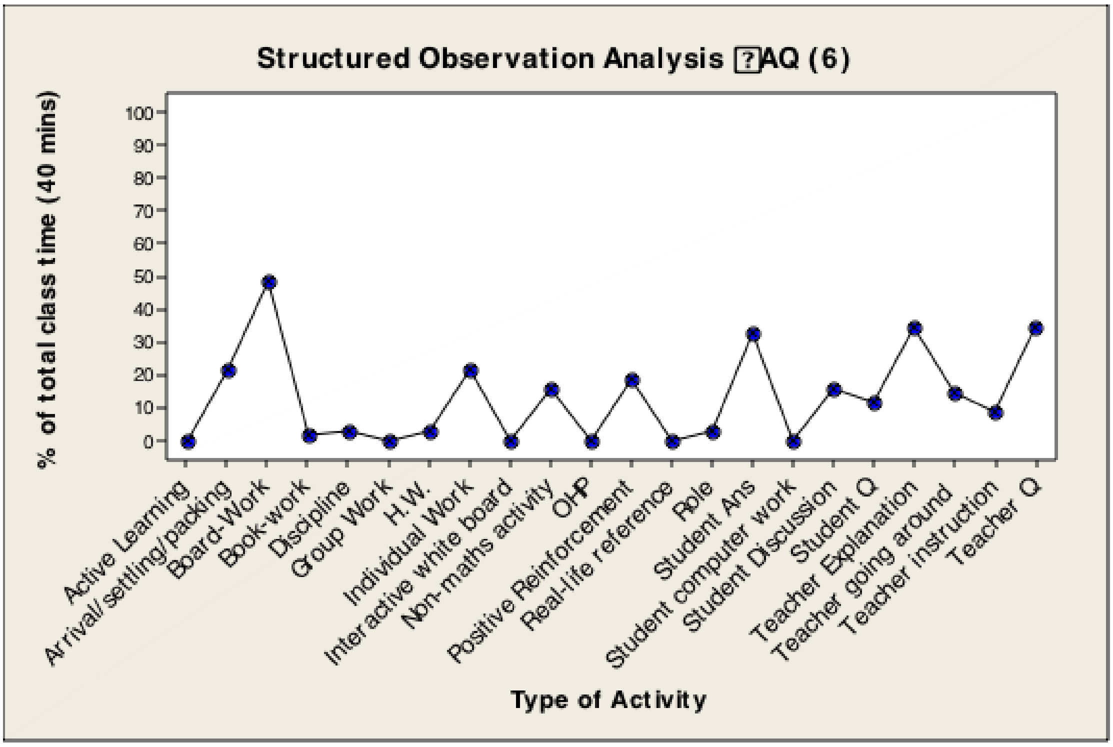


Figure 10: Graphical analysis of the observed group 6 lesson

7.2.8. Analysis of the seven structured observations

The following table considers the activities noted in the structured observations of the seven groups involved:

Type of Activity	Pilot	Group 1	Group 2	Group 3	Group 4	Group 5	Group 6	Mean per class
Role-call	3/72	2/80	1/80	2/80	-	-	2/70	1.89%
Arrival/settling/packing	6/72	13/80	6/80	6/80	14/80	8/80	15/70	12.65%
Discipline	-	4/80	-	-	8/80	-	2/70	2.55%
H.W.	56/72	-	32/80	6/80	33/80	14/80	2/70	26.7%
Active Learning	-	-	-	-	-	-	-	0%
Book-work	15/72	24/80	36/80	31/80	22/80	31/80	1/70	28.89%
Board-Work	**53/72**	**20/80**	**57/80**	**41/80**	**36/80**	**42/80**	**34/70**	**52.46%**
Individual Work	**14/72**	**39/80**	**37/80**	**36/80**	**22/80**	**44/80**	**15/70**	**37.63%**
Group Work	-	-	-	-	5/80	-	-	0.89%

Type of Activity	Pilot	Group 1	Group 2	Group 3	Group 4	Group 5	Group 6	Mean per class
Teacher Explanation	**45/72**	**12/80**	**64/80**	**31/80**	**45/80**	**46/80**	**24/70**	**46.61%**
Student Q	11/72	4/80	15/80	4/80	12/80	6/80	8/70	11.13%
Teacher Q	26/72	23/80	34/80	24/80	17/80	-	24/70	27.56%
Student Ans	11/72	4/80	15/80	4/80	12/80	6/80	8/70	11.13%
Positive Reinforcement	-	22/80	4/80	26/80	7/80	17/80	13/70	16.23%
OHP	-	-	-	-	-	-	-	0%
Interactive white board	-	-	-	-	-	-	-	0%
Student computer work	-	-	-	-	-	-	-	0%
Real-life reference	-	7/80	-	-	-	-	-	1.25%
Non-maths activity	1/72	3/80	-	2/80	1/80	5/80	11/70	4.41%
Teacher going around	9/72	31/80	6/80	25/80	20/80	19/80	10/70	21.56%
Teacher instruction	-	16/80	12/80	13/80	9/80	16/80	6/70	13.01%
Student discussion	7/72	1/80	-	-	3/80	-	11/70	4.35%

Table 21: Analysis of the seven groups involved in the structured observation

In the above analysis of the seven observed class groups (including the pilot structured observation) the most frequently occurring teaching and learning activities are highlighted. These are:

1. Board-work (52.46%);
2. Teacher explanation (46.61%); and
3. Individual work (37.63%).

The term 'board-work' involves the teacher writing on the white-board to illustrate solutions to mathematical questions. The fact that the two most frequently occurring teaching and learning activities, across observation of the seven class groups, are based primarily on teacher activity, suggests that the teacher is the focus in the Irish mathematics classroom. It is interesting that the third most common classroom activity is 'individual work' which involves the students working on their own. Activities that did not occur at all in the observed mathematics lessons are:

- ‘Active learning’;
- The use of an ‘overhead projector’;
- The use of an ‘interactive white board’; and
- Computer work of any kind, particularly ‘student computer work’.

7.2.9. Reflection on the Findings from the Structured Observations:

The structured observations of the seven mathematics lessons suggest that Irish mathematics teaching may be heavily influenced by the behaviourist philosophy. While the author is aware that seven observations is not a basis on which to determine the nature of teaching in Ireland it is interesting that the lessons observed are similar in terms of the teaching and learning activities addressed and those which do not occur. Individual work is valued over group work, the textbook is a key feature of the mathematics lesson and the teacher is the centre of all teaching and learning activity. Despite observing both male and female teachers, with varying levels of teaching experience, the overriding impression from the observed lessons is that mathematics teaching in Ireland varies little from classroom to classroom and from school to school. Again, this assumes that the lessons observed are typical of what is occurring in mathematics lessons throughout Ireland.

Despite the fact that the teachers observed (including the student teacher) had undergone training for the incoming ‘Project Maths’ curriculum, the author observed no direct influence from this training on the teaching and learning activities in the mathematics lessons. It will be interesting to reconsider mathematical teaching styles in Ireland in the coming decade to see if ‘Project

Maths' alters teaching habits in Irish classrooms. The author is of the opinion that teaching and learning methods in Ireland must undergo a significant overhaul if Irish mathematics education is to be altered and improved.

7.3 The Semi-Structured Interviews

The following section considers the findings from the semi-structured interviews.

7.3.1. Semi-Structured Interview: Teacher A

The school involved in the first interview, school 1, provided two class groups (group 1 and group 6) for the Research project. Both of the class groups were small, ordinary level mathematics classes. In both instances they represent the bottom stream in ordinary level (on two opposite sides of the time-table). The school is a large school with 1150-1250 students and eighty full-time teachers. It is a secondary school with an open-intake policy and a strong academic reputation. It is an established school, founded in the late 1800's, with a strong sporting tradition. The school was initially a male only establishment but in recent years girls were welcomed and now account for a third of the student population. As a result of the large number of students in the school, and the fact that there is a compulsory obligation for all students to study mathematics in each year group, there are 24 teachers in the school who are qualified to teach mathematics. In second year there are ten mathematics class groups. The year group is divided in two to facilitate timetabling of staff, with five mathematics classes and mathematics teachers operating at any one time.

The teacher in question, teacher A, has been teaching full-time for the last six years. Teacher A has a higher level degree in science with a specialisation in chemistry and a master's degree in Information Technology. The second year class are the only mathematics group that Teacher A is teaching during the academic year in question, but in the past she has had a significant amount of mathematics in her timetable and is a respected higher level, Junior Certificate and ordinary level, Leaving Certificate mathematics teacher. The class observed and assessed as part of the data collection are an ordinary level mathematics class with a significant number of students that require one-on-one assistance. For this reason there are only nine students in the mathematics class, and in addition to the teacher there are three 'special needs assistants' (SNAs) for three individual students with various recognised learning and behavioural difficulties. The students are currently studying the ordinary level Junior Certificate mathematics syllabus. Teacher A would hope that the majority of the students would sit the ordinary level mathematics paper in the Junior Certificate at the end of their third year. It is very probable, however, that one or two of the students may end up sitting the foundation level Junior Certificate paper in mathematics. This will not be decided by the student, under advice from Teacher A, until after the trial examinations which will be held approximately three months before the actual Junior Certificate examinations commence. Teacher A may advise the student to sit the foundation level paper at this stage if the student received a very low score, significantly below the pass mark of 40%, in their trial examination in ordinary level mathematics.

Teacher A was enthusiastic, when interviewed, about teaching mathematics and advancing her own teaching and learning skills. Teacher A was very comfortable with the concept of being involved in a structured observation and was a willing and enthusiastic interviewee. Topics discussed with Teacher A included the difficulty regarding punctuality and attendance with some students in the class group and the disruptive nature of this behaviour.

Teacher A discussed some of the class activities that she utilises when teaching mathematics in an effort to encourage understanding and student participation – these include many examples of authentic problems involving real-life activities. However, Teacher A was keen to point out that the mathematical content of these activities is at a relatively basic level, and many of the students struggle with mathematics despite real-life references. An example given by Teacher A was the use of real-life scenarios involving arithmetic and the problems students have in decimalisation. When asked to calculate the cost of three cups of coffee at €2.10 each, one particular student was adamant that the total cost of the coffee is €63. Despite discussion with the teacher regarding the unlikelihood that three cups of coffee could possibly cost that much the student in question was reluctant to accept otherwise. Despite these difficulties Teacher A was optimistic about the value of mathematical skills for all students and enthusiastic about the role she could play in this development.

7.3.2. Semi-Structured Interview: Teacher B

School 2 is a community college situated in a village and has a primarily rural catchment area, with a significant minority of students travelling 20 miles from the nearest large town. School 2 was built less than twenty years ago and benefits from excellent facilities and a dynamic, enthusiastic principal. The school replaced two established single-sex schools in one purpose built, co-educational facility. The school has an upper limit for student enrolment of 325 and this results in an intimate and familiar atmosphere. The student-teacher ratio averages out at approximately 12-1. The school is exceptionally well maintained and has a very strict work-ethos and code of conduct for students.

The teacher, Teacher B, of the class involved in the research is an experienced teacher of over thirty years experience. Teacher B describes her methods of teaching mathematics as ‘traditional’ and ‘old-fashioned’. Teacher B explains that she is very aware that her experience in the classroom has led to the development of a very particular teaching style which relies heavily on ‘chalk and talk’. Despite this self-description of a ‘traditional’ teacher, the author found the teacher in question, Teacher B, to be an extremely open-minded, enthusiastic individual who was at pains to advance with the incoming ‘Project Maths’ curriculum. Teacher B demonstrated to the author some of the information technology resources she uses in the class including Geo-gebra for co-ordinate geometry explanations and online resources for algebra. The author was heartened to see an experienced teacher with such passion and openness for change.

As the school in question is small there are only two mathematics class groups in second year: one higher level class and an ordinary level class. The class involved in the research is the higher level class and as it is the only class in second year studying mathematics at this level there was a greater ability range than may be found in a higher level class in a larger school. Teacher B described the class as consisting of students that are extremely able down to students who realistically would eventually end up sitting the ordinary level, Junior Certificate examination. Discipline, Teacher B explained, was not an issue of any note with this particular class and this reflected the general ethos of the school which was very strictly run. Teacher B also explained that the small size of the school led to increased familiarity with the student body which reduced the occurrence of unnoticed discipline issues.

7.3.3. Semi-Structured Interview: Teacher C

Group 3 is a community school situated in a medium sized, industrial town in the west of Ireland. The school was built as part of a public-private partnership and is a well maintained facility. The teacher in question, Teacher C, is an experienced mathematics teacher and head of the mathematics department in the school.

The class involved in the research, and taught by Teacher C, are a higher-level class. There are 150 second year students, and approximately 700 students in the school in total. There are fifty-four teachers in the school, nine of whom are qualified to teach mathematics. Students are taught a common-level mathematics program in first year, which covers roughly one third of the ordinary level mathematics course. Teacher C explained that students are examined with a common mathematics examination at the end of first year and their result from this examination, in conjunction with teacher advice based on in-class tests throughout the year, determines the level of mathematics to be followed in second year: higher, ordinary or foundation.

Teacher C explained the structured of mathematics class groups within the school in general and for second year in particular. In second year there are six mathematics class groups plus a reduced class group for students with identified learning difficulties which significantly affect their mathematical performance. Teacher C described the mathematics teachers in the school as enthusiastic and passionate. Three of the six mainstream mathematics class groups are higher level mathematics class groups, and the remaining three are ordinary level.

Approximately seven or eight students sit the foundation level, Junior Certificate mathematics paper in the school each year. Teacher C explained that these foundation level students would be examination candidates that may have a particular learning difficult or social, personal and/or health issues that may have impacted on regular school attendance. The students who opt for the foundation level paper in the Junior Certificate usually follow the ordinary level course until some stage late in third year. For Leaving Certificate students there is the option of following the 'Leaving Certificate Applied' curriculum which is offered in a minority of schools, School 3 being one of them. Teacher C described 'Leaving Certificate Applied' (LCA) as an excellent option for students who wish to follow a less academic route and prepare for a vocational working life. The LCA option was first implemented in the school more than a decade ago and Teacher C described it as very successful. The LCA programme is a very valid option for students who may struggle significantly at academic subjects, to include mathematics, at Junior Certificate level.

The class taught by Teacher C, and involved in this research project, is the higher level mathematics class group that contains the most mathematically able students in the year group. There are thirty students in the class and Teacher C speaks of them all in glowing terms. Teacher C is a dynamic, vibrant woman with a significant amount of teaching experience and is the head of the mathematics department in the school. Teacher C is passionate about mathematics and sharing her mathematical knowledge with the students she teaches. All second year mathematics class groups have five class periods per week, and Teacher C explains that her students utilise the class time provided for learning at a very fast pace. Teacher C explains that at this mathematical level she rarely has to deal with discipline issues as the students are so eager to learn. This raises an interesting question regarding teacher, school and societal expectation regarding mathematical ability and discipline. In practical terms, very able students possibly are more eager to learn as it is more likely to be a positive experience; therefore there is less need to numb the pain of the mathematics class with disruptive behaviour.

7.3.4. Semi-Structured Interview: Teacher D

School 4 is a community college in a medium sized town situated in a rural location and drawing from a rural catchment area. The school has a student population of 600 students, with approximately 130 students in second year. The class observed are an ordinary level mathematics class with a reasonably wide ability range.

The teacher of the class group, Teacher D, is a male teacher with over twenty years teaching experience. Teacher D is an enthusiastic teacher who is very interested in the concept of student-centred learning, deferring to students regarding decisions made, and gentle discipline. Teacher D explained how he treats students as equals and bases his teaching on a practice of guided learning rather than directive learning. Teacher D explained that he did not have any major discipline issues in the class but does allow, and indeed encourage, discussion and interaction in his mathematics class. This, he explained, sometimes led to issues with noise control and off-subject discussion but Teacher D believes that this is worth the benefits to be gained through active learning.

Teacher D has a very forward-thinking attitude to the teaching and learning of mathematics, but is curtailed to the same extent as others by the didactic element required for successful examination preparation. Teacher D uses a text-book on a regular basis and, like all Irish teachers involved in the research, is a regular proponent of the 'chalk and talk' movement. Information Technology resources are not utilised on a regular basis with this mathematics class.

Teacher D explained that he has some reservations regarding the implementation of the new 'Project Maths' curriculum. These reservations are primarily due to a sense of being ill-prepared to teach the new curriculum without more substantial training. However, Teacher D described curriculum change as an essential aspect of improving mathematical performance in Ireland. Teacher D was particularly interested in the real life applications of mathematics and described how he runs the mathematics and science club in the school in an effort to promote mathematical (and scientific) applications.

Teacher D was the only Irish mathematics teacher interviewed who had spent time teaching mathematics in another country. Teacher D described his time spent working as both a mathematics teacher and head of the mathematics department at a school in the United States of America. Teacher D expressed his opinion that Irish mathematics education could benefit from an awareness of advancement in mathematics education in other countries. While complementing Irish mathematics education as regards the high content level and the mathematical knowledge imparted, Teacher D explained that he spends a far more significant proportion of his class time in Ireland on didactic teaching in comparison to his teaching experience in the U.S.A.

7.3.5. Semi-Structured Interview – Teacher E

School 5 is a religious controlled secondary school in a large town. The school was founded in the 1800's and has a strong academic reputation. The school was previously run by a religious order but no longer has any of the religious order teaching. The school does not have an open admissions policy, with all students required to sit an aptitude test prior to acceptance. The school is medium sized with a student population of 700-800 students and 47 teaching staff. In total there are ten mathematics teachers currently teaching mathematics in the school, and there are a further two members of teaching staff that are qualified to teach mathematics but are not currently doing so.

Teacher E is a mathematics and physical education teacher with over twenty-five years experience. Teacher E explained the structure of mathematics class groups in her school and described it as well structured and effective. There are four, second year mathematics classes and five mathematics class periods per week in second year. Three of the second year mathematics class groups are following the higher level course, and one the ordinary level course. Teacher E described the general standard of mathematics as being reasonably high but believed that mathematics knowledge in general had deteriorated in the time she had been teaching.

Teacher E explained that students are taught in common mathematics classes throughout first year. During this time they are regularly given common assessments and all grades are noted. At the end of first year the mathematics teacher makes a recommendation as to whether each particular student should proceed with the higher or ordinary level course. The recommendation is based on what the teacher has observed in class throughout the year and on the series of common tests that the student has sat. However all students are allowed follow the higher level course if they wish to do so, no student is forced to follow the ordinary level course against their wishes.

The class group involved in the research are the most mathematically able of the three higher level groups. Teacher E described this class group as being a 'joy to teach' but showed less enthusiasm for teaching mathematics to less able students. Teacher E explained that there were fewer discipline issues when teaching a more mathematically able class group.

Teacher E showed a willingness to incorporate new mathematics teaching techniques and was enthusiastic about the incoming 'Project Maths' curriculum. Teacher E explained how she is adapting her teaching at all levels and for all year groups in preparation for the implementation of the new curriculum. She explained that this was causing some difficulty as not all of the new techniques she tried were equally effective but despite this Teacher E remained determined that 'Project Maths' would be a success for her class groups.

7.3.6. Semi-Structured Interview: Teacher F

Teacher F is also based at School 1 (as is Teacher A). As noted above School 1 is a large co-educational secondary school situated in a large town. The teacher in question, Teacher F, is an established teacher with close to thirty years teaching experience. Teacher F teaches science and mathematics to all year groups in the school.

The class observed and assessed, Group 6, are the lowest stream on one side of the time-table (as discussed earlier, the school is so large each year group is split into two sections for ease of timetabling). The class consists of ten students and the teacher explained that attendance is generally poor with a minority of students rarely attending class. Teacher F also described student behavioural issues and frequent disrupted mathematics lessons. There were three students in the class who had a very poor attendance history and caused disruption on the rare occasions that they are present.

Despite this, Teacher F described her fondness for the mathematics class and described the immense pleasure she gained in teaching a class group where she felt all progress was significant and valuable. Teacher F explained that with this particular class group she felt that she was teaching them valuable life skills rather than solely imparting mathematical knowledge as she felt is sometimes the case with a higher level class group.

Teacher F described her teaching style as very 'traditional' and expressed a level of trepidation regarding the incoming 'Project Maths' curriculum. Her reservation stemmed from a sense that she was not fully prepared to teach the new curriculum, despite attending the mandatory in-service workshops. Teacher F also expressed some reservation at the idea of teaching mathematics without a core textbook to rely on.

7.3.7. Semi-Structured Interview: Teacher G (Massachusetts)

The school from Massachusetts is a public 'middle school' with approximately 600 students. 'Middle school' covers 6^{th}, 7^{th} and 8^{th} grade. The students assessed for the purpose of this research are 8^{th} graders at the time of testing. There are approximately 200 8^{th} grade students in the school, and the mean age is 13.5 at the time of testing (January). The school is situated in a town on the outskirts of Boston with a predominately middle-class population. As discussed in Chapter 6, Massachusetts is the top performing U.S. state in terms of the international assessments carried out by TIMSS. In TIMSS 2007 eighth grade students from Massachusetts scored sixth in the world in mathematics.

Teacher G, is the head of mathematics within the school. He is an experience mathematics teacher with more than twenty years experience and holds a master's degree in mathematics education. Teacher G was the only mathematics teacher interviewed in Massachusetts. The mathematics teachers in the school elected that Teacher G speak on their behalf due to the fact that the researcher could not physically visit the school and all interviews would occur by telephone and email. After an initial telephone interview it was decided that email correspondence was best. Six email interviews occurred in total with Teacher G providing information about the school, the mathematics classes taught in eighth grade and the teaching and learning methods used.

The number of students from Massachusetts that participated in both tests is 71. All students that study ‘Algebra 1’ were asked to participate (50% of eight grade, therefore approximately 100 students). Five mathematics teachers teach the class groups that participated in the research. All teachers hold a qualification in teaching, with various levels of mathematical qualifications. The following table provides an overview of the teachers involved in the study:

Teacher	**Teaching Experience**	**Gender**	**Qualifications**
Teacher 1 (Teacher G)	22 years	Male	B.A. in mathematics and English M.A. in mathematics education
Teacher 2	4 years	Male	B.Ed. science education
Teacher 3	7 years	Female	B.A. History M.Ed. Education
Teacher 4	31 years	Female	B.Sc. chemistry
Teacher 5	2 years	Female	B.Sc. maths, physics

Table 22: Overview of Massachusetts’ Teachers

The most mathematically able eighth grade students did not participate in the research project, as these students do not study ‘Algebra 1’ but ‘Advanced Topics’. Eight percent of eighth-graders take the ‘Advanced Topics’ course. The least mathematically able eighth graders did not participate in the study either. These students are also not part of the cohort that study ‘Algebra 1’, instead these students study ‘Algebra A’ which covers the first one-third of ‘Algebra 1’. Forty two percent of eighth-graders take the ‘Algebra A’ course.

The average class size in the participating school is 18 students. There are ten mathematics class groups in eighth grade to provide for the 200 students. There are five mathematics class periods per week for eighth grade students. The length of each mathematics class is 49 minutes. Topics covered in eighth grade ‘Algebra 1’ include the following:

- Equations;
- Algebraic properties;
- Inequalities;
- Systems of equations and inequalities;
- Exponents;
- Quadratics;
- Radicals;
- Probability;
- Proportions; and
- Ratios.

Teacher G explained that each Algebra 1 class participates in a project based on an open-ended question that takes approximately six weeks work. Each individual class teacher decides on the question and the topic it relates to. Group-work is encouraged within the class structure and the classrooms are physically designed to facilitate this with 4-6 smaller white boards located around the classroom for student use. Textbooks are used by the teachers, as are online computer programs that work on the concept of improving student fluency.

Teacher G identified the following activities as occurring frequently in the five class groups involved in the research:

- Group work;
- Individual work;

- Continuous assessment;
- Class tests;
- Discovery learning;
- Project work;
- Problem-solving involving open-ended questions;
- Teaching and learning involving the use of a text-book;
- Teacher explanation while standing at a white-board;
- Computer-work and the use of other information technology resources; and
- Reinforcement through questions and answers.

There appears to be a wider range of teaching and learning activities used in the mathematics lessons in the Massachusetts' school. Without further investigation it is difficult to know how frequently the activities above occur in mathematics lessons or how representative they are of teaching in Massachusetts. As each school district in Massachusetts is responsible for their own curriculum, it is possible that more variation occurs between schools than may occur in Ireland.

7.3.8. Reflection on the Findings from the Semi-Structured Interviews

The qualitative data offered by the semi-structured interviews provides a valuable insight into teaching and teacher attitudes towards mathematics education in Ireland. All the Irish teachers involved in this aspect of the research were enthusiastic and forthcoming. The author notes that the fact that all of the interviewed teachers volunteered to participate in the study may contribute towards this enthusiasm and is not necessarily a reflection of Irish mathematics teaching in general. The following are some of the notable findings from the interviewing process with the Irish teachers:

- Enthusiasm towards mathematics education in general and teaching in particular;
- A respect for the young people they teach;
- A willingness to participate in the research project;
- A sense of confidence in their teaching ability;
- Regular descriptions of their personal teaching style as being 'traditional';
- A sense of trepidation towards the implementation of 'Project Maths' and descriptions of feeling unprepared to teach the new curriculum.

The interview data provided by the Massachusetts school through the initial telephone interview, and subsequent email correspondence, differed slightly as the head of the mathematics department, Teacher G, was speaking on behalf of the individual mathematics teachers. The value of this information was in providing the author with an insight into the teaching and learning of mathematics in the school, and the structure provided within the school to facilitate this. Interestingly, Teacher G described a greater variety of mathematics teaching and learning activities than those described and/or observed within the Irish groups. Activity learning and the use of information technology were described as being common place. Project work was also used. This would suggest that mathematics students are exposed to a broader range of teaching and learning activities in the Massachusetts' school.

7.4 Conclusion

The data collected through the structured observation of the mathematics lessons with the Irish research groups involved and the interviews with the Irish teachers, plus the head of the mathematics department in the school in Massachusetts, provide a valuable insight into teaching and learning habits in mathematics. The Irish mathematics lessons observed are interesting in how little teaching and learning activity varied between classrooms. One would imagine that younger teachers, new to the profession and recently trained in the newest mathematical education techniques as part of their teacher training would be more innovative in their approach than more established teachers, but this did not appear to be the case. All the Irish mathematics teachers observed taught in a method heavily influenced by the behaviourist philosophy and there was little time spent on group-work, active learning or the use of resources such as computers.

The interviewing process provided the Irish teachers with an opportunity to explain their classroom practices and their attitude towards mathematics education. For the most part, the teachers agreed that the observed mathematics lessons were typical of teaching and learning activity in their mathematics classes. All the teachers interviewed were enthusiastic and spoke of a willingness to adapt. However, the overall impression the author got regarding the incoming 'Project Maths' syllabus was a sense of unease and trepidation on the part of the mathematics teachers interviewed. This appeared to be due to feeling unprepared and unsure as to what the curriculum entailed. The author is of the opinion that substantial support must be offered on a continuous basis, with a focus on teaching pedagogy, if 'Project Maths' is to be a success.

8.0 <u>Chapter 8: Data Analysis of tests</u>

8.1 Introduction

The following section considers the quantitative analysis of the results for the administered tests. As previously explained, the author implemented two tests in the data collection phase of the research. The tests consisted of a 'Realistic' test involving open-ended questions and a 'Traditional' test with closed-ended questions. The 'Traditional' test consisted of questions from the arithmetic, statistics and algebraic sections of the Junior Certificate mathematics examination. The questions administered from the Junior Certificate were from the Ordinary Level examination. The 'Realistic' test involved a problem-solving scenario where the students had to make decisions based on the information provided. Evidence of reflection and a demonstration of understanding were required in the open-ended questions asked in the 'Realistic' test. The mathematics involved in the 'Realistic' test are of a similar standard to those needed to solve the closed-ended questions in the 'Traditional' test, and involved similar mathematics concepts. The identification of the standard of mathematics required to successfully answer the questions in both tests was validated by a group involving 'experts' in the area of teaching mathematics at second-level in Ireland.

The research question is '*Can students transfer the mathematical knowledge learned in the classroom to successfully solve realistic, authentic mathematical problems'*. The phrase 'achievement levels' refers to the mean scores for each test. The author simplifies the hypothesis for testing and seeks to reject or accept the following:

- Null hypothesis: '*There is no difference in achievement levels between the traditional test (test=0) and the realistic test (test=1)*'; and

- Alternative hypothesis: '*There is a difference in achievement levels between the traditional test (test=0) and the realistic test (test=1)*'.

The null hypothesis is tested by considering the statistical significance and correlation coefficient of the scores attributed to students in both test types. Two methods of statistical testing were used:

- A two-sample t-test; and

- A one-way ANOVA.

Both test methods considered the Traditional test versus the Realistic test. The purpose of the t-test is to assess if the means of the two groups are statistically different from each other. The author decided on a two-sample t-test as it is an effective method of considering small sample sizes. The objective is to make inferences about the difference between the two population means. When the sample size is small it is important that the original populations are normal. If the original sample for each population is normal than the difference between the means of both populations will be normal, even for small sample sizes (Mendenhall et al, 2009: 399).

The use of the t-test for small populations is effective as long as the following four assumptions are met:

- The samples are randomly selected;

- The samples are independent;

- The populations should be moderately normal; and

- The population variance should be reasonably similar (Mendenhall et al, 2009:405).

The t-test and the ANOVA effectively test the same thing. The author uses both tests as a form of validation and reliability. The MINITAB statistical computer package was used for data analysis and graphical representations. All results were tested for a 95% confidence level (the assumption being that there is a 5% possibility that the scores are different due to chance – the 95% confidence level accounts for this and seeks to eliminate the possibility). Therefore the author sets the alpha-level for $p=0.05$. If $p\leq 0.05$ the author rejects the null hypothesis and states that there is a difference in performance between the Realistic and the Traditional tests. If $p>0.05$ the author fails to reject the null hypothesis, and states that there is no difference in mathematical achievement in the two tests ('Traditional' versus 'Realistic').

8.2 The Pilot Study

The pilot study considered a group of thirty-two students in a higher level, second-year mathematics class. The students are the top class within the higher level class groups in second year and are, according to their mathematics teacher, extremely able with a couple of students who struggle. The purpose of testing the students in the pilot study is to consider the effectiveness of the tests, and the ability of the tests to address the research question.
The analysis seeks to either accept or reject the following:

- The Null Hypothesis: *'That there is no difference in achievement levels in the 'Traditional' and the 'Realistic' test for all pilot students';*
- The Alternative Hypothesis: *'That there is a difference in achievement levels in the 'Traditional' and the 'Realistic' test for all pilot students'.*

The confidence level is set at 95% to account for any difference that may arise by chance – the alpha level of 0.05 goes some way towards eliminating this risk.

8.2.1. Pilot Study Descriptive Statistics (t=0, t=1)

Descriptive Statistics: Traditional Test, Realistic Test

Variable	Test	N	N*	Mean	SE Mean	StDev	Minimum	Q1	Median	Q3
Trad Test	0	30	2	93.82	1.58	8.64	60.83	89.55	97.50	100.00
Realistic	1	30	2	52.07	3.49	19.11	12.00	34.00	57.00	66.00

Variable	Test	Maximum
Trad Test	0	100.00
Realistic	1	84.00

Figure 11: Descriptive Statistics for Pilot study (t=0, t=1)

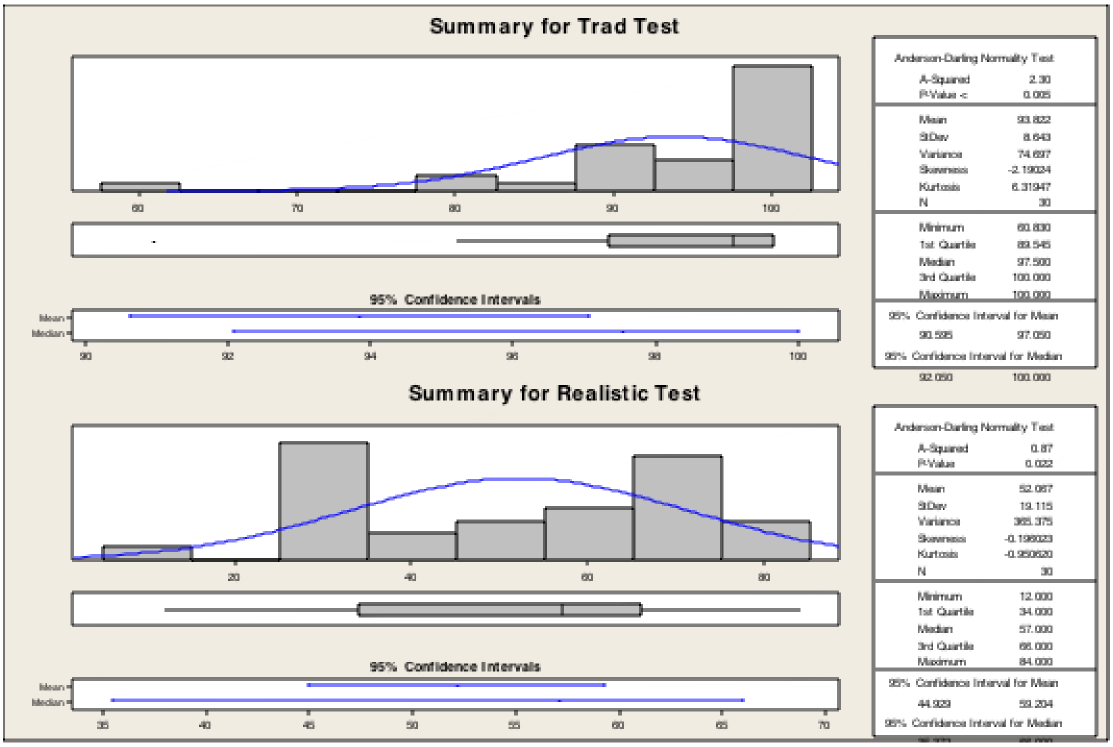


Figure 12: Graphical Summary of descriptive statistics for pilot study (t=0, t=1)

8.2.2. Pilot study (t=0, t=1) hypothesis test (two sample t-test)

The approach taken to determine the difference between the mean responses for the 'Traditional' and 'Realistic' tests at the 5% level of significance is as follows:

- Assess the data sets for equal variance;

- Conduct a two-sample t-test, with or without equal variance, based on the variance findings; and

- Conclude from the t-test whether the difference is significant at the appropriate level.

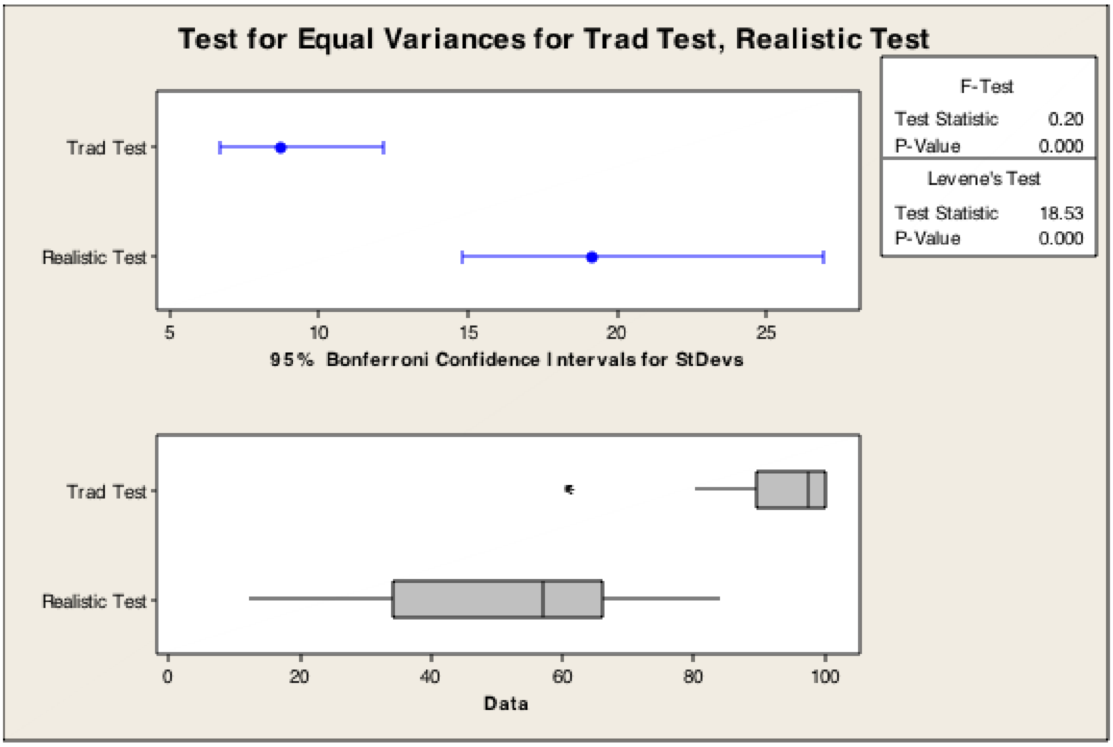
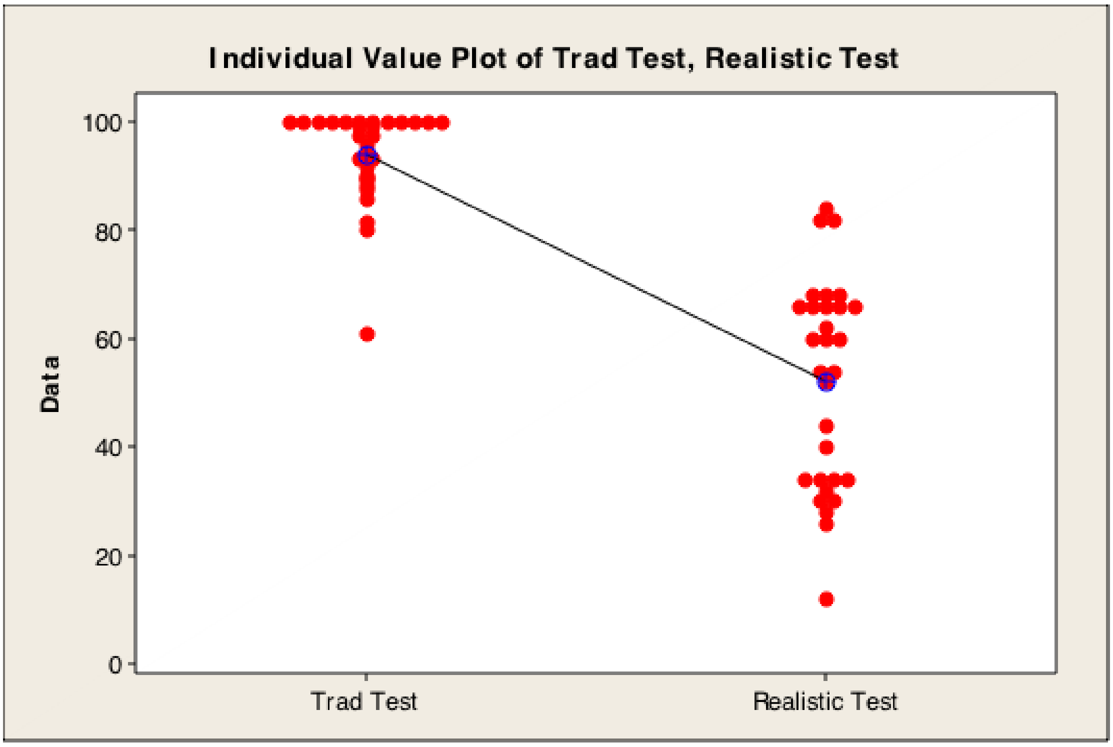


Figure 13: Graphical summary of test for equal variance for pilot study (t=0, t=1)

Minitab provides a test statistic and P-value for both the F-Test and Levene's test (fig. 13). It is of note that both P-values of 0.000 and 0.000 are less than 0.05. Therefore, this result is significant and there is sufficient evidence to conclude that the variances are not equal. Based on the outcome of the test for equal variance, we can reject the possibility of equal variance based on the p-values of less than 0.05.

```
Two-Sample T-Test and CI: Traditional Test, Realistic Test
Difference = mu (Trad Test) - mu (Realistic Test)
Estimate for difference:  41.76
95% CI for difference:  (34.01, 49.50)
T-Test of difference = 0 (vs not =): T-Value = 10.90  P-Value = 0.000
DF = 40
```

Figure 14: Two sample t-test statistics of Pilot study (t=0, t=1)



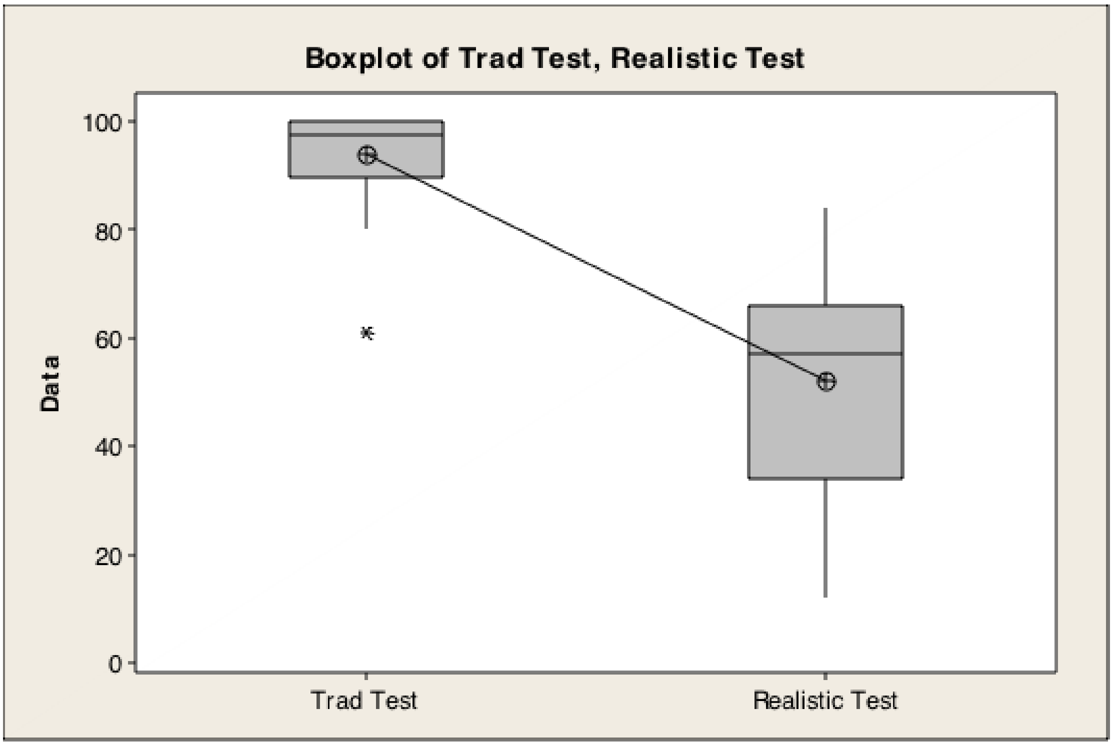


Figure 15: Pilot study two sample t-test Individual and box-plot graphics (t=0, t=1)

Based on the outcome of the two sample t-test (fig. 14), the author notes the estimate difference is 41.76, which would indicate that there is a considerable difference between the performance responses of the 'Traditional' and 'Realistic' tests. Based on the p-value of 0.000, the author rejects the null hypothesis at the 5% level of significance and concludes that there is a difference between the results of the 'Traditional' and 'Realistic' tests.

8.2.3. Pilot study (t=0, t=1) hypothesis test (one way ANOVA)

One-way ANOVA: Result versus Test

```
Source  DF     SS     MS       F      P
Test     1  26153  26153  118.86  0.000
Error   58  12762    220
Total   59  38915

S = 14.83   R-Sq = 67.21%   R-Sq(adj) = 66.64%

                         Individual 95% CIs For Mean Based on
                         Pooled StDev
Level   N   Mean  StDev  ---------+---------+---------+---------+
0      30  93.82   8.64                               (---*--)
1      30  52.07  19.11  (---*--)
                         ---------+---------+---------+---------+
                                 60        75        90       105

Pooled StDev = 14.83

Tukey 95% Simultaneous Confidence Intervals
All Pairwise Comparisons among Levels of Test

Individual confidence level = 95.00%

Test = 0 subtracted from:

Test   Lower  Center   Upper  ---+---------+---------+---------+------
1     -49.42  -41.76  -34.09  (----*----)
                              ---+---------+---------+---------+------
                               -45       -30       -15         0
```

Figure 16: Pilot study One way ANOVA statistics (t=0, t=1)

The ANOVA output of immediate interest (fig. 16) is the F-test statistic. As the associated P-value is 0.000, one can reject the null hypothesis and conclude that the means of the two samples are statistically different.

Minitab also generates confidence intervals (CIs) for the mean of both tests (fig. 16); the confidence intervals for this study do not demonstrate an overlapping of the intervals for the test samples. Additionally, the post-hoc testing performed using the Tukey test provides confidence intervals for the difference in the pair of means under evaluation. From this analysis, it can be concluded that there is a significant difference between the performance of 'test

0' and 'test 1' as the interval goes from – 49.42 to – 34.09 and zero is not in the interval. In this instance, where the 'test 0' has been subtracted from 'test 1' and the resultant CI contains negative values, one can equate that 'test 1' had significantly lower results. The centre point of the CI is – 41.76 and is the estimated mean difference between the test groups.

In order to test that the ANOVA assumptions were not violated; a residual plot was created (fig. 17). The standard assumptions are as follows:

- The relationship between Y and X must be linear;
- The values are normally distributed; and
- The values of random error are independent.

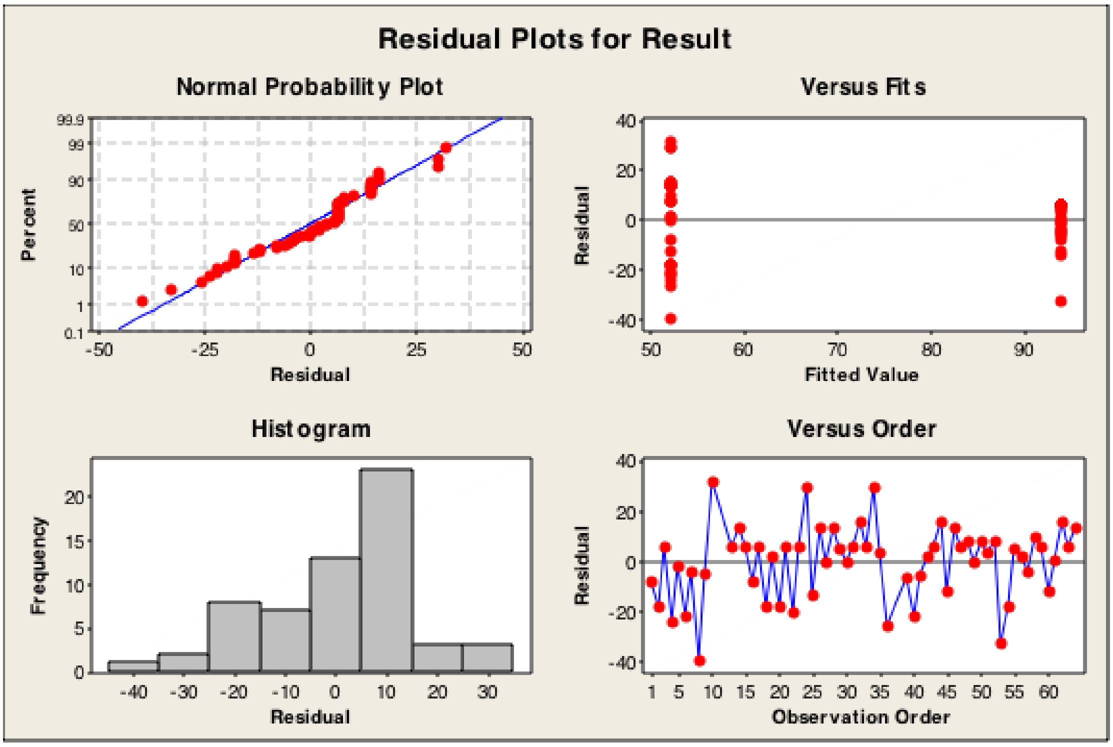


Figure 17: Pilot One way ANOVA Residual plot graphics (t=0, t=1)

Interpretation of the residual plots:

‘Residuals represent experimental error, the basic variability of an experiment, and should have an approximately normal distribution with a mean of 0 and the same variation for each treatment group’ (Mendenhall et al, 2009:488).

- The normal probability plot of residuals: Plots the residuals from each observation against the expected value of the residual had it come from a normal distribution. All plotted values appear in a straight line if the residuals are approximately normal (Mendenhall et al, 2009: 489). In this instance the values are in a reasonably straight line which suggests that there is no reason to state that the assumptions have been breached.
- The plot of residual versus fit: This graph plots the residual values against the expected value of the observation using the experimental design implemented. The plot is used to check for constant variance. The data appears to have a random pattern and as a result there is no reason to state that the assumptions have been breached.
- The histogram of residuals: The plot is used to check distribution. Viewed in conjunction with the normal plot, the histogram appears to support the normal distribution and as a result there is no reason to state that the assumptions have been breached.
- Residual versus overall plot: This plot is used to check for observational influence. The data appears to have a random pattern around the central line and as a result there is no reason to state that the assumptions have been breached.

The overall conclusion, based on the residual plots for the results, is that there is no evidence to suggest that standard assumptions of the regression have been violated.

8.2.4. Pilot study (t=0, t=1) Correlation test

```
Pearson correlation of Traditional Test and Realistic Test = 0.149
P-Value = 0.431
```

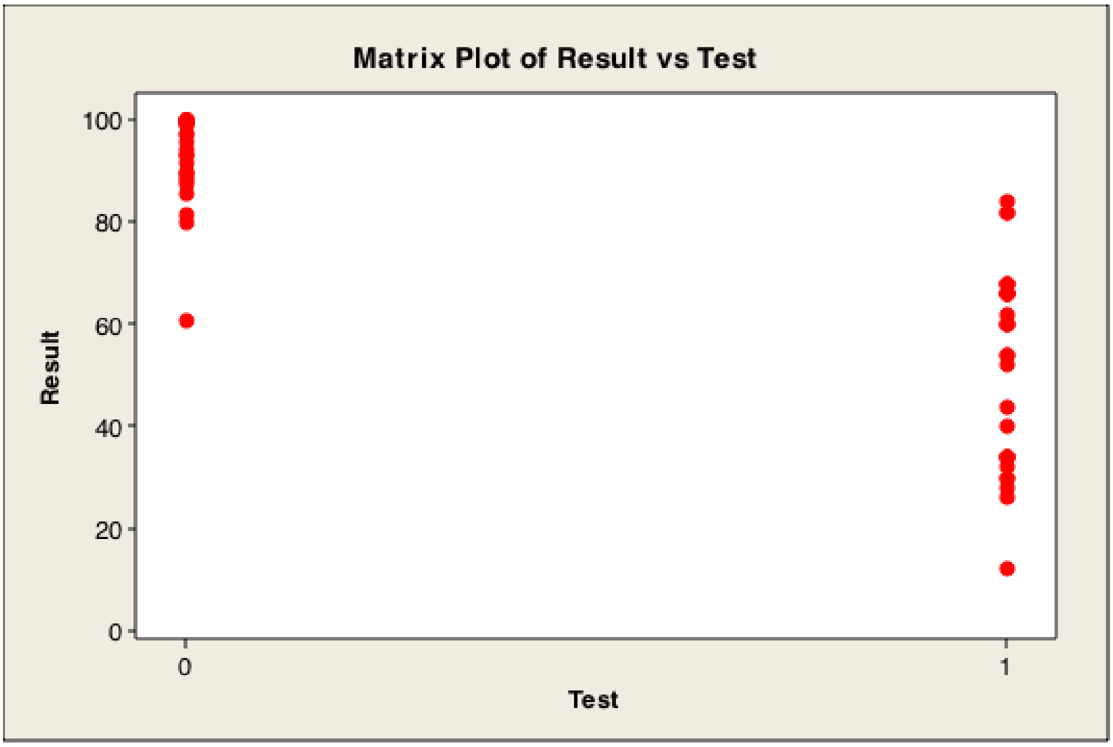


Figure 18: Pilot study Correlation statistics and matrix plot graphics (t=0, t=1)

There is sufficient evidence to support the lack of presence of linear correlation between the two variable tests, as demonstrated by the r-value of 0.149 and a P-value of 0.431 (fig. 18).

8.2.5. Interpretation of Results (Pilot Study)

As the p-value (for two-tailed significance) for the t-test is less than 0.5, p=0.000, the author rejects the null hypothesis '*That there is no difference in achievement levels in the 'Traditional' test and the 'Realistic' test for all pilot students'*. Hence, the analysis of the test results for all students (n=30) indicates that there is significant difference between performance in the 'Traditional' test and the 'Realistic' test. Students performed at a significantly higher level in the 'Traditional' test in comparison to results gained in the 'Realistic' test. The 'Traditional' test (M=93.82, S.D.=8.64) scored considerably higher than the 'Realistic' test (M=52.1, S.D.=19.1). The estimate for difference between the two tests is 41.76 in favour of the 'Traditional' test (fig. 14).

8.2.6. Adaption made to the Tests after analysis the Pilot study

Following the initial marking stage of the 'Realistic' test the author made the decision to alter the test as follows:

- To alter the format of the test so that it was divided into easy to read sub-sections; and
- Emphasise the questions so that students are aware of what they are being asked.

The author was happy to continue with the 'Traditional' test as initially designed.

8.3 The Main Study

The following table provides the coding and legend given to each of the groups involved in the testing process. The MA group (group 6) represents the students from the United States in the state of Massachusetts; the other five groups (number 1-5) represent the Irish class groups.

Group number	Group name
1	MS (Ordinary Level)
2	KD (Higher Level)
3	SC (Higher level)
4	MH (Ordinary Level)
5	RC (Higher level)
6	AQ (Ordinary Level)
7	MA (Massachusetts)

Gender number	Gender type
0	Male
1	Female

Test number	Test type
0	Traditional
1	Realistic

Level number	Level type
0	Ordinary (MS, MH, AQ)
1	Higher (KD, SC, RC)
2	USA

Region number	Region name
0	Ireland
1	USA

Table 23: Coded Identifiers of Main study Groups

8.3.1. Traditional versus Realistic Test (All Students)

The following graphical and numerical information provides the statistical analysis of the comparison of the results for student performance between the two tests, the 'Traditional' and the 'Realistic'.

The statistical analysis, provided by a two-sample t-test and a one-way ANOVA, considers the between test performance for all students involved in the research (n=157) and seeks to determine if there is a difference in test performance. The students involved in the research were from both Ireland and the U.S. state of Massachusetts. The statistical analysis seeks to consider mean individual test performance and if there is a significant difference between them. The tests implemented are:

- The 'Traditional' Test (test=0): which involves closed-ended questions based on the Junior Certificate curriculum and the Junior Certificate examination. The topics tested are based on the Algebraic and Arithmetic sections of the Junior Certificate course. The majority of Irish students involved in the study should have covered the topics necessary for successful completion of the 'Traditional' test by the stage of test implementation. The minority that may not would be those students who need extra assistance to complete the Junior Certificate syllabus, and indeed terminal examination (but these students would have covered a significant proportion of the desired material). The marking scheme is based on that provided for the Junior Certificate examination and marks are awarded for effort and correct steps. However, as is typical of the Junior Certificate assessment style, there is only one correct answer for each question asked.

- The ‘Realistic’ Test (test=1): is based on an authentic, problem-solving scenario. The questions asked require logical thought, reasoning and reflection, and ask for evidence of these skills. The questions are asked in an open-ended manner and may have more than one correct answer. The ‘Realistic’ test requires the research participant (the student) to read more which may be an issue for students with compromised literacy skills. However, all students are allowed ask for assistance in the reading of the test and in Irish schools those students with significant learning difficulties have the assistance of an S.N.A (a special needs assistant) who may assist in the reading also. The mathematical skills necessary for successful completion of the ‘Realistic’ test are similar to those for the ‘Traditional’ test: namely algebraic and arithmetic skills. The mathematical skills required for both tests (test=0 and test=1) are of a similar level (this is verified by a group of ‘experts’ in the area of mathematics education).

The analysis seeks to either accept or reject the following:

- The Null Hypothesis: ‘*That there is no difference in achievement levels in the ‘Traditional’ (test=0) and the ‘Realistic’ test (test=1) for all students’;*

- The Alternative Hypothesis: ‘*There is a difference in achievement levels in the ‘Traditional’ (test=0) and the Realistic’ test (test=1) for all students’.*

The confidence level is set at 95% to account for any difference that may arise by chance – the alpha level of 0.05 goes some way towards eliminating this risk.

8.3.1.1. ‘Traditional’ versus ‘Realistic’ test descriptive statistics (t=0, t=1)

Descriptive Statistics: Results

Variable	Test	N	N*	Mean	SE Mean	StDev	Minimum	Q1	Median	Q3
Results	0	157	21	75.56	1.62	20.31	0.00	64.17	80.00	91.67
	1	157	11	55.33	1.59	20.57	0.00	45.00	61.67	70.00

Variable	Test	Maximum
Results	0	100.00
	1	96.67

Figure 19: Descriptive Statistics for Traditional vs. Realistic study

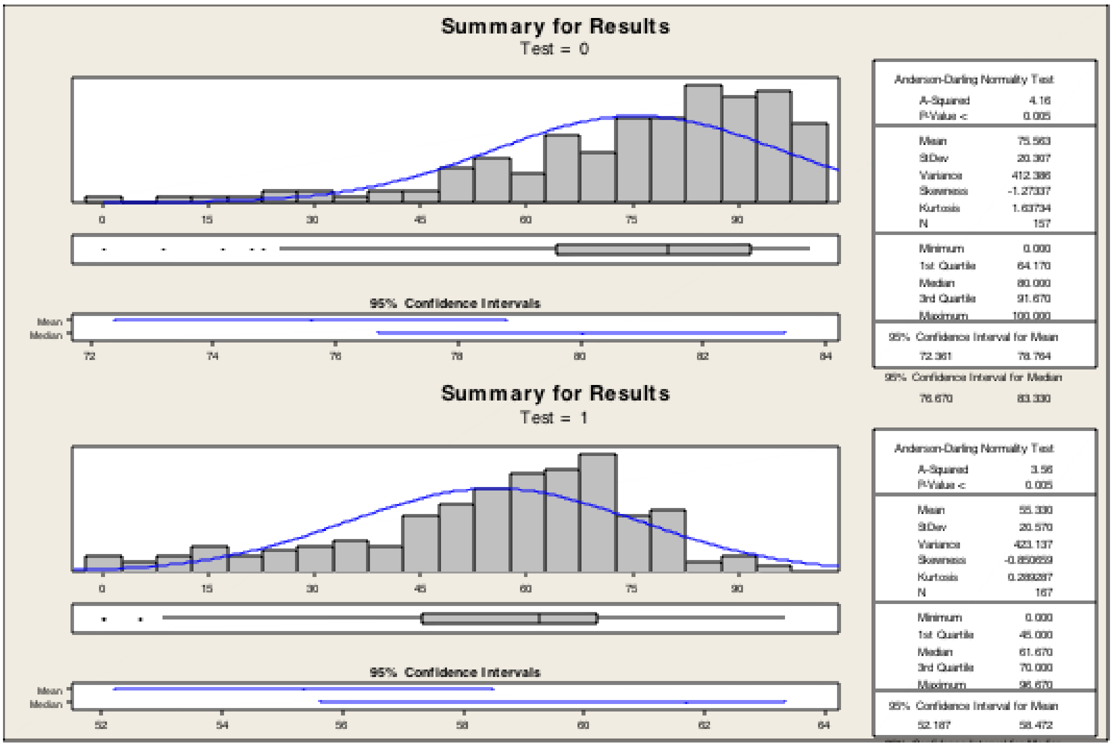


Figure 20: Graphical Summary of descriptive statistics for Traditional vs. Realistic study

8.3.1.2. ‘Traditional’ versus ‘Realistic’ Hypothesis Test (Two sample t-test; t=0, t=1)

The approach taken to determine the difference between the mean responses for the ‘Traditional’ and ‘Realistic’ tests at the 5% level of significance is as follows:

- Assess the data sets for equal variance;
- Conduct a two-sample t-test, with or without equal variance, based on the variance findings; and
- Conclude from the t-test whether the difference is significant at the appropriate level.

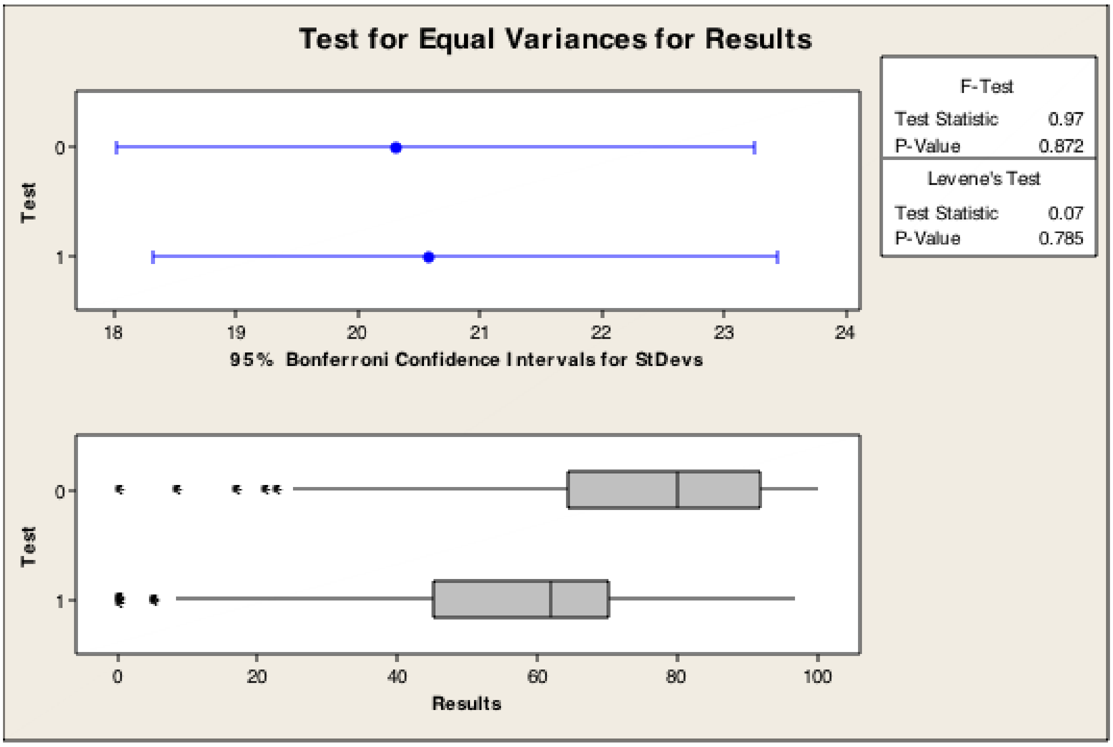


Figure 21: Graphical summary of test for equal variance for Traditional vs. Realistic study

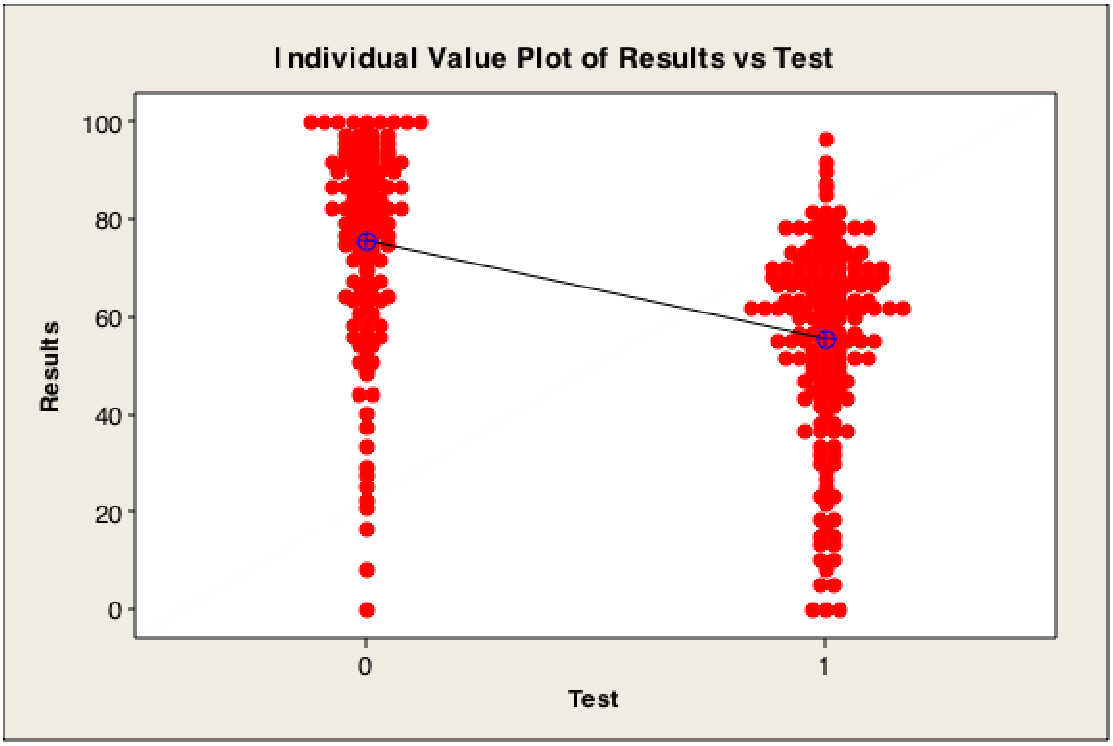
Minitab provides a test statistic and P-value for both the F-Test and Levene’s test (fig. 21). It is of note that both P-values of 0.872 and 0.785 are greater than

0.05. Therefore, this result is not significant and there is sufficient evidence to conclude that the variances are equal.

Two-Sample T-Test and CI: Results, Test

```
Two-sample T for Results
Test      N  Mean  StDev  SE Mean
0       157  75.6   20.3      1.6
1       157  55.3   20.6      1.6
Difference = mu (0) - mu (1)
Estimate for difference:  20.23
95% CI for difference:  (15.76, 24.70)
T-Test of difference = 0 (vs not =): T-Value = 8.90  P-Value = 0.000
DF = 322
Both use Pooled StDev = 20.4433
```

Figure 22: Two sample t-test statistics Traditional vs. Realistic study



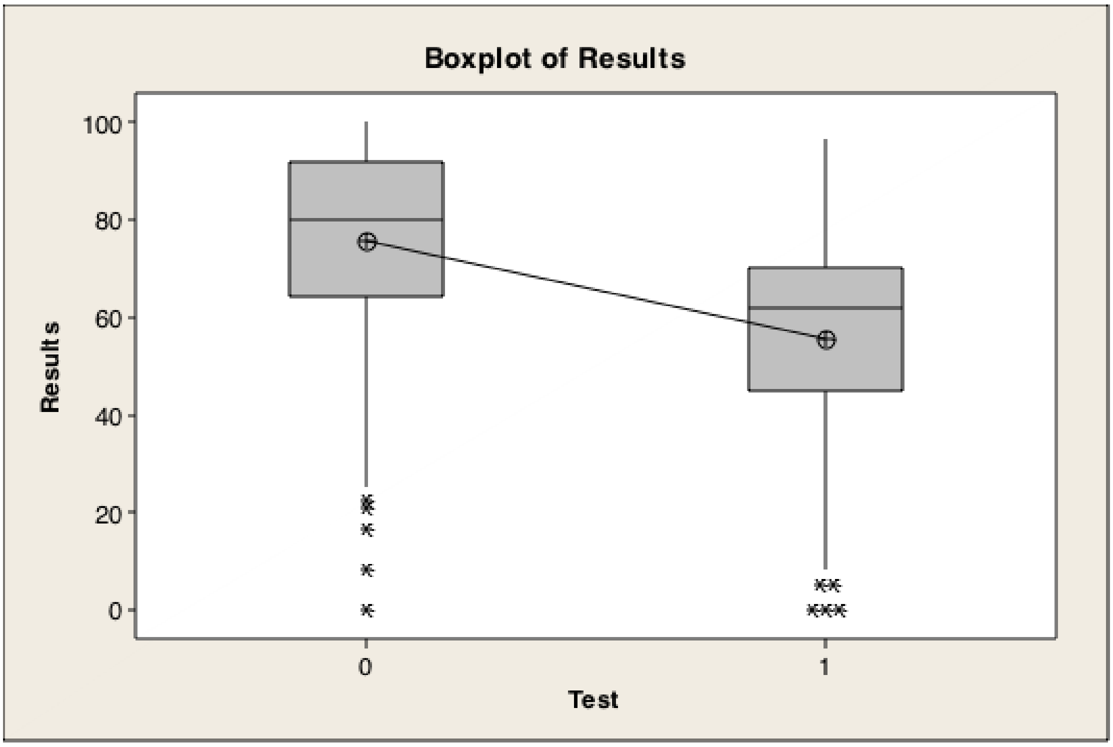


Figure 23: Two sample t-test Individual and box-plot graphics Traditional vs. Realistic study

Based on the outcome of the two sample t-test, the author notes the estimate difference is 20.23, which would indicate that there is a considerable difference between the performance responses of the 'Traditional' and 'Realistic' tests (fig. 22). Based on the p-value of 0.000, the author rejects the null hypothesis at the 5% level of significance and concludes that there is a difference between the results of the 'Traditional' and 'Realistic' tests.

8.3.1.3. Traditional versus Realistic study (t=0, t=1) Hypothesis Test (One way ANOVA)

One-way ANOVA: Results versus Test

```
Source   DF      SS     MS      F      P
Test      1   33128  33128  79.27  0.000
Error   322  134573    418
Total   323  167701

S = 20.44   R-Sq = 19.75%   R-Sq(adj) = 19.51%

                           Individual 95% CIs For Mean Based on
                           Pooled StDev
Level    N   Mean  StDev  -----+---------+---------+---------+----
0      157  75.56  20.31                              (----*----)
```

```
1       157  55.33  20.57  (---*---)
                           -----+---------+---------+---------+----
                             56.0      63.0      70.0      77.0

Pooled StDev = 20.44
Tukey 95% Simultaneous Confidence Intervals
All Pairwise Comparisons among Levels of Test
Individual confidence level = 95.00%
Test = 0 subtracted from:

Test    Lower  Center   Upper      -+---------+---------+---------+-----
---
1      -24.70  -20.23  -15.76     (-----*----)
                                  -+---------+---------+---------+-----
---
                               -24.0     -16.0      -8.0       0.0
```

Figure 24: One way ANOVA statistics Traditional vs. Realistic study

The ANOVA output of immediate interest, as outlined above, is the F-test statistic. As the associated P-value is 0.000, one can reject the null hypothesis and conclude that the means of the two samples are statistically different.

Minitab also generates confidence intervals (CIs) for the mean of both tests; the confidence intervals for this study do not demonstrate an overlapping of the intervals for the test samples (fig. 24). Additionally, the post-hoc testing performed using the Tukey test provides confidence intervals for the difference in the pair of means under evaluation. From this analysis, it can be concluded that there is a significant difference between the performance of 'test 0' and 'test 1' as the interval goes from $-$ 24.70 to $-$ 15.76 and zero is not in the interval. In this instance, where the 'test 0' has been subtracted from 'test 1' and the resultant CI contains negative values, one can equate that 'test 1' had significantly lower results. The centre point of the CI is $-$ 20.23 and is the estimated mean difference between the test groups.

In order to test that the ANOVA assumptions were not violated; a residual plot was created (fig. 25). The standard assumptions are as follows:

- The relationship between Y and X must be linear;
- The values are normally distributed; and
- The values of random error are independent.

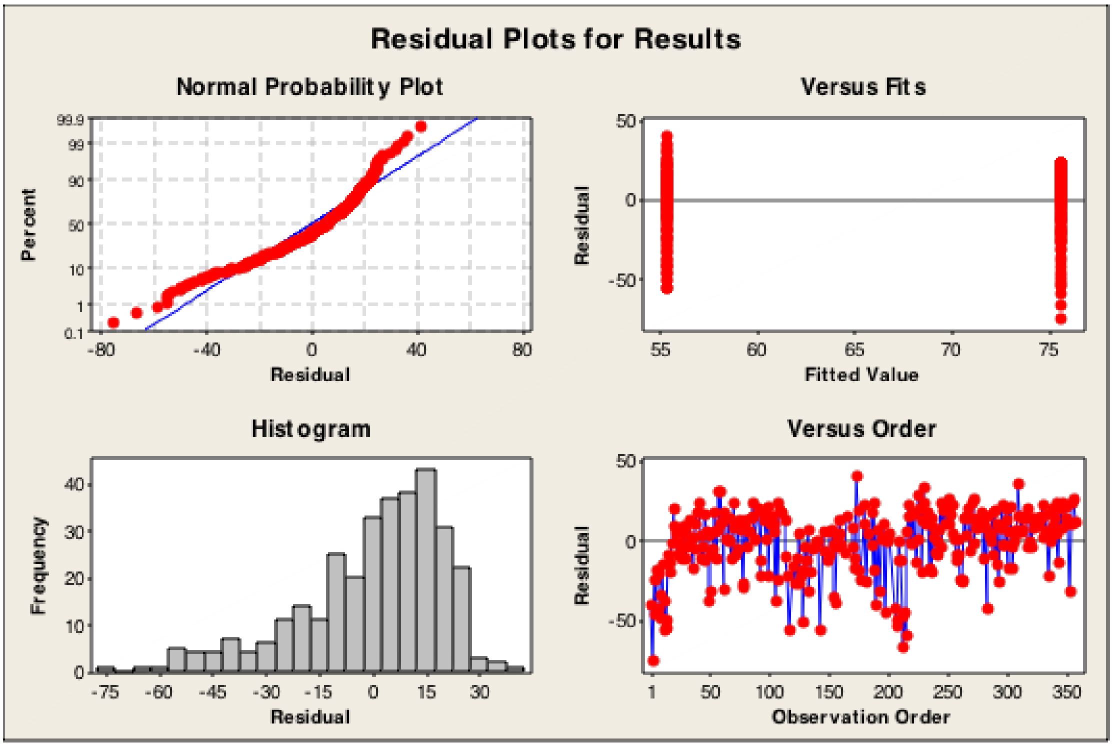


Figure 25: One way ANOVA Residual plot graphics Traditional vs. Realistic study

Interpretation of the residual plots:

- The normal probability plot of residuals: Plots the residuals from each observation against the expected value of the residual had it come from a normal distribution. All plotted values appear in a straight line if the residuals are approximately normal (Mendenhall et al, 2009: 489). In this instance the values are in a reasonably straight line which suggests that there is no reason to state that the assumptions have been breached.
- The plot of residual versus fit: This graph plots the residual values against the expected value of the observation using the experimental design implemented. The plot is used to check for constant variance. The data appears to have a random pattern and as a result there is no reason to state that the assumptions have been breached.
- The histogram of residuals: The plot is used to check distribution. Viewed in conjunction with the normal plot, the histogram appears to support the normal distribution and as a result there is no reason to state that the assumptions have been breached.

- Residual versus overall plot: This plot is used to check for observational influence. The data appears to have a random pattern around the central line and as a result there is no reason to state that the assumptions have been breached.

The overall conclusion, based on the residual plots for the results (fig. 25), is that there is no evidence to suggest that standard assumptions of the regression have been violated.

8.3.1.4. Traditional versus Realistic study (t=0, t=1) Correlation Study

Correlations: Results, Test

```
Pearson correlation of Results and Test = -0.444
P-Value = 0.000
```

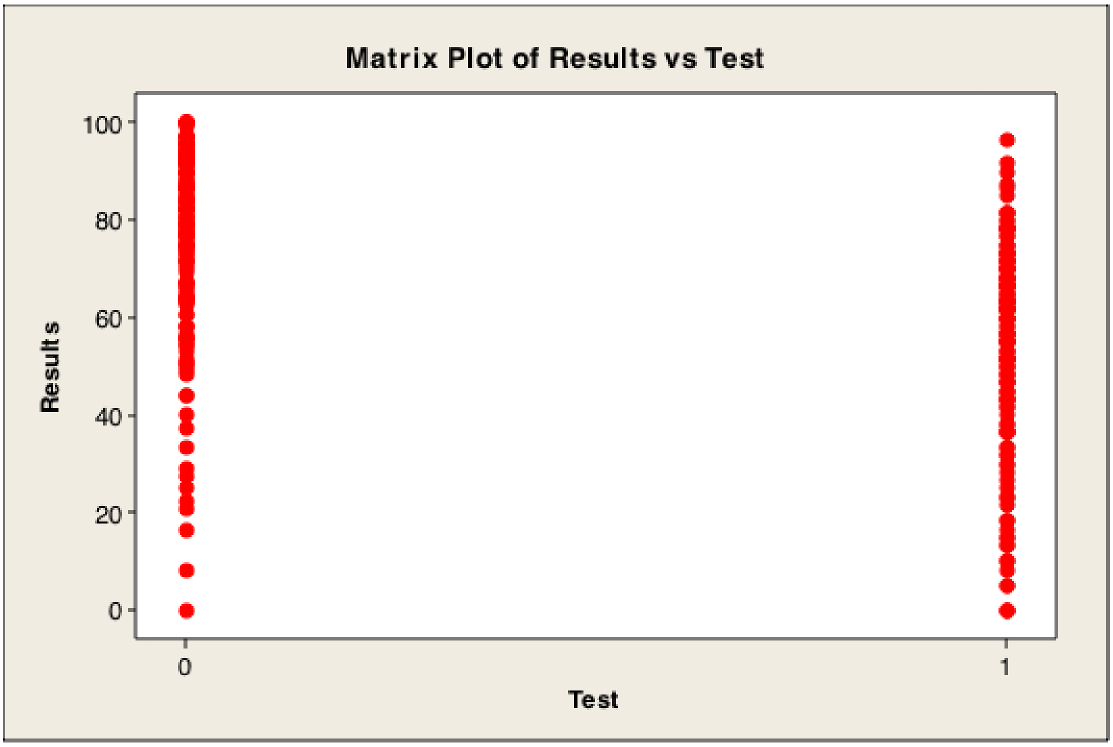


Figure 26: Traditional vs. Realistic study Correlation statistics matrix plot graphics (t=0, t=1)

There is sufficient evidence to suggest a slight negative correlation between the two variable tests, as demonstrated by the r-value of -0.444 and a P-value of 0.000 (fig. 26).

8.3.1.5. Interpretation of results (Realistic vs. Traditional – All Students)

As the p-value (for two-tailed significance) for the t-test is less than 0.5 (p=0.000), the author rejects the null hypothesis: '*That there is no difference in achievement levels in the 'Traditional' (test=0) and the 'Realistic' test (test=1) for all students'*. Hence, the analysis of the test results for all students (N=157) that participated in the research (in both Ireland and Massachusetts) indicates that there is a significance difference between performance in the traditional test (test=0) and the realistic test (test=1). This is indicated with a P-value of 0.000 (a P-Value of less than 0.05 indicates no co-relation as confidence testing, and the alpha-level, was set at a 95% confidence level).

Students performed at a significantly higher level in the 'Traditional' test compared to the 'Realistic' test. The 'Traditional' test, test=0, (M=75.6, S.D. 20.3) scored higher than the 'Realistic' test, test=1, (M=46.5, SD=21.7). The estimate for difference between performance in the two tests is 20.23 in favour of the traditional test (as demonstrated above in the two-sample t-test results) (fig. 22). This rejects the null hypothesis: *'There is no difference in achievement levels between the traditional test (test=0) and the realistic test (test=1)'*.It is worth noting that a 20.23% mean difference is very substantial. This would indicate that students of all ability levels, genders, and from both Massachusetts and Ireland, perform at a higher achievement level in the 'Traditional' test. The Pearson correlation coefficient is -0.444 which indicates there is a reasonable weak, negative correlation between the 'Traditional' and 'Realistic' tests (fig. 26)).

8.3.1.6. Test analysis for different categories

The comparison between two groups from different countries is always a difficult process as no two groups from different education systems are ever truly equitable. The particularly high performance of the state of Massachusetts in international assessment provides an interesting point of comparison for Irish students. It is interesting to consider Irish performance when compared to participants in an education system that is considered to provide well educated mathematics students. The author considered a number of different comparisons between the research participants in order to initiate debate and raise questions about various influencing factors. The author is interested in considering the break-down of achievement difference for the following groups:

- *Ireland versus Massachusetts*: The author wishes to consider the difference in achievement levels for Ireland only, Massachusetts only, and Ireland versus Massachusetts on the traditional test. The author then plans to tabulate the results and remark on any noticeable difference in achievement patterns between the two groups.

- *Gender – Male versus female:* The author hopes to consider test achievement with respect to gender. The author plans to consider male results only and female results only, and compare the difference in achievement levels for the two groups.

- *Ability Grouping based on 'Traditional' test performance*. The author considers all the students involved in the testing process (N=157) in three ability categories:

  -Results ≥80%,

  -Results between 60% and 80%, and

  -Results under 60%.

The author based these categories on test performance in the 'Traditional' test as this is the test that would be associated with ability in Irish assessment.

- *Level of Junior Certificate Course Studied:* For Irish students the author considered the level of course studied at Junior cycle. For all Irish students involved in the study this was either higher or ordinary level. There is a third (less difficult) Junior Certificate level, foundation level. However, while some of the students involved in this research may eventually sit the foundation level paper on the day of the Junior Certificate examination, at the time of the testing for this research all such students were studying the ordinary level course. The decision to eventually sit the foundation level paper would be made, on advice from the teacher, shortly before the terminal assessment in June of third year. The three ordinary level teachers involved in the research stress that they would avoid this route if at all possible as sitting the foundation level paper at Junior Certificate level restricts the mathematical options open to the students at senior cycle.

8.3.2. Ireland versus Massachusetts Test Results

The author considered the results for both tests from Ireland and Massachusetts in various ways:

- Irish test results for Irish student performance in the 'Traditional' (test=0) versus 'Realistic' (test-1);
- Massachusetts' test results for student performance in the 'Traditional' (test=0) versus 'Realistic' (test=1);
- Ireland versus Massachusetts for all test results (i.e. overall performance – all Irish test results versus all Massachusetts' test results);

- Ireland, Higher level, (test=0) versus Massachusetts (test=1) for the 'Traditional' test; and

- Ireland, Higher level, (test=0) versus Massachusetts (test=1) for the 'Realistic' test.

The null hypothesis varied from scenario to scenario but all t-test and ANOVA analysis seeks to determine if there is a statistical difference between the two scenarios.

8.3.2.1. Irish Test Study (Realistic vs. Traditional)

The following statistical analysis considers the Irish test results for both tests ('Realistic' versus 'Traditional'). The analysis seeks to either accept or reject the following:

- The Null Hypothesis: '*That there is no difference in achievement levels between the realistic and traditional tests*';

- The Alternative Hypothesis: '*There is a difference in achievement levels between the realistic and traditional tests*'.

The confidence level is again set at 95% to account for any difference that may arise by chance – the alpha level of 0.05 goes some way towards eliminating this risk.

8.3.2.1.1. Irish study Descriptive statistics (t=0, t=1)

Descriptive Statistics: Results for Region = 0

Variable	Test	N	N*	Mean	SE Mean	StDev	Minimum	Q1	Median	Q3
Results	0	86	21	72.17	2.46	22.80	0.00	63.33	78.33	88.54
	1	96	11	46.55	2.21	21.68	0.00	32.09	50.00	61.67

Variable	Test	Maximum
Results	0	100.00
	1	96.67

Figure 27: Descriptive statistics of Irish study (t=0, t=1)

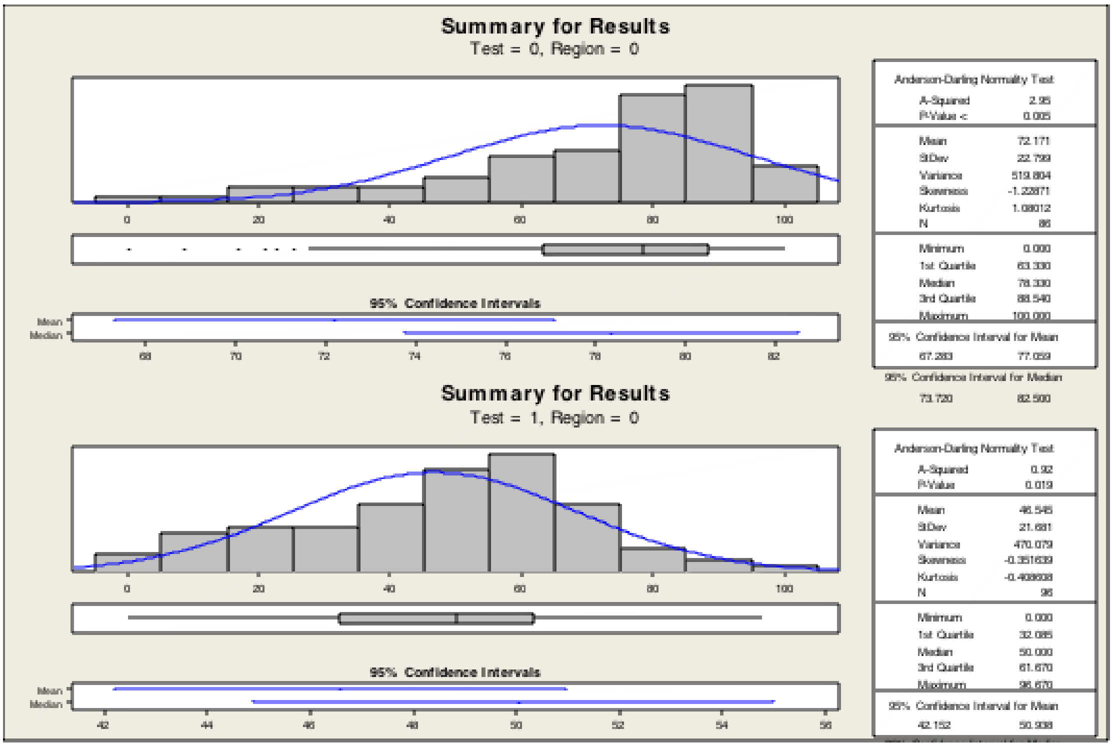


Figure 28: Graphical Summary of descriptive statistics of Irish study (t=0, t=1)

8.3.2.1.2. Irish study (t=0, t=1) hypothesis test (Two sample t-test)

The approach taken to determine the difference between the mean responses for the 'Traditional' and 'Realistic' tests at the 5% level of significance is as follows:

- Assess the data sets for equal variance;

- Conduct a two-sample t-test, with or without equal variance, based on the variance findings; and
- Conclude from the t-test whether the difference is significant at the appropriate level.

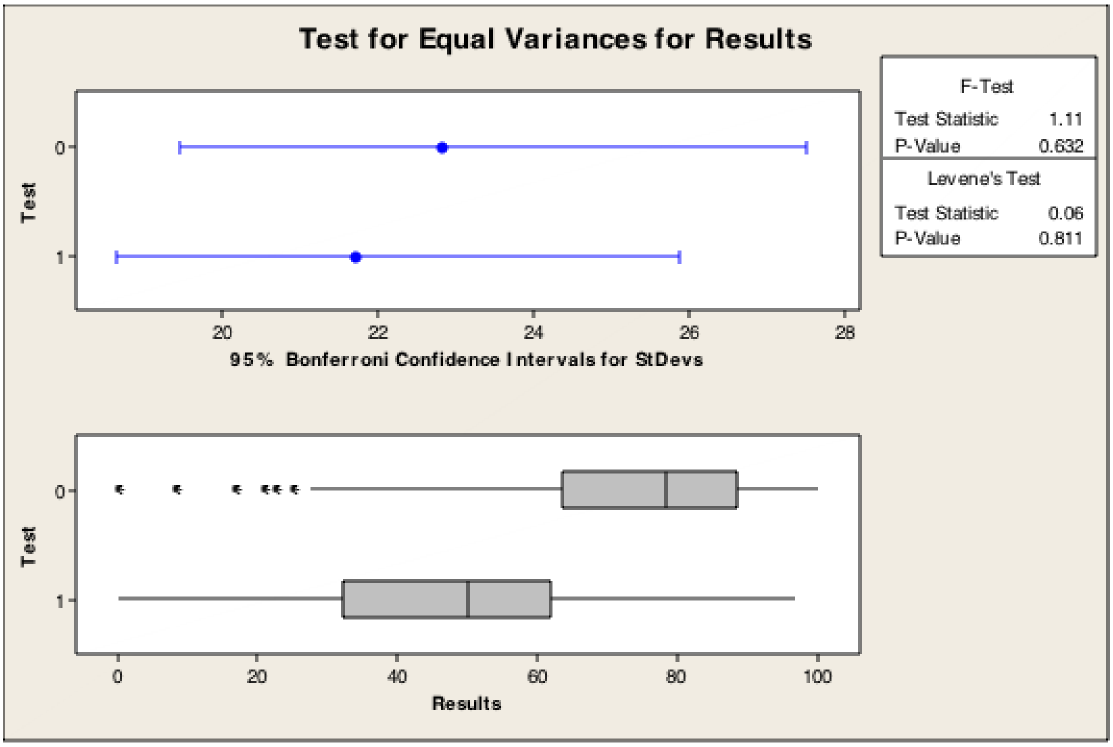


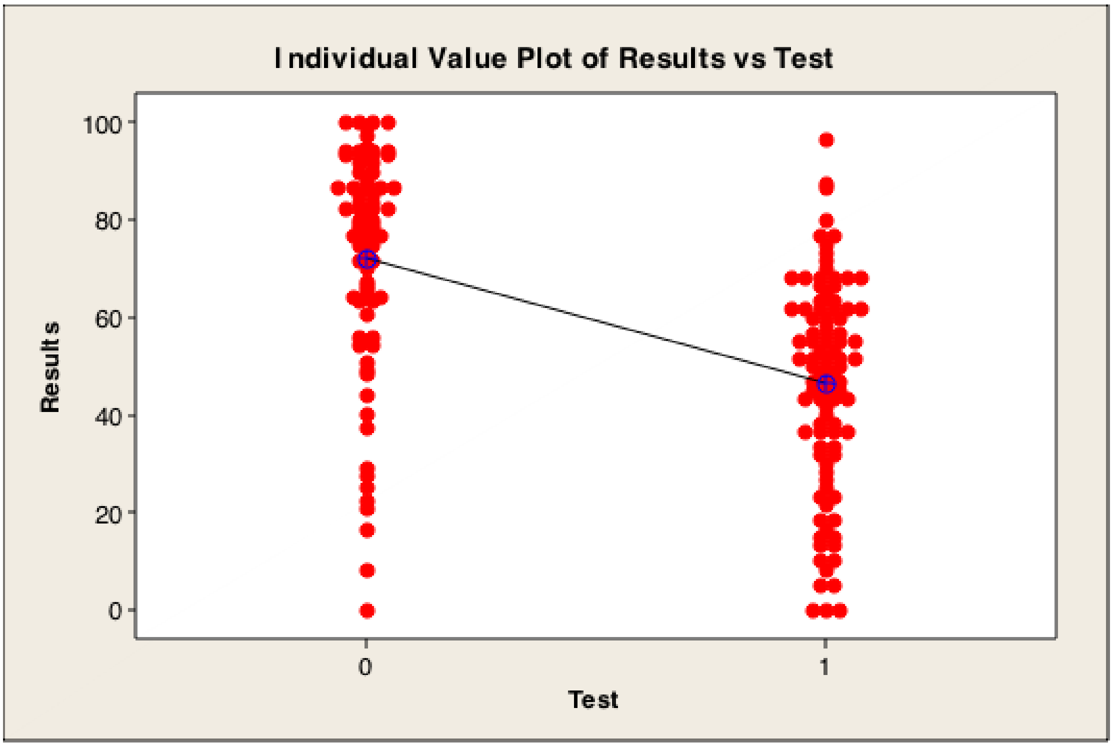
Figure 29: Graphical summary of test for equal variance of Irish study (t=0, t=1)

Minitab provides a test statistic and P-value for both the F-Test and Levene's test. It is of note that both P-values of 0.632 and 0.811 are greater than 0.05 (fig. 29). Therefore, this result is not significant and there is sufficient evidence to conclude that the variances are equal.

Two-Sample T-Test and CI: Results, Test

```
Two-sample T for Results
Test    N  Mean  StDev  SE Mean
0      86  72.2   22.8      2.5
1      96  46.5   21.7      2.2
Difference = mu (0) - mu (1)
Estimate for difference:  25.63
95% CI for difference:  (19.12, 32.13)
T-Test of difference = 0 (vs not =): T-Value = 7.77  P-Value = 0.000
DF = 180
Both use Pooled StDev = 22.2162
```

Figure 30: Two sample t-test statistics Traditional vs. Realistic study



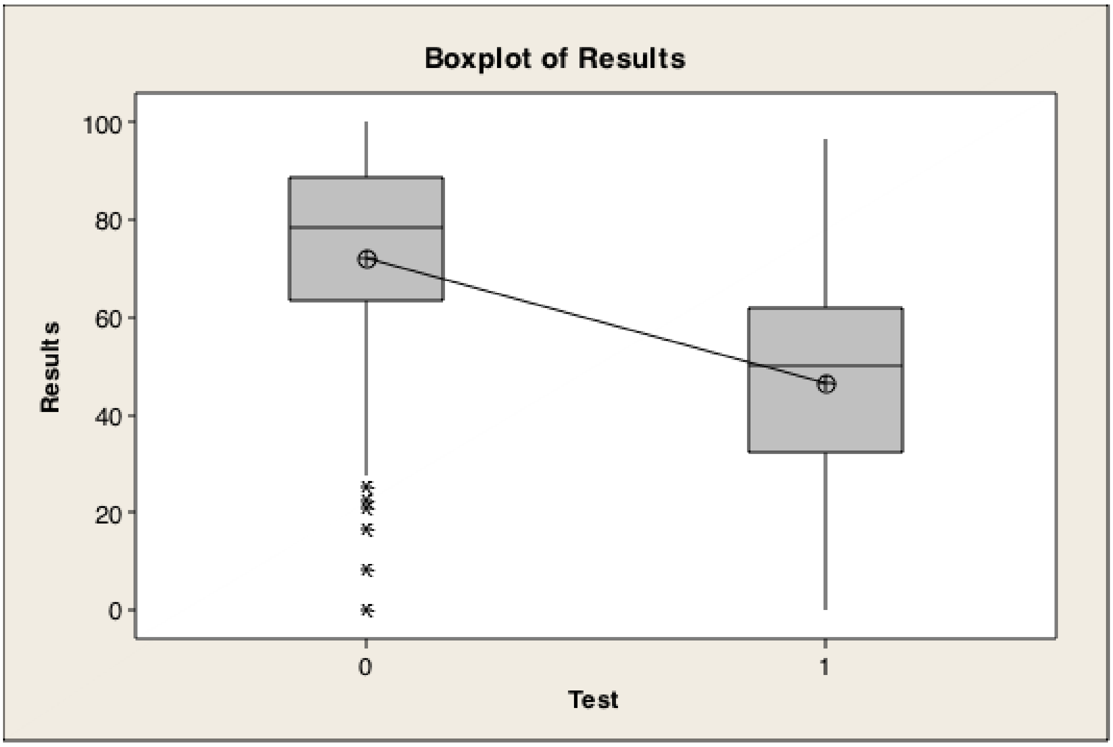


Figure 31: Two sample t-test Individual and box-plot graphics of Irish study (t=0, t=1)

Based on the outcome of the two sample t-test, the author notes the estimate difference is 25.63, which would indicate that there is a considerable difference between the performance responses of the 'Traditional' and 'Realistic' tests (fig. 30). Based on the p-value of 0.000, the author rejects the null hypothesis at the

5% level of significance and concludes that there is a difference between the results of the ‘Traditional’ and ‘Realistic’ tests for Irish students.

8.3.2.1.3. Irish study (t=0, t=1) hypothesis test (One way ANOVA)

```
One-way ANOVA: Results versus Test

Source   DF      SS     MS      F      P
Test      1   29788  29788  60.35  0.000
Error   180   88841    494
Total   181  118629

S = 22.22   R-Sq = 25.11%   R-Sq(adj) = 24.69%


                         Individual 95% CIs For Mean Based on
                         Pooled StDev
Level   N   Mean  StDev  --------+---------+---------+---------+-
0      86  72.17  22.80                           (----*----)
1      96  46.55  21.68  (----*---)
                         --------+---------+---------+---------+-
                                50        60        70        80

Pooled StDev = 22.22


Tukey 95% Simultaneous Confidence Intervals
All Pairwise Comparisons among Levels of Test

Individual confidence level = 95.00%


Test = 0 subtracted from:

Test   Lower  Center   Upper  --+---------+---------+---------+-------
1     -32.13  -25.63  -19.12  (-----*------)
                              --+---------+---------+---------+-------
                              -30       -20       -10         0
```

Figure 32: One way ANOVA statistics of Irish study (t=0, t=1)

The ANOVA output of immediate interest, as outlined above, is the F-test statistic. As the associated P-value is 0.000, one can reject the null hypothesis and conclude that the means of the two samples are statistically different.

Minitab also generates confidence intervals (CIs) for the mean of both tests; the confidence intervals for this study do not demonstrate an overlapping of the intervals for the test samples (fig. 32). Additionally, the post-hoc testing performed using the Tukey test provides confidence intervals for the difference in the pair of means under evaluation. From this analysis, it can be concluded that there is a significant difference between the performance of 'test 0' and 'test 1' as the interval goes from – 32.13 to – 19.12 and zero is not in the interval. In this instance, where the 'test 0' has been subtracted from 'test 1' and the resultant CI contains negative values, one can equate that 'test 1' had significantly lower results. The centre point of the CI is – 25.63 and is the estimated mean difference between the test groups (fig. 32).

In order to test that the ANOVA assumptions were not violated; a residual plot was created (fig. 33). The standard assumptions are as follows:

- The relationship between Y and X must be linear;
- The values are normally distributed; and
- The values of random error are independent.

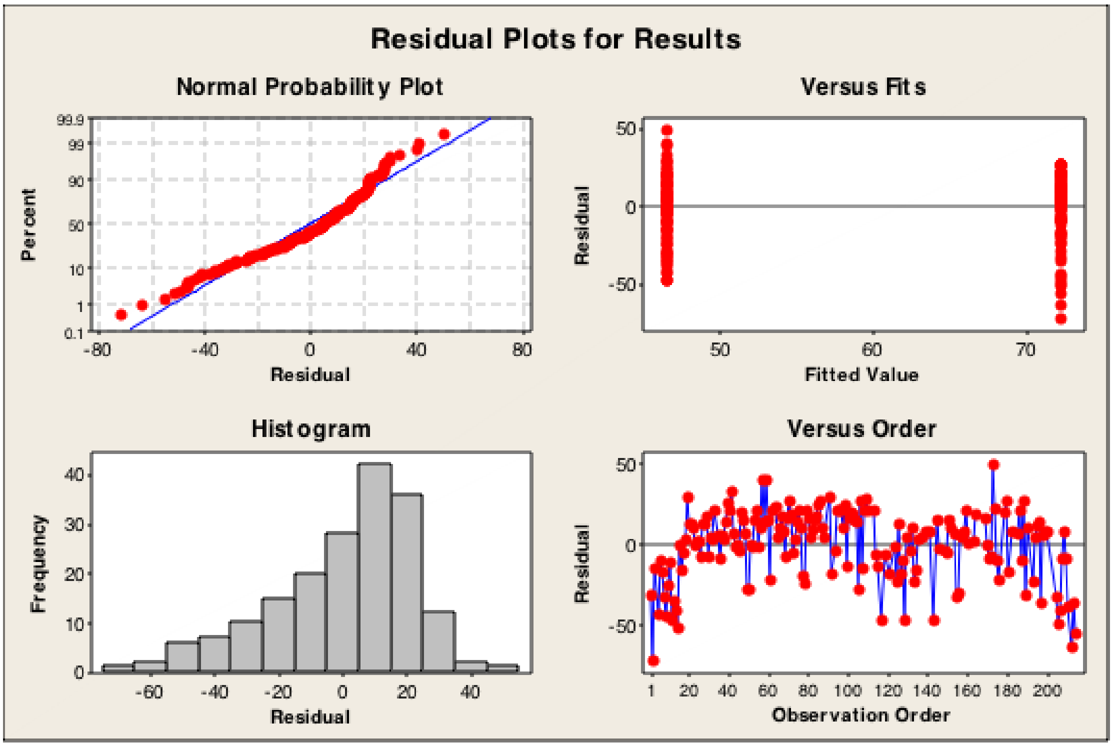


Figure 33: One way ANOVA Residual plot graphics of Irish study (t=0, t=1)

Interpretation of the residual plots:

- The normal probability plot of residuals: Plots the residuals from each observation against the expected value of the residual had it come from a normal distribution. All plotted values appear in a straight line if the residuals are approximately normal (Mendenhall et al, 2009: 489). In this instance the values are in a reasonably straight line which suggests that there is no reason to state that the assumptions have been breached.

- The plot of residual versus fit: This graph plots the residual values against the expected value of the observation using the experimental design implemented. The plot is used to check for constant variance. The data appears to have a random pattern and as a result there is no reason to state that the assumptions have been breached.
- The histogram of residuals: The plot is used to check distribution. Viewed in conjunction with the normal plot, the histogram appears to support the normal distribution and as a result there is no reason to state that the assumptions have been breached.
- Residual versus overall plot: This plot is used to check for observational influence. The data appears to have a random pattern around the central line and as a result there is no reason to state that the assumptions have been breached.

The overall conclusion, based on the residual plots for the results, is that there is no evidence to suggest that standard assumptions of the regression have been violated.

8.3.2.1.4. Irish study (t=0, t=1) (Correlations)

Correlations: Results, Test, Region

```
Pearson correlation of Results and Region = 0.322
P-Value = 0.000
```

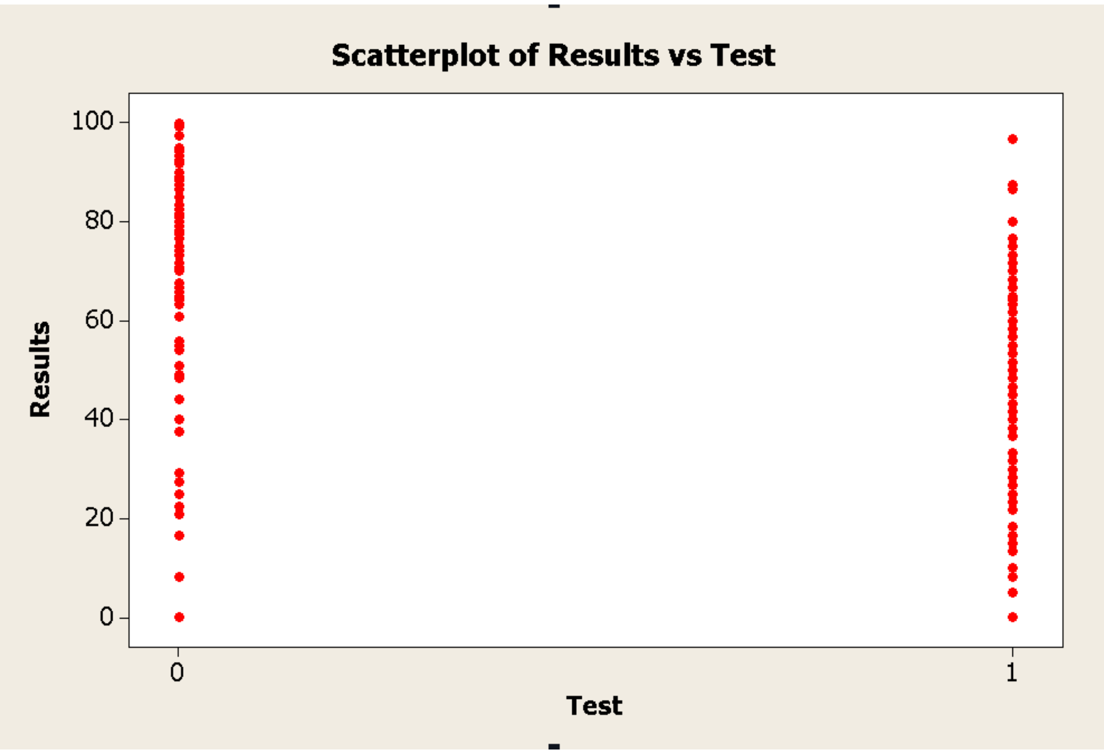


Figure 34: Correlation statistics and matrix plot of Irish study (t=0, t=1)

8.3.2.1.5. Implication of Results (Irish results)

The two-sample t-test for the Irish results showed a significant difference between performance in the 'Traditional' test and performance in the 'Realistic' test. The author rejects the null hypothesis '*That there is no difference in achievement levels between the realistic and traditional tests'* as the t-value is statistically significant. This is demonstrated by a P-Value of 0.000. Test 0, 'Traditional', (M=72.2, SD=22.8) scored higher than Test 1, 'Realistic' (m=46.5, SD=21.7). Irish students performed significantly better in the 'Traditional' test with the t-test showing an 'estimate for difference' of 25.63 in favour of the 'Traditional' test (fig. 30).

8.3.2.2. Massachusetts' Study Results (Realistic vs. Traditional)

The following statistical analysis considers the Massachusetts' test results for both tests ('Realistic' versus 'Traditional'). The analysis seeks to either accept or reject the following:

- The Null Hypothesis: '*That there is no difference in achievement levels between the realistic and traditional tests*';

- The Alternative Hypothesis: '*There is a difference in achievement levels between the realistic and traditional tests*'.

The confidence level is again set at 95% to account for any difference that may arise by chance – the alpha level of 0.05 goes some way towards eliminating this risk.

8.3.2.2.1. Massachusetts study Descriptive statistics (t=0, t=1)

Descriptive Statistics: Results

Variable	Test	N	N*	Mean	SE Mean	StDev	Minimum	Q1	Median	Q3
Results	0	71	0	79.67	1.90	16.02	33.33	67.50	83.33	92.50
	1	71	0	67.21	1.28	10.80	30.00	61.67	68.33	73.33

Variable	Test	Maximum
Results	0	100.00
	1.	91.67

Figure 35: Descriptive statistics of Massachusetts study (t=0, t=1)

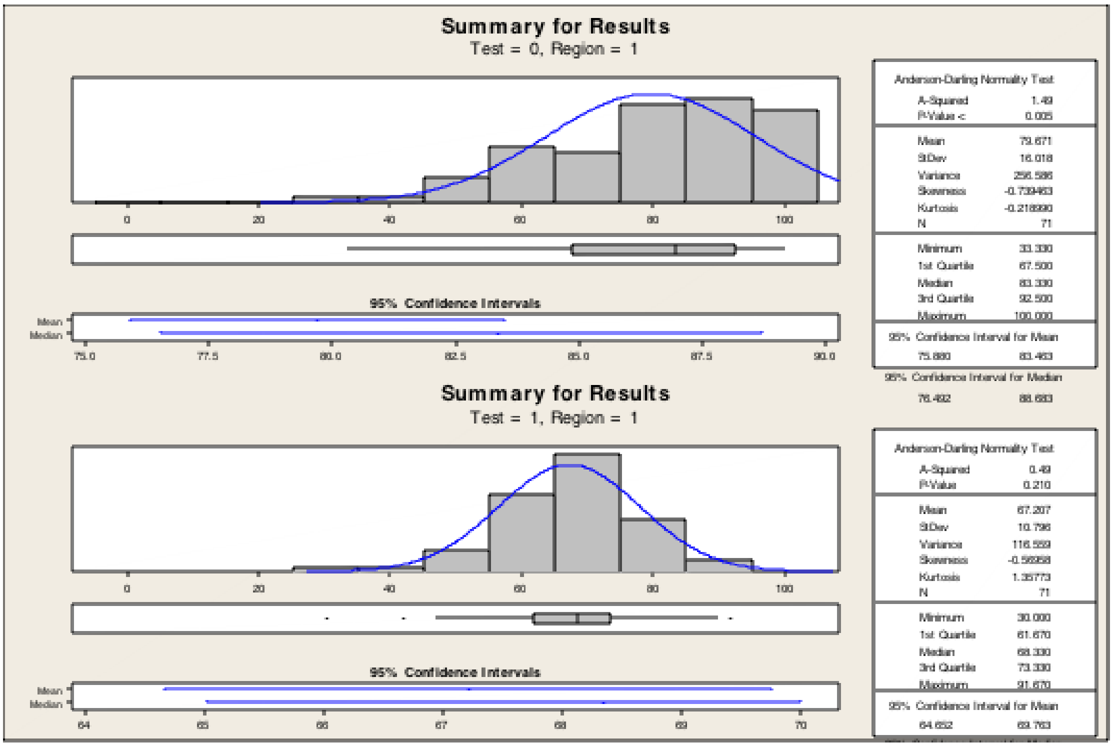


Figure 36: Graphical Summary of descriptive statistics of Massachusetts study (t=0, t=1)

8.3.2.2.2. Massachusetts study (t=0, t=1) hypothesis test (Two sample t-test)

The approach taken to determine the difference between the mean responses for the 'Traditional' and 'Realistic' tests at the 5% level of significance is as follows:

- Assess the data sets for equal variance;
- Conduct a two-sample t-test, with or without equal variance, based on the variance findings; and
- Conclude from the t-test whether the difference is significant at the appropriate level.

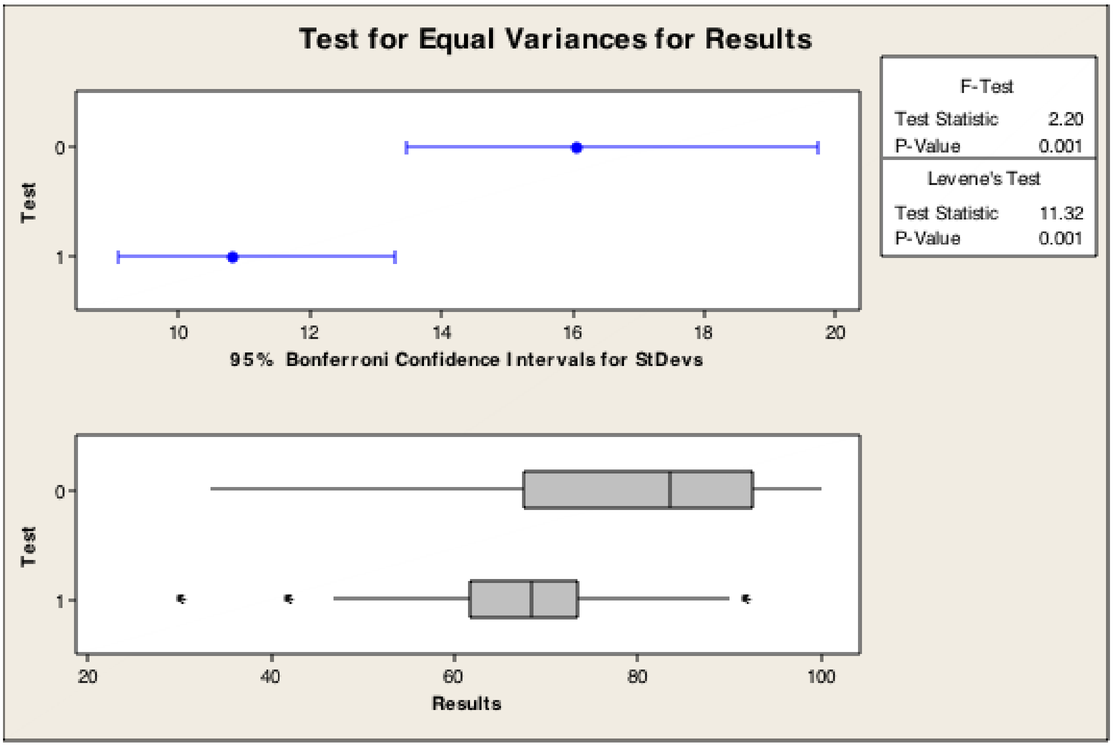


Figure 37: Graphical summary of test for equal variance of Massachusetts study (t=0, t=1)

Minitab provides a test statistic and P-value for both the F-Test and Levene's test. It is of note that both P-values of 0.001 and 0.001 are less than 0.05 (fig. 37). Therefore, this result is significant and there is sufficient evidence to conclude that the variances are not equal.

Two-Sample T-Test and CI: Results, Test

```
Two-sample T for Results
Test    N  Mean  StDev  SE Mean
0      71  79.7   16.0      1.9
1      71  67.2   10.8      1.3
Difference = mu (0) - mu (1)
Estimate for difference:  12.46
95% CI for difference:  (7.93, 17.00)
T-Test of difference = 0 (vs not =): T-Value = 5.44  P-Value = 0.000
DF = 122
```

Figure 38: Two sample t-test statistics Traditional vs. Realistic study

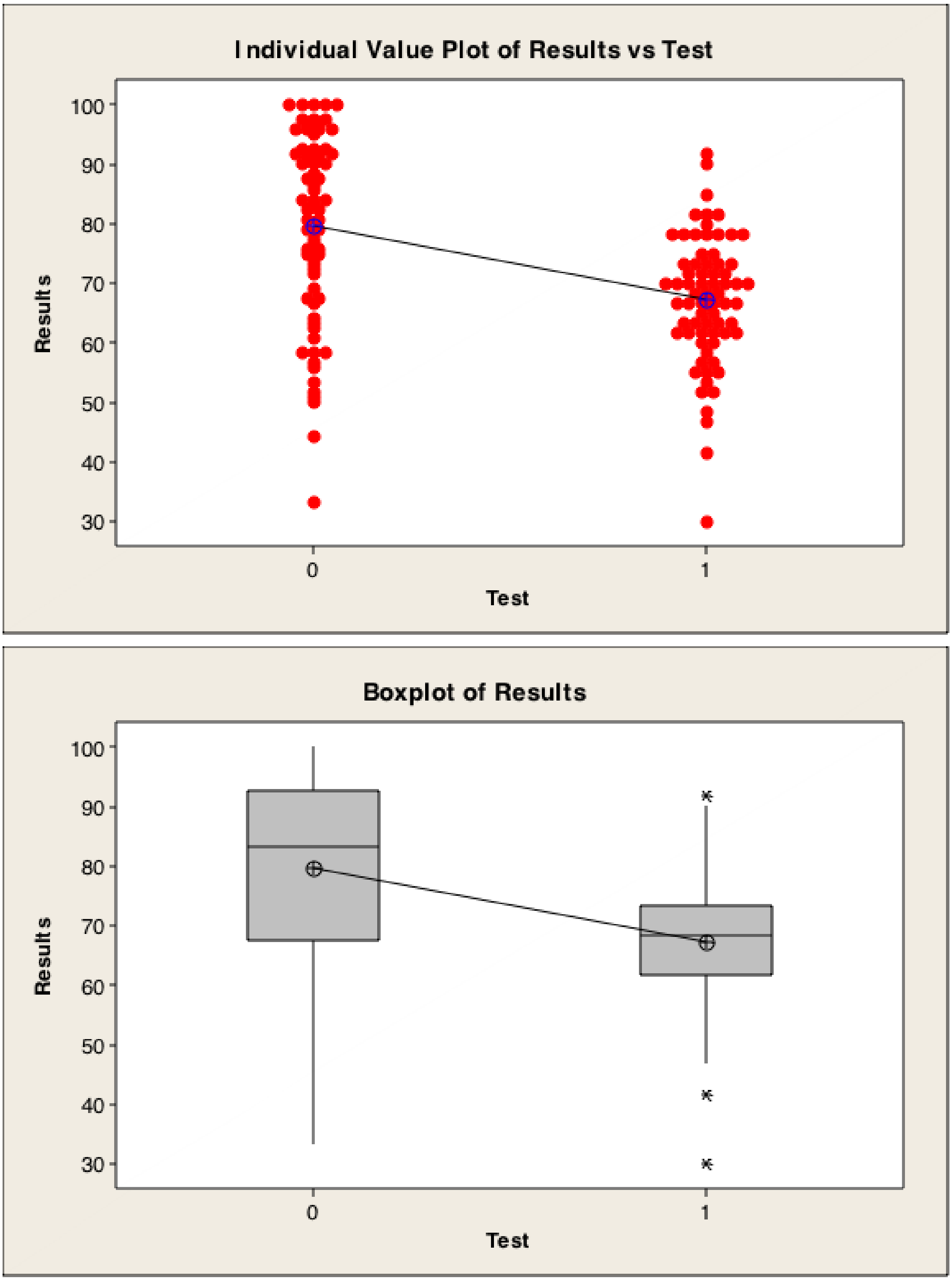


Figure 39: Two sample t-test Individual and box-plot graphics of Massachusetts study (t=0, t=1)

Based on the outcome of the two sample t-test, the author notes the estimate difference is 12.46, which would indicate that there is a significant difference between the performance responses of the 'Traditional' and 'Realistic' tests (fig. 38). Based on the p-value of 0.000, the author rejects the null hypothesis at the 5% level of significance and concludes that there is a difference between the results of the 'Traditional' and 'Realistic' tests for Massachusetts' students.

8.3.2.2.3. Massachusetts study (t=0, t=1) hypothesis test (One way ANOVA)

One-way ANOVA: Results versus Test

```
Source   DF     SS    MS      F      P
Test      1   5515  5515  29.56  0.000
Error   140  26120   187
Total   141  31635

S = 13.66   R-Sq = 17.43%   R-Sq(adj) = 16.84%

                         Individual 95% CIs For Mean Based on
                         Pooled StDev
Level   N   Mean  StDev   --+---------+---------+---------+-------
0      71  79.67  16.02                            (-----*------)
1      71  67.21  10.80   (-----*------)
                          --+---------+---------+---------+-------
                         65.0      70.0      75.0      80.0

Pooled StDev = 13.66

Tukey 95% Simultaneous Confidence Intervals
All Pairwise Comparisons among Levels of Test
Individual confidence level = 95.00%

Test = 0 subtracted from:
Test    Lower  Center  Upper  --------+---------+---------+---------+-
1      -17.00  -12.46  -7.93  (------*-------)
                              --------+---------+---------+---------+-
                                  -12.0      -6.0       0.0       6.0
```

Figure 40: One way ANOVA statistics of Massachusetts study (t=0, t=1)

The ANOVA output of immediate interest, as outlined above, is the F-test statistic. As the associated P-value is 0.000, one can reject the null hypothesis and conclude that the means of the two samples are statistically different.

Minitab also generates confidence intervals (CIs) for the mean of both tests; the confidence intervals for this study do not demonstrate an overlapping of the intervals for the test samples (fig. 40). Additionally, the post-hoc testing performed using the Tukey test provides confidence intervals for the difference in the pair of means under evaluation. From this analysis, it can be concluded that there is a significant difference between the performance of 'test 0' and 'test 1' as the interval goes from – 17.00 to – 7.93 and zero is not in the interval. In this instance, where the 'test 0' has been subtracted from 'test 1' and the resultant CI contains negative values, one can equate that 'test 1' had significantly lower results. The centre point of the CI is – 12.46 and is the estimated mean difference between the test groups.

In order to test that the ANOVA assumptions were not violated; a residual plot was created (fig. 41). The standard assumptions are as follows:

- The relationship between Y and X must be linear;
- The values are normally distributed; and
- The values of random error are independent.

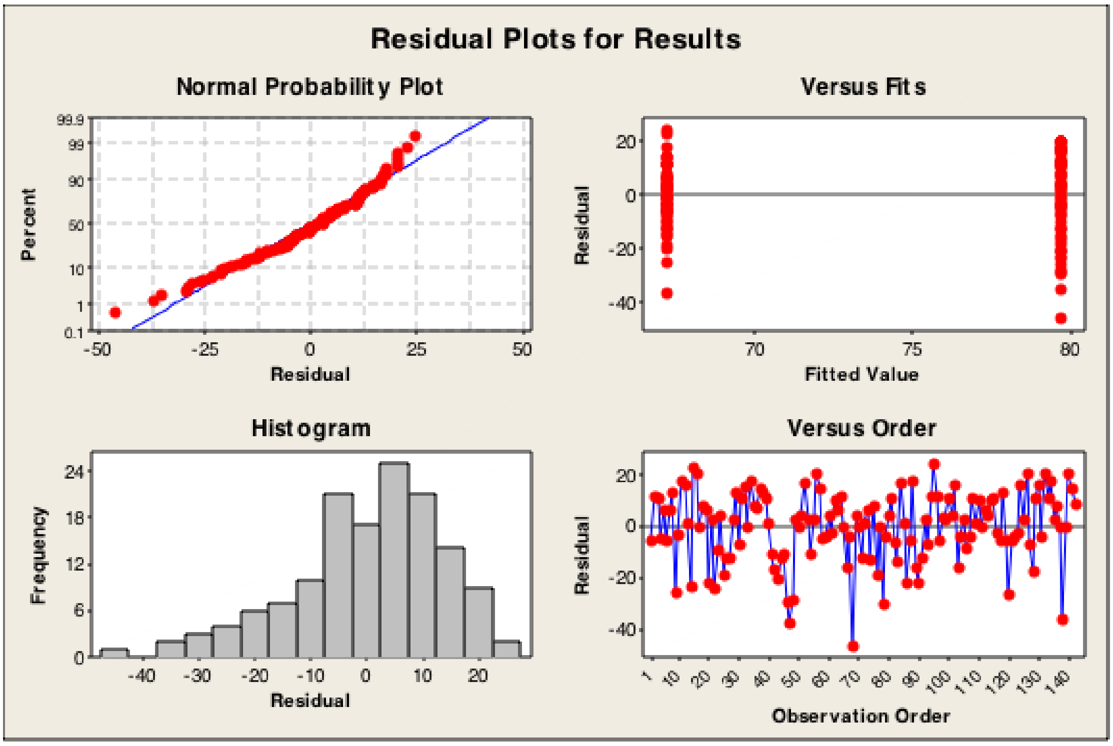


Figure 41: ANOVA Residual plot graphics of Massachusetts study (t=0, t=1)

Interpretation of the residual plots:

- The normal probability plot of residuals: Plots the residuals from each observation against the expected value of the residual had it come from a normal distribution. All plotted values appear in a straight line if the residuals are approximately normal (Mendenhall et al, 2009: 489). In this instance the values are in a reasonably straight line which suggests that there is no reason to state that the assumptions have been breached.
- The plot of residual versus fit: This graph plots the residual values against the expected value of the observation using the experimental design implemented. The plot is used to check for constant variance. The data appears to have a random pattern and as a result there is no reason to state that the assumptions have been breached.
- The histogram of residuals: The plot is used to check distribution. Viewed in conjunction with the normal plot, the histogram appears to support the normal distribution and as a result there is no reason to state that the assumptions have been breached.
- Residual versus overall plot: This plot is used to check for observational influence. The data appears to have a random pattern around the central line and as a result there is no reason to state that the assumptions have been breached.

The overall conclusion, based on the residual plots for the results, is that there is no evidence to suggest that standard assumptions of the regression have been violated.

8.3.2.2.4. Massachusetts study (t=0, t=1) (Correlations)

Correlations: Test, Results

```
Pearson correlation of Test and Results = -0.418
P-Value = 0.000
```

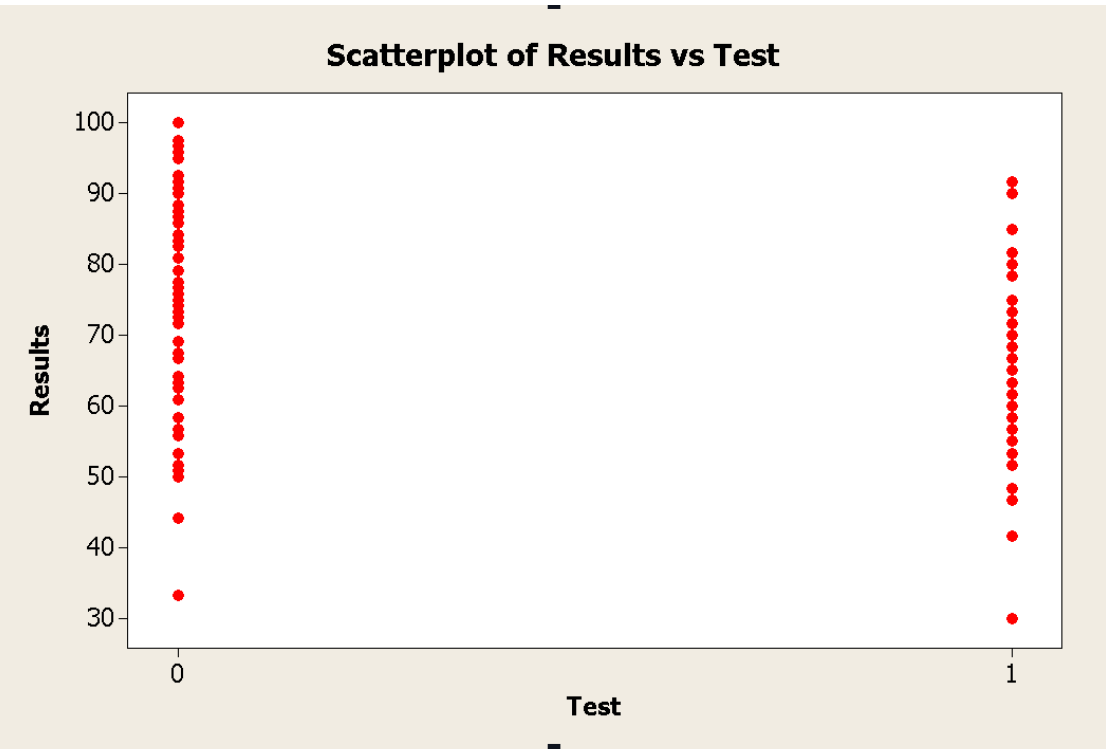


Figure 42: Correlation matrix plot graphics of Massachusetts study (t=0, t=1)

There is sufficient evidence to support the presence of moderate negative linear correlation between the two variable tests, as demonstrated by the r-value of -4.18 and a P-value of 0.00 (fig. 42).

8.3.2.2.5. Implication of Results (Massachusetts)

The results from Massachusetts indicate that there is a significant difference in student performance between the two tests. The author therefore rejects the null hypothesis: 'That there is no difference in achievement levels between the 'Traditional' test (test=0) and the 'Realistic' test (test=1)'. The significant difference in student performance between the ‘Traditional’ and ‘Realistic’ tests is indicated by a T-Value=5.44 and a P-value=0.000. As the P-value ≤0.05 this indicates a difference. As the T-Value is >0 it is apparent that the mean score in test=0 (Traditional) is higher than the mean score in test=1 (Realistic). Test 0 (M=79.7, SD=16.0) scored higher than Test 1 (M=67.2, SD=10.8) (fig. 38).

8.3.2.2.6. Implication of Irish Test Results in comparison to Massachusetts Test Results

	Number	Traditional (test=0)	Realistic (test=1)	p-value
Ireland	86	M=72.2, SD=22.6	M=46.5, SD=21.7	0.000
Massachusetts	71	M=79.7, SD=16	M=67.2, SD=10.8	0.000

Table 24: Irish and Massachusetts test results comparison

The above table (table 24) illustrates the scores from the t-test for Ireland and Massachusetts (‘Traditional’ test versus ‘Realistic’ test). Despite the fact that the p-value for both groups is the same (p=0.000), indicating that there is a difference in performance between student achievement in the two tests, it is interesting to note that the difference is far greater for the Irish group than for the group from Massachusetts. The author finds it interesting that despite the fact that one would assume that Irish students are more familiar with a traditional style assessment, due to the fact that it is based on actual assessment questions from the Junior Certificate examination which assesses the Irish junior cycle curriculum, students from Massachusetts outperformed the Irish students in both the ‘Traditional’ test and the ‘Realistic’ test. The mean difference for Irish students between both tests is 25.63, in favour of the traditional test (fig. 30). The mean difference for students from Massachusetts is significantly smaller with a mean of 12.46 (fig. 38). This would indicate that Irish students find it more difficult to transfer the mathematics they are familiar with to problem-solving scenarios where reflection and mathematical understanding are required.

From an Irish point of view the out-performance of Irish students by those from Massachusetts is a worrying indicator of Irish mathematical achievement. This is especially worrying when one considers that the students involved in the research, from both groups, have the same mean age (≈13.5). However it should be noted that the least able students from the Massachusetts school do not study 'Algebra 1' (the general mathematics course) so did not sit the examination. Saying this it should be noted that the majority of the Irish students involved in the study are following the higher level course (N=68) compared to the ordinary level course (N=18). Indeed it should also be stated that following the ordinary level course in mathematics at Junior Cycle is not an indicator of compromised mathematical ability. Many proficient students, who do not consider themselves particularly mathematically gifted, follow the ordinary level course. The author will consider the traditional test results from both groups in closer details.

8.3.2.3. Ireland versus Massachusetts (overall test results)

The following statistical analysis considers the Irish test results (test=0) versus the Massachusetts' tests results (test=1) over all tests (Irish test results for both 'Traditional' and 'Realistic' tests versus Massachusetts' test results for both 'Traditional' and 'Realistic'). The analysis seeks to either accept or reject the following:

- The Null Hypothesis: '*That there is no difference in achievement levels, in both the 'Traditional' and 'Realistic' tests, between Ireland and Massachusetts*';

- The Alternative Hypothesis: '*There is a difference in achievement levels, in both the 'Traditional' and 'Realistic' tests, between Ireland and Massachusetts*'.

The confidence level is again set at 95% to account for any difference that may arise by chance – the alpha level of 0.05 goes some way towards eliminating this risk.

8.3.2.3.1. Ireland versus Massachusetts Descriptive statistics (overall test results)

Descriptive Statistics: Results (Ireland versus Massachusetts for all)

Variable	Region	N	N*	Mean	SE Mean	StDev	Minimum	Q1	Median
Results	0	182	32	58.65	1.90	25.60	0.00	43.33	61.67
	1	142	0	73.44	1.26	14.98	30.00	63.33	72.91

Variable	Region	Maximum	Q3
Results	0	100.00	79.17
	1	100.00	84.38

Figure 43: Descriptive Statistics of Ireland versus Massachusetts study (overall test results)

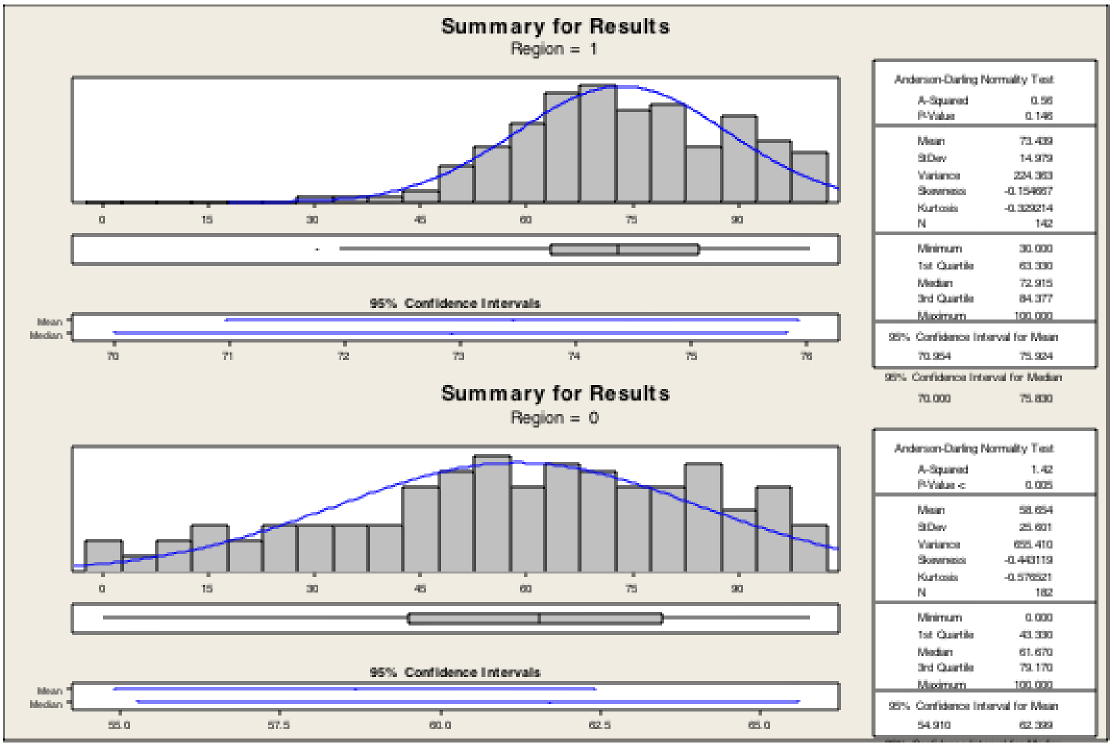


Figure 44: Graphical Summary of descriptive statistics for Ireland versus Massachusetts study (overall test results)

8.3.2.3.2. Ireland versus Massachusetts, overall test results hypothesis test (Two sample t-test)

The approach taken to determine the difference between the mean responses for the 'Traditional' and 'Realistic' tests at the 5% level of significance is as follows:

- Assess the data sets for equal variance;
- Conduct a two-sample t-test, with or without equal variance, based on the variance findings; and
- Conclude from the t-test whether the difference is significant at the appropriate level.

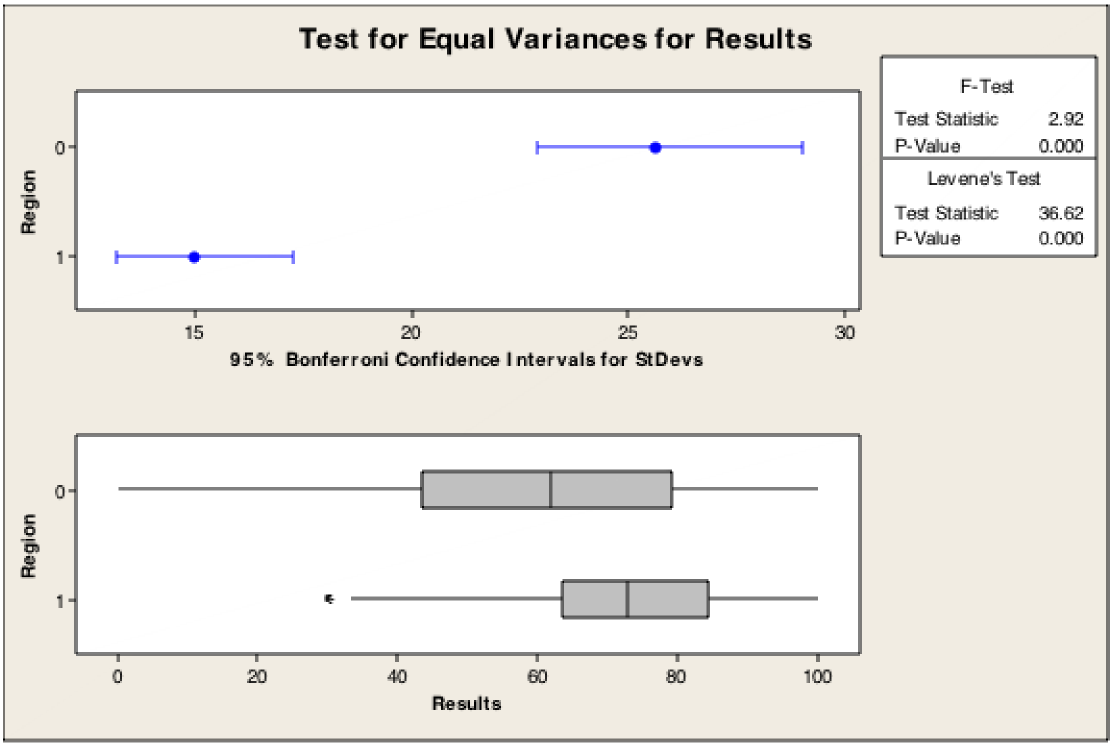


Figure 45: Graphical summary of test for equal variance for Ireland versus Massachusetts study (overall test results)

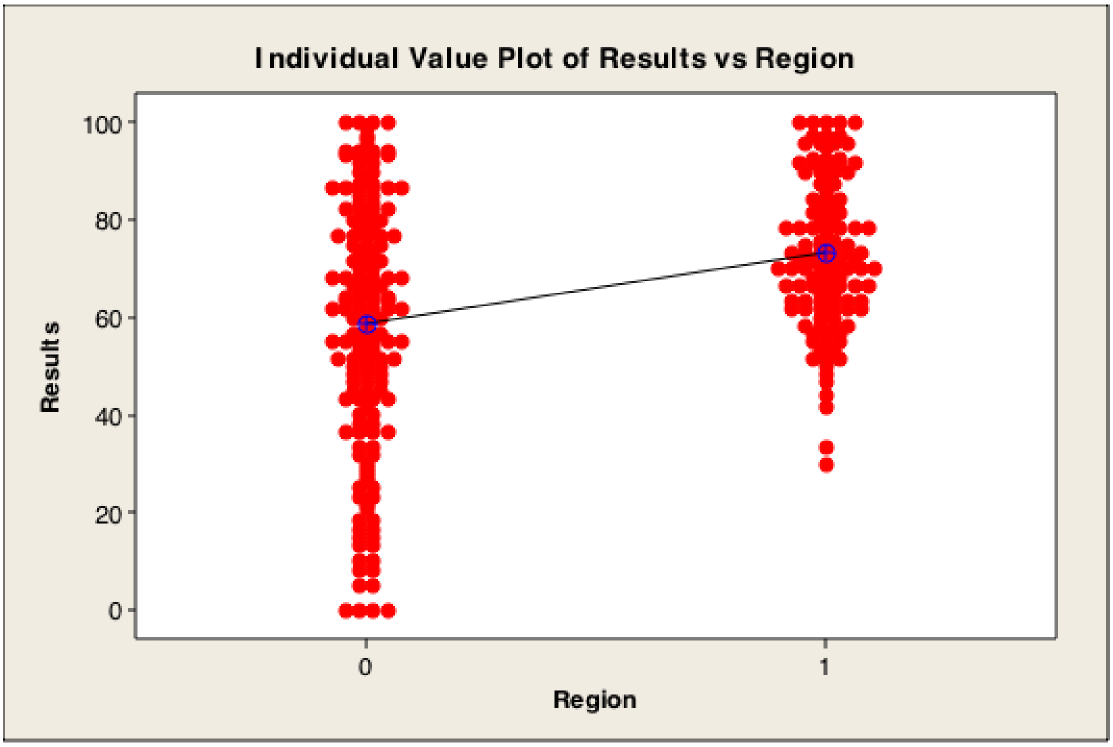
Minitab provides a test statistic and P-value for both the F-Test and Levene's test (fig. 45). It is of note that both P-values of 0.000 and 0.000 are less than

0.05. Therefore, this result is significant and there is sufficient evidence to conclude that the variances are not equal.

Two-Sample T-Test and CI: Results, Region

```
Two-sample T for Results
Region      N  Mean  StDev  SE Mean
0         182  58.7   25.6      1.9
1         142  73.4   15.0      1.3
Difference = mu (0) - mu (1)
Estimate for difference:  -14.79
95% CI for difference:  (-19.26, -10.31)
T-Test of difference = 0 (vs not =): T-Value = -6.50  P-Value = 0.000
DF = 300
```

Figure 46: sample t-test statistics Traditional vs. Realistic study



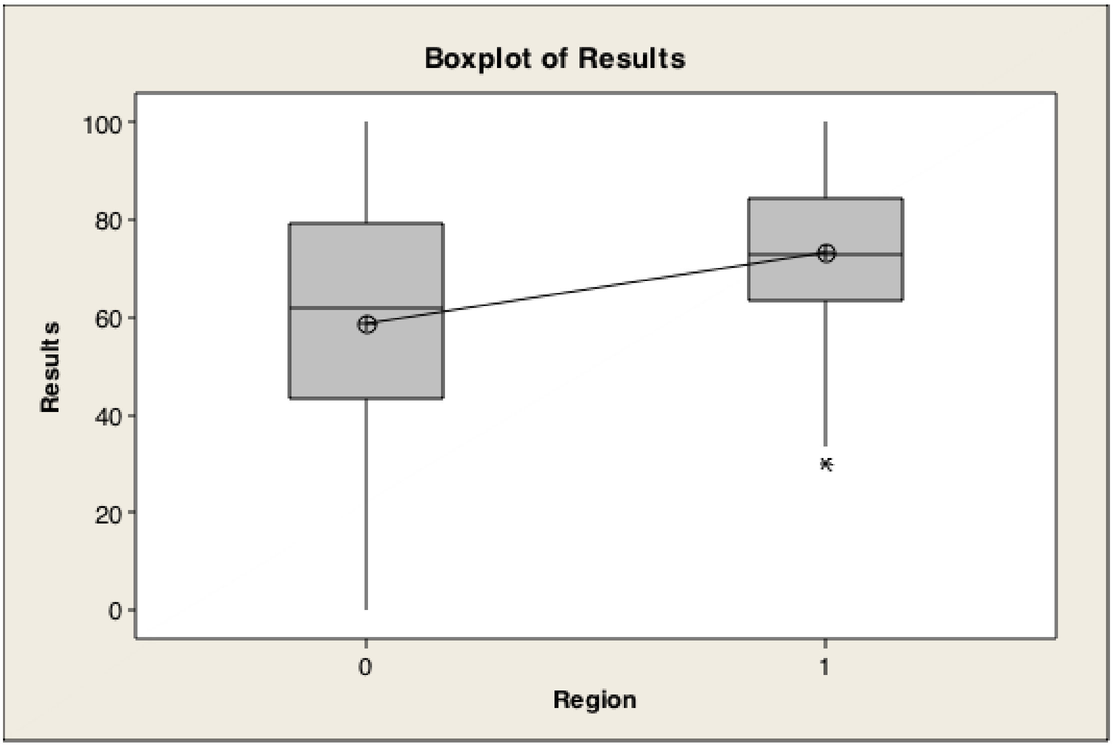


Figure 47: Ireland versus Massachusetts study two sample t-test Individual and box-plot graphics (overall test results)

Based on the outcome of the two sample t-test, the author notes the estimate difference is -14.79, which would indicate that there is a considerable difference between the performance responses of the Irish and Massachusetts tests (fig. 46). Based on the p-value of 0.000, the author rejects the null hypothesis at the 5% level of significance and concludes that there is a difference between the results of the Irish and Massachusetts overall test performance.

8.3.2.3.3. Ireland versus Massachusetts, overall test results hypothesis test (One way ANOVA)

One-way ANOVA: Results versus Region

Source	DF	SS	MS	F	P
Region	1	17437	17437	37.36	0.000
Error	322	150264	467		
Total	323	167701			

```
S = 21.60   R-Sq = 10.40%   R-Sq(adj) = 10.12%
```

```
                          Individual 95% CIs For Mean Based on
```

```
                                 Pooled StDev
Level     N   Mean  StDev  -------+---------+---------+---------+--
0       182  58.65  25.60  (----*----)
1       142  73.44  14.98                         (-----*-----)
                           -------+---------+---------+---------+--
                               60.0      66.0      72.0      78.0

Pooled StDev = 21.60


Tukey 95% Simultaneous Confidence Intervals
All Pairwise Comparisons among Levels of Region

Individual confidence level = 95.00%


Region = 0 subtracted from:

Region  Lower  Center  Upper  -------+---------+---------+---------+--
1       10.03   14.79  19.54                       (------*------)
                              -------+---------+---------+---------+--
                                   0.0       7.0      14.0      21.0
```

Figure 48: Ireland versus Massachusetts study One way ANOVA statistics (overall test results)

The ANOVA output of immediate interest, as outlined above, is the F-test statistic. As the associated P-value is 0.000, one can reject the null hypothesis and conclude that the means of the two samples are statistically different.

Minitab also generates confidence intervals (CIs) for the mean of both tests (fig. 47); the confidence intervals for this study do not demonstrate an overlapping of the intervals for the test samples. Additionally, the post-hoc testing performed using the Tukey test provides confidence intervals for the difference in the pair of means under evaluation. From this analysis, it can be concluded that there is a significant difference between the performance of ‘test 0’ and ‘test 1’ as the interval goes from 10.03 to 19.54 and zero is not in the interval. In this instance, where the ‘test 0’ has been subtracted from ‘test 1’ and the resultant CI contains positive values, one can equate that ‘test 1’ had significantly higher results. The centre point of the CI is 14.79 and is the estimated mean difference between the test groups.

In order to test that the ANOVA assumptions were not violated; a residual plot was created (fig. 48). The standard assumptions are as follows:

- The relationship between Y and X must be linear;
- The values are normally distributed; and
- The values of random error are independent.

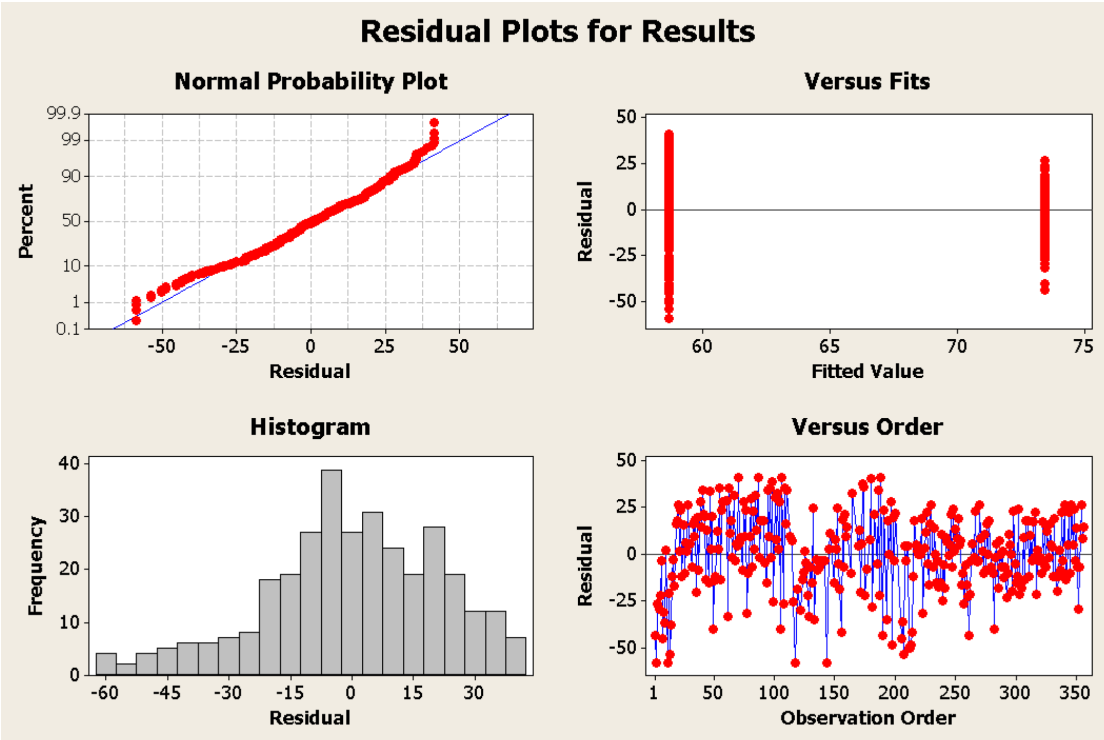


Figure 49: Ireland versus Massachusetts study One way ANOVA Residual plot graphics (overall test results)

Interpretation of the residual plots:

- The normal probability plot of residuals: Plots the residuals from each observation against the expected value of the residual had it come from a normal distribution. All plotted values appear in a straight line if the residuals are approximately normal (Mendenhall et al, 2009: 489). In this instance the values are in a reasonably straight line which suggests that there is no reason to state that the assumptions have been breached.
- The plot of residual versus fit: This graph plots the residual values against the expected value of the observation using the experimental design implemented. The plot is used to check for constant variance. The data appears to have a random pattern and as a result there is no reason to state that the assumptions have been breached.
- The histogram of residuals: The plot is used to check distribution. Viewed in conjunction with the normal plot, the histogram appears to support the normal distribution and as a result there is no reason to state that the assumptions have been breached.

- Residual versus overall plot: This plot is used to check for observational influence. The data appears to have a random pattern around the central line and as a result there is no reason to state that the assumptions have been breached.

The overall conclusion, based on the residual plots for the results, is that there is no evidence to suggest that standard assumptions of the regression have been violated.

8.3.2.3.4. Ireland versus Massachusetts, overall test results Correlations test

Correlations: Results, Region

```
Pearson correlation of Results and Region = 0.322
P-Value = 0.000
```

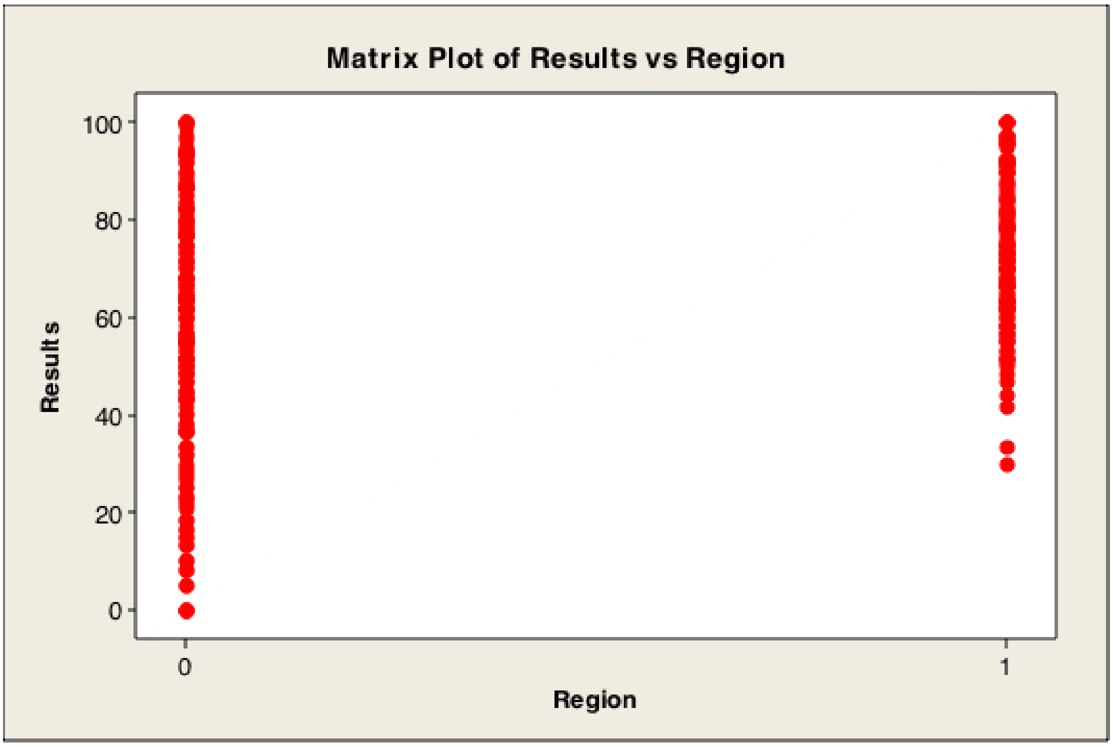


Figure 50: Ireland versus Massachusetts study Correlation matrix plot graphics (overall test results)

There is sufficient evidence to support the presence of a very slight positive correlation between the two variable tests, as demonstrated by the r-value of 0.322 and a P-value of 0.000 (fig. 49).

8.3.2.3.5. Implications of Results (Ireland versus Massachusetts for all).

The t-test for all tests finds a significant difference in performance between Ireland and Massachusetts with a t-value=-6.5 and a p-value=0.000 (fig. 46). The negative t-value indicates that the students in Massachusetts scored significantly higher over both tests than the Irish students. The fact that the p-value is less than 0.05 indicates a significant difference between the scores of each group. As a result the null hypothesis '*that there is no difference in achievement between Ireland and Massachusetts'* is rejected. Irish test results, test=0, (M=58.7, SD=25.6) scored lower than the Massachusetts test results, test=1, (M=73.4, SD 15). The mean difference is 14.59 in favour of the Massachusetts results. The Pearson Correlation of 0.322 indicates that there is a weak positive correlation between test performance in Ireland and test performance in Massachusetts.

8.3.2.4. Ireland Higher Level versus Massachusetts (Traditional test)

The following statistical analysis considers the Irish test results for Higher level students (test=0) versus the Massachusetts' tests results (test=1) for the 'Traditional' test only. The author was interested in comparing the 'Traditional' test for both groups as this is the test that is directly based on the Junior Certificate examination. As a result it provides an interesting indicator of how Irish students perform in the test that they are being prepared for on a daily basis in comparison with students from Massachusetts who are unfamiliar with the Irish assessment style. The author eliminated the Irish Ordinary level results from this analysis as the Massachusetts results do not include work from their least mathematically able students (due to the fact that they do not study the ‘Algebra 1’ course which was a timetabling requirement by the school itself). The analysis seeks to either accept or reject the following:

- The Null Hypothesis: ‘*That there is no difference in achievement levels in the ‘Traditional’ test between Irish Higher level students (test=0) and students from Massachusetts (test=1)’;*

- The Alternative Hypothesis: ‘*There is a difference in achievement levels in the ‘Traditional’ test between Irish Higher level students (test=0) and students from Massachusetts (test=1).*

The confidence level is again set at 95% to account for any difference that may arise by chance – the alpha level of 0.05 goes some way towards eliminating this risk.

8.3.2.4.1. Ireland Higher Level versus Massachusetts Descriptive statistics (Traditional test)

Descriptive Statistics: Results (Ireland versus Massachusetts traditional test)

Variable	Region	N	N*	Mean	SE Mean	StDev	Minimum	Q1	Median	Q3
Results	0	68	11	80.42	1.69	13.91	40.00	73.54	82.50	91.67
	1	71	0	79.67	1.90	16.02	33.33	67.50	83.33	92.50

Variable	Region	Maximum
Results	0	100.00

Figure 51: Descriptive Statistics for Ireland Higher Level versus Massachusetts study (Traditional test results)

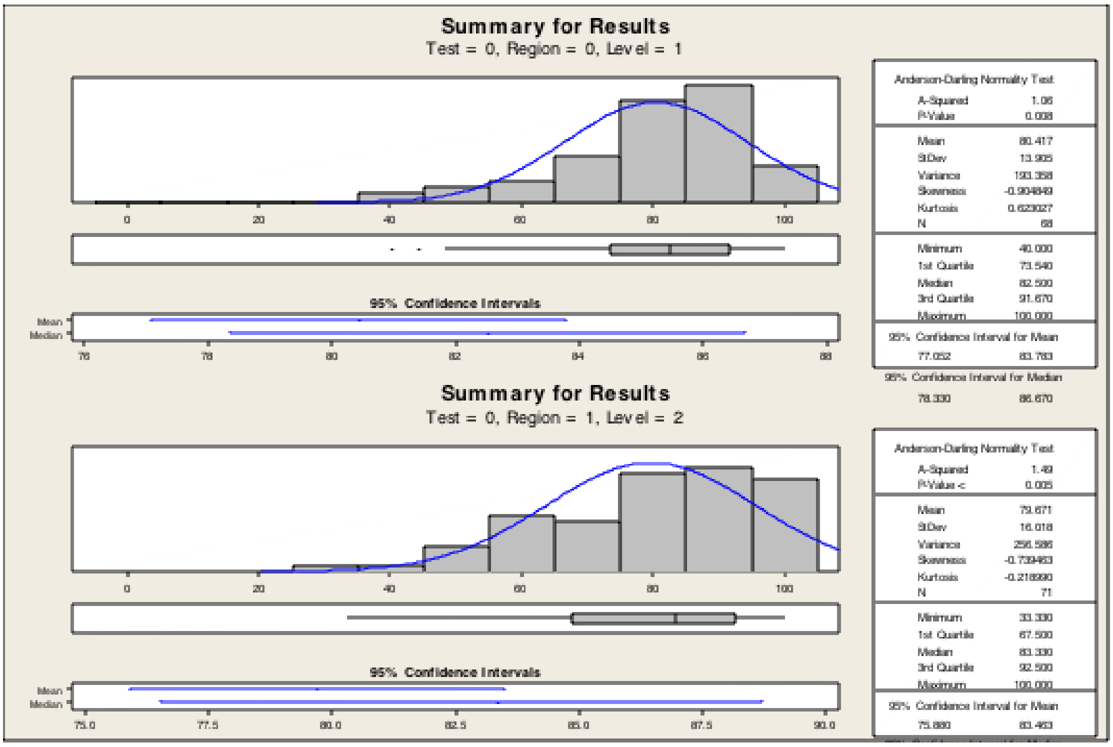


Figure 52: Graphical Summary of descriptive statistics for Ireland higher level versus Massachusetts study (Traditional test results)

8.3.2.4.2. Ireland Higher Level versus Massachusetts hypothesis test traditional test (Two sample t-test)

The approach taken to determine the difference between the mean responses for the ‘Traditional’ and ‘Realistic’ tests at the 5% level of significance is as follows:

- Assess the data sets for equal variance;
- Conduct a two-sample t-test, with or without equal variance, based on the variance findings; and
- Conclude from the t-test whether the difference is significant at the appropriate level.

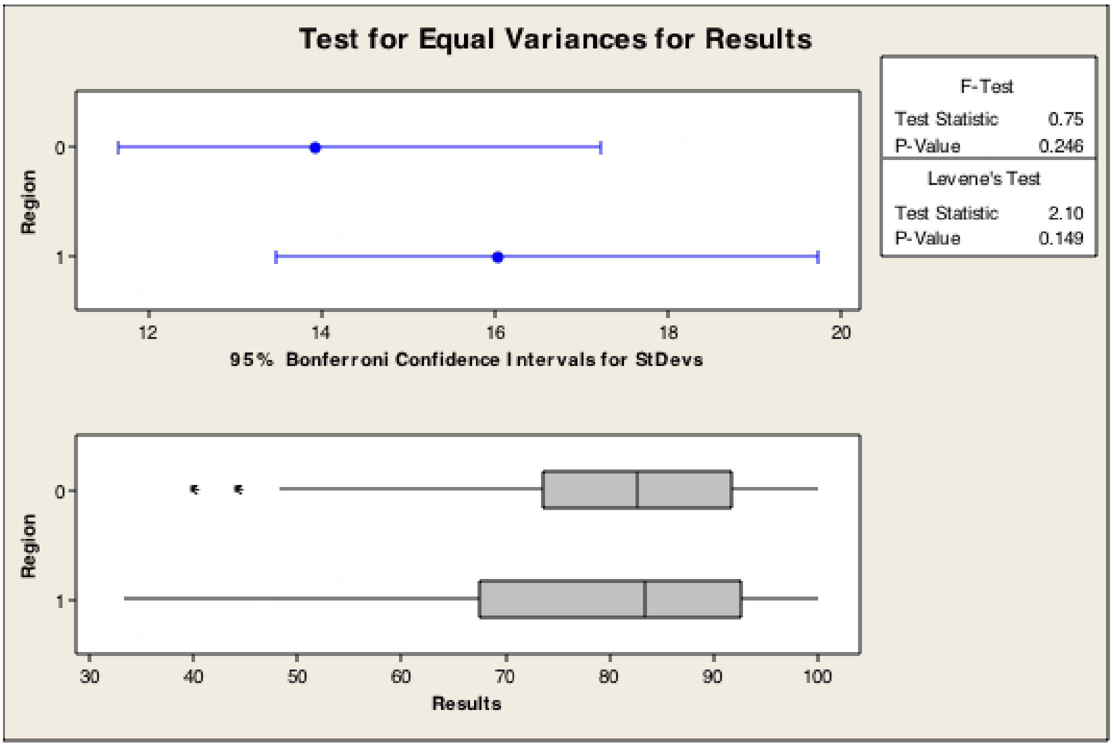


Figure 53: Graphical summary of test for equal variance for Ireland higher level versus Massachusetts study (Traditional test results)

Minitab provides a test statistic and P-value for both the F-Test and Levene's test. It is of note that both P-values of 0.246 and 0.149 are greater than 0.05 (fig. 53). Therefore, this result is not significant and there is sufficient evidence to conclude that the variances are equal.

Two-Sample T-Test and CI: Results, Region

```
Two-sample T for Results
Region    N  Mean  StDev  SE Mean
0        68  80.4   13.9      1.7
1        71  79.7   16.0      1.9
Difference = mu (0) - mu (1)
Estimate for difference:  0.75
95% CI for difference:  (-4.29, 5.79)
T-Test of difference = 0 (vs not =): T-Value = 0.29  P-Value = 0.770
DF = 137
Both use Pooled StDev = 15.0221
```

Figure 54: Two sample t-test statistics of Ireland higher level versus Massachusetts study (Traditional test results)

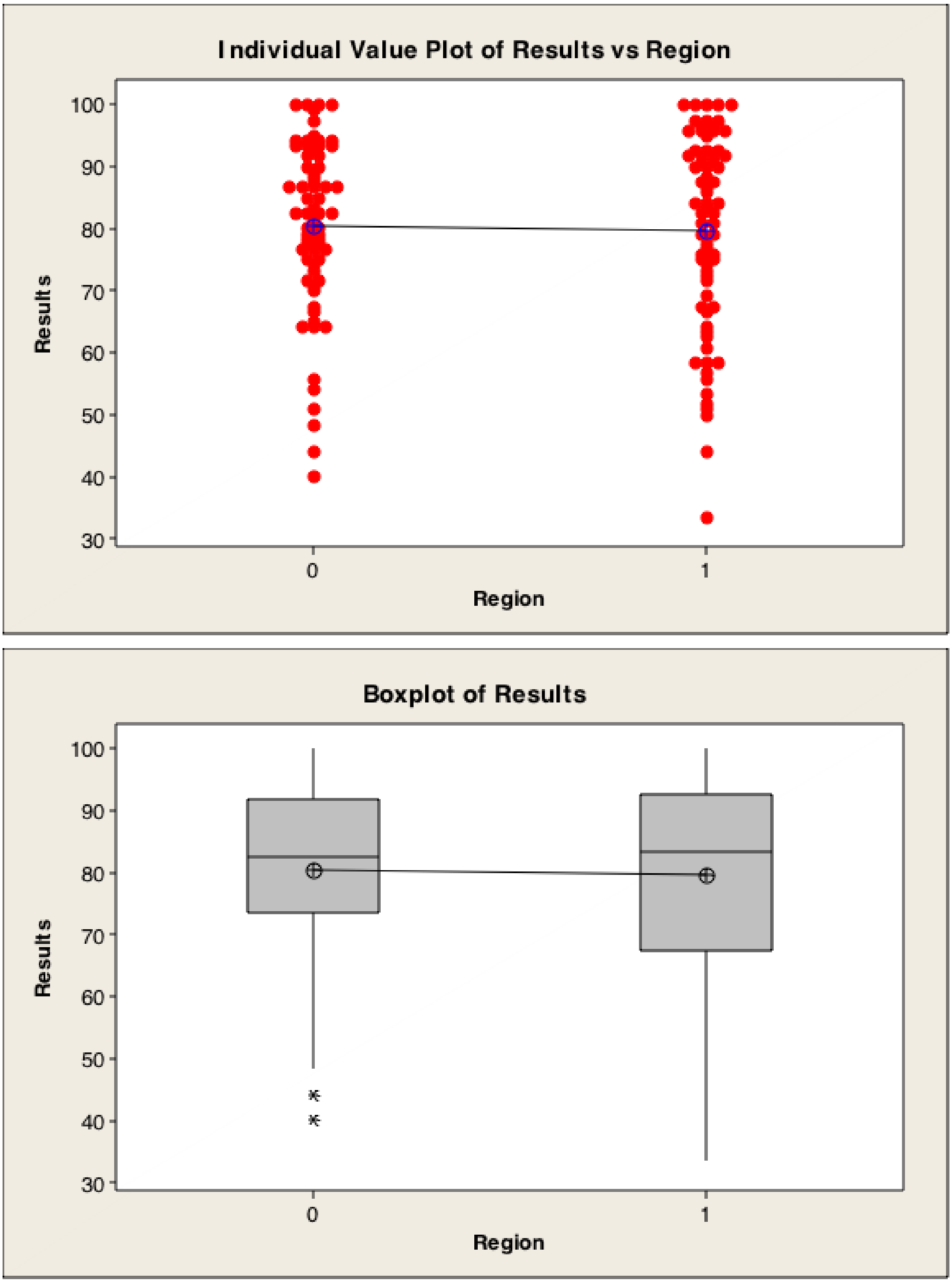


Figure 55: Two sample t-test Individual and box-plot graphics of Ireland higher level versus Massachusetts study (Traditional test results)

Based on the outcome of the two sample t-test, the author notes the estimate difference is 0.75, which would indicate that there is not a significant difference between the performance responses of the Irish higher level students and the Massachusetts' tests. Based on the p-value of 0.770, the author fails to reject the null hypothesis at the 5% level of significance and concludes that there is no significant difference between the results of the Irish higher level students and the students in Massachusetts.

8.3.2.4.3. Ireland Higher Level versus Massachusetts hypothesis test traditional test (One-way ANOVA)

One-way ANOVA: Results versus Region

```
Source    DF     SS   MS     F      P
Region     1     19   19  0.09  0.770
Error    137  30916  226
Total    138  30935
S = 15.02   R-Sq = 0.06%   R-Sq(adj) = 0.00%

                          Individual 95% CIs For Mean Based on
                          Pooled StDev
Level   N   Mean  StDev  ---------+---------+---------+---------+
0      68  80.42  13.91     (-----------------*-----------------)
1      71  79.67  16.02  (-----------------*-----------------)
                         ---------+---------+---------+---------+
                               78.0      80.0      82.0      84.0

Pooled StDev = 15.02

Tukey 95% Simultaneous Confidence Intervals
All Pairwise Comparisons among Levels of Region
Individual confidence level = 95.00%
Region = 0 subtracted from:
Region  Lower  Center  Upper  ---------+---------+---------+---------+
1       -5.79   -0.75   4.29  (----------------*---------------)
                              ---------+---------+---------+---------+
                                    -3.0       0.0       3.0       6.0
```

Figure 56: One way ANOVA statistics of Ireland higher level versus Massachusetts study (Traditional test results)

The ANOVA output of immediate interest, as outlined above, is the F-test statistic. As the associated P-value is 0.770, one can fail to reject the null hypothesis and conclude that the means of the two samples are not statistically different.

Minitab also generates confidence intervals (CIs) for the mean of both tests; the confidence intervals for this study do not demonstrate an overlapping of the intervals for the test samples (fig. 56). Additionally, the post-hoc testing performed using the Tukey test provides confidence intervals for the difference in the pair of means under evaluation. From this analysis, it can be concluded that there is no significant difference between the performance of 'test 0' and 'test 1' as the interval goes from – 5.79 to 4.29 and zero is in the interval. The centre point of the CI is – 0.75 and is the estimated mean difference between the test groups.

In order to test that the ANOVA assumptions were not violated; a residual plot was created (fig. 57). The standard assumptions are as follows:

- The relationship between Y and X must be linear;
- The values are normally distributed; and
- The values of random error are independent.

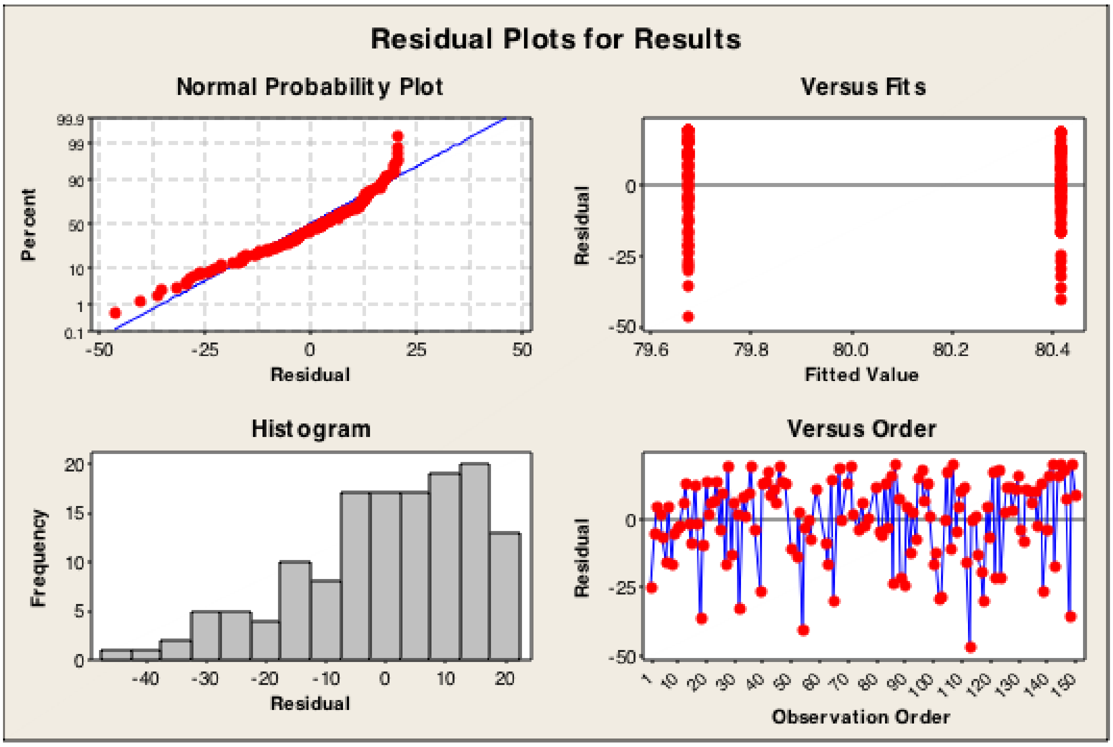


Figure 57: One way ANOVA Residual plot graphics (t=0, t=1) statistics for Ireland higher level versus Massachusetts study (Traditional test results)

Interpretation of the residual plots:

- The normal probability plot of residuals: Plots the residuals from each observation against the expected value of the residual had it come from a normal distribution. All plotted values appear in a straight line if the residuals are approximately normal (Mendenhall et al, 2009: 489). In this instance the values are in a reasonably straight line which suggests that there is no reason to state that the assumptions have been breached.

- The plot of residual versus fit: This graph plots the residual values against the expected value of the observation using the experimental design implemented. The plot is used to check for constant variance. The data appears to have a random pattern and as a result there is no reason to state that the assumptions have been breached.
- The histogram of residuals: The plot is used to check distribution. Viewed in conjunction with the normal plot, the histogram appears to support the normal distribution and as a result there is no reason to state that the assumptions have been breached.
- Residual versus overall plot: This plot is used to check for observational influence. The data appears to have a random pattern around the central line and as a result there is no reason to state that the assumptions have been breached.

The overall conclusion, based on the residual plots for the results, is that there is no evidence to suggest that standard assumptions of the regression have been violated.

8.3.2.4.4. Ireland Higher Level versus Massachusetts hypothesis test traditional test (Correlation study)

Correlations: Results, Region

```
Pearson correlation of Results and Region = -0.025
P-Value = 0.770
```

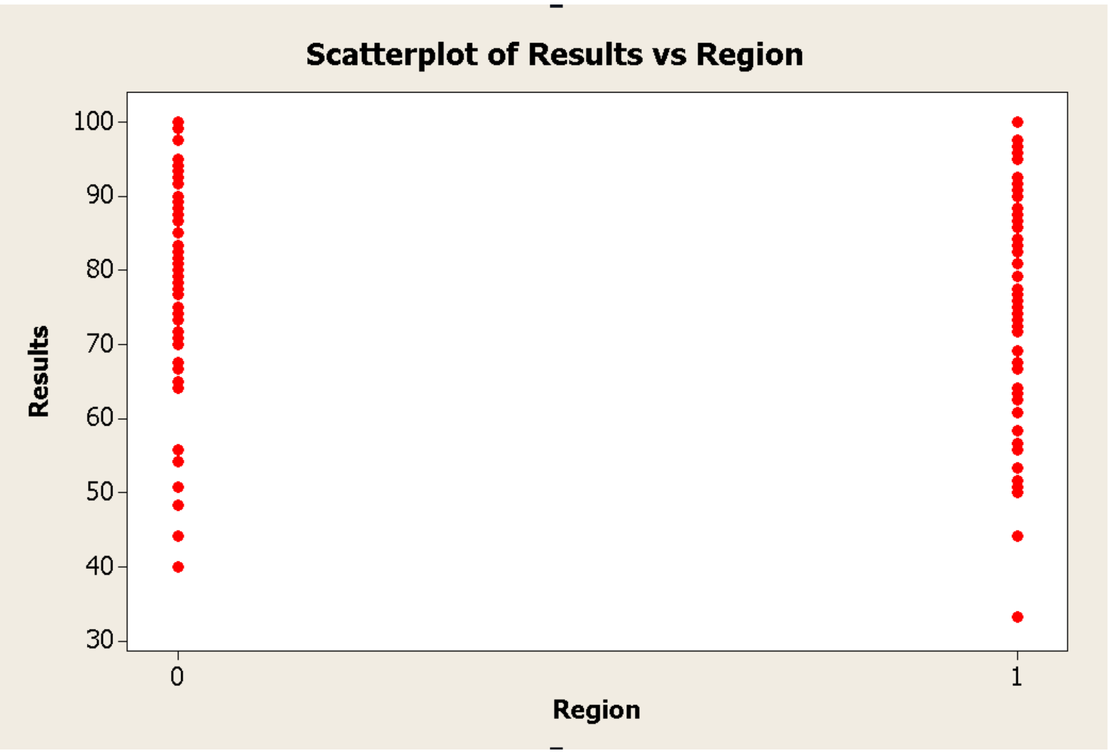


Figure 58: Correlation matrix plot graphics of Ireland Higher Level versus Massachusetts traditional test

There is sufficient evidence to support the lack of presence of linear correlation between the two variable tests, as demonstrated by the r-value of -0.025 and a P-value of 0.770 (fig. 58).

8.3.2.4.5. Implication of Results (H.L. Irish results vs. Massachusetts)

The t-test analysis of the Irish Higher level results versus the Massachusetts results for the 'Traditional' test show no difference. The p-value is 0.770, as this is greater than the alpha-value of 0.5 it suggests there is no difference in student performance between regions. Therefore we fail to reject the null hypothesis '*That there is no difference in achievement levels in the 'Traditional' test between Irish Higher level students (test=0) and students from Massachusetts (test=1)'.* Irish Higher level students, test=0, (M=80.4, S.D.=13.9) score marginally higher than students from Massachusetts, test=1, (M=79.7, S.D.=16.0). There is a very small difference between the mean scores for each region of 0.75. The author is surprised that the students from Massachusetts performed at such a comparable level with Irish Higher level students. One would accept that in an assessment system that students are being especially prepared for they would hold a distinct advantage. It is important to note that not only is the 'Traditional' test based on the Irish Junior Certificate curriculum, but the test questions are directly selected from the Junior Certificate assessment. In contrast the author remain reasonably unfamiliar with the content in the Massachusetts Algebra 1 curriculum, outside of the main topics the students cover while studying this eighth grade course. This would suggest, to some extent, that students from Massachusetts' are demonstrating a greater mathematical ability as the content, the format style, the presentation of the questions, the wording etc. are not familiar to students in the United States to the same extent as they would be to Irish students.

8.3.2.5. Ireland Higher Level versus Massachusetts (Realistic Tests)

The following statistical analysis considers the Irish test results for Higher level students (test=0) versus the Massachusetts' tests results (test=1)for the 'Realistic' test only. The author eliminated the Irish Ordinary level results from this analysis as the Massachusetts results do not include work from the least mathematically able students (due to the fact that they do not study the 'Algebra 1' course which was a timetabling requirement by the school itself). The analysis seeks to either accept or reject the following:

- The Null Hypothesis: '*That there is no difference in achievement levels in the 'Traditional' test between Irish Higher level students (test=0) and students from Massachusetts (test=1)*';

- The Alternative Hypothesis: '*There is a difference in achievement levels in the 'Traditional' test between Irish Higher level students (test=0) and students from Massachusetts (test=1).*

The confidence level is again set at 95% to account for any difference that may arise by chance – the alpha level of 0.05 goes some way towards eliminating this risk.

8.3.2.5.1. Ireland Higher Level versus Massachusetts Descriptive statistics (Realistic test)

Descriptive Statistics: Results (Ireland versus Massachusetts Realistic test)

Variable	Region	N	N*	Mean	SE Mean	StDev	Minimum	Q1	Median	Q3
Results	0	68	11	53.11	2.23	18.41	10.00	43.33	55.00	66.25
	1	71	0	67.21	1.28	10.80	30.00	61.67	68.33	73.33

Variable	Region	Maximum
Results	0	96.67
	1.	91.67

Figure 59: Descriptive Statistics for Ireland Higher Level versus Massachusetts study (Realistic test results)

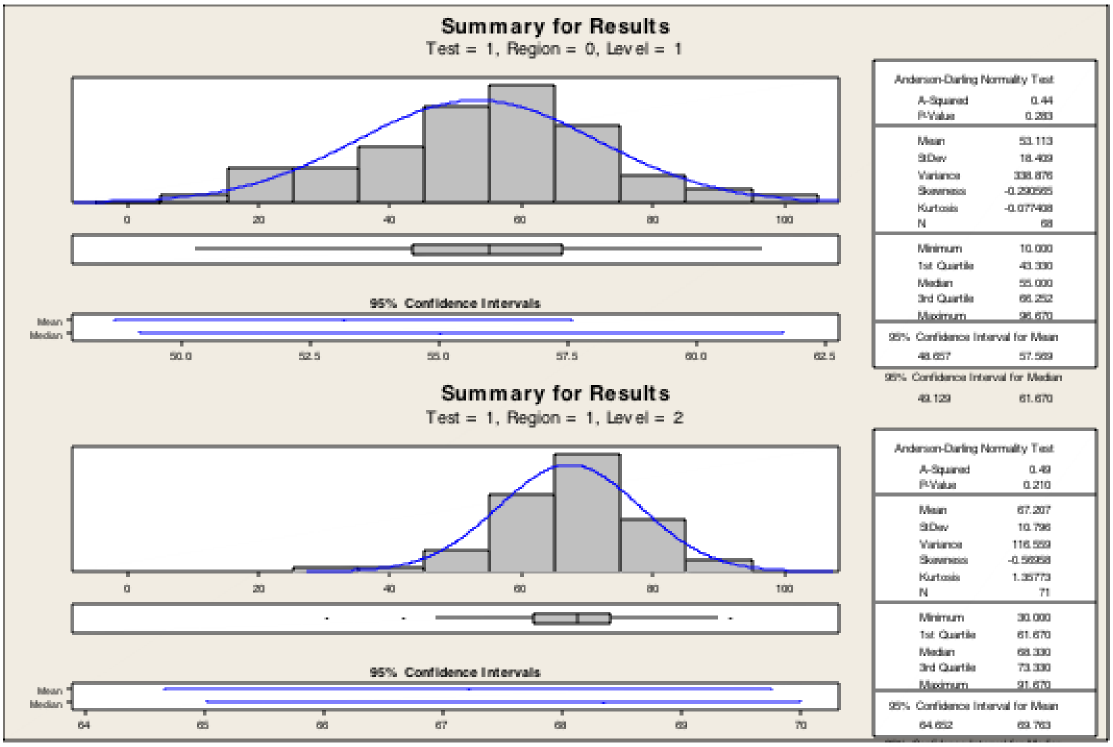


Figure 60: Graphical summary of descriptive Statistics for Ireland Higher Level versus Massachusetts study (Realistic test results)

8.3.2.5.2. Ireland Higher Level versus Massachusetts hypothesis test - two sample t-test (Realistic test))

The approach taken to determine the difference between the mean responses for the 'Traditional' and 'Realistic' tests at the 5% level of significance is as follows:

- Assess the data sets for equal variance;
- Conduct a two-sample t-test, with or without equal variance, based on the variance findings; and
- Conclude from the t-test whether the difference is significant at the appropriate level.

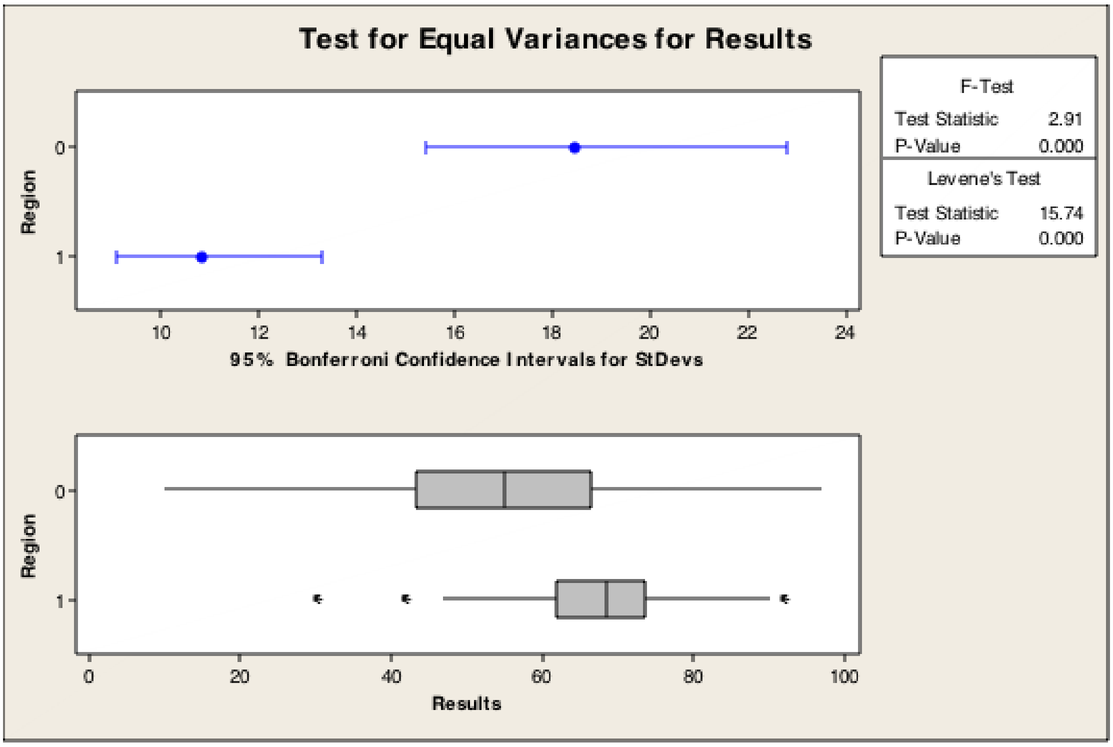


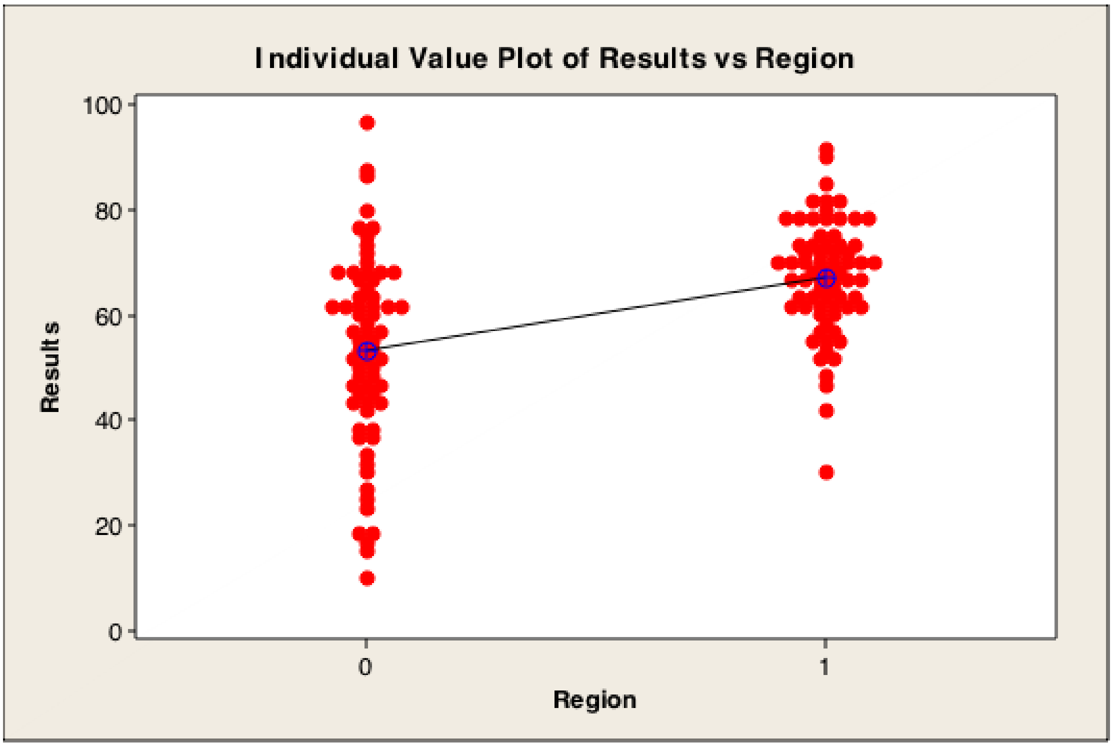
Figure 61: Graphical summary of test for equal variance for Ireland Higher Level versus Massachusetts study (Realistic test results)

Minitab provides a test statistic and P-value for both the F-Test and Levene's test. It is of note that both P-values of 0.000 and 0.000 are less than 0.05 (fig. 61). Therefore, this result is significant and there is sufficient evidence to conclude that the variances are not equal.

Two-Sample T-Test and CI: Results, Region

```
Two-sample T for Results
Region    N  Mean  StDev  SE Mean
0        68  53.1   18.4      2.2
1        71  67.2   10.8      1.3
Difference = mu (0) - mu (1)
Estimate for difference:  -14.09
95% CI for difference:  (-19.20, -8.99)
T-Test of difference = 0 (vs not =): T-Value = -5.48  P-Value = 0.000
DF = 107
```

Figure 62: Two sample t-test statistics for Ireland Higher Level versus Massachusetts study (Realistic test results)



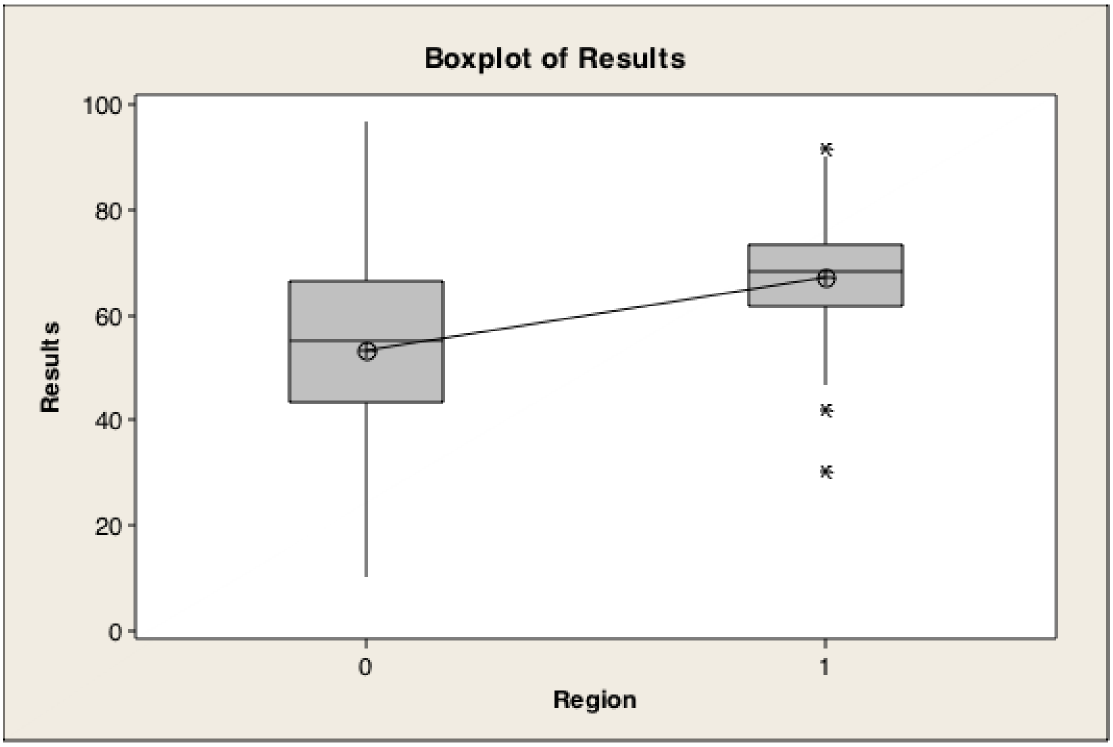


Figure 63: Two sample t-test Individual and box-plot graphics for Ireland Higher Level versus Massachusetts study (Realistic test results)

Based on the outcome of the two sample t-test, the author notes the estimate difference is -14.09, which would indicate that there is a difference between the performance responses of the Irish, higher level students and the

Massachusetts' students on the 'Realistic' test (fig. 62). Based on the p-value of 0.000, the author rejects the null hypothesis at the 5% level of significance and concludes that there is a difference between the results in the 'Realistic' test between the Irish, higher level students and the students in Massachusetts.

8.3.2.5.3. Ireland Higher Level versus Massachusetts hypothesis test: one-way ANOVA (Realistic test))

One-way ANOVA: Results versus Region

```
Source   DF     SS    MS      F      P
Region    1   6900  6900  30.63  0.000
Error   137  30864   225
Total   138  37763

S = 15.01   R-Sq = 18.27%   R-Sq(adj) = 17.67%

                         Individual 95% CIs For Mean Based on
                         Pooled StDev
Level   N   Mean  StDev  -------+---------+---------+---------+--
0      68  53.11  18.41  (-----*-----)
1      71  67.21  10.80                        (-----*-----)
                         -------+---------+---------+---------+--
                             54.0      60.0      66.0      72.0

Pooled StDev = 15.01

Tukey 95% Simultaneous Confidence Intervals
All Pairwise Comparisons among Levels of Region

Individual confidence level = 95.00%

Region = 0 subtracted from:

Region  Lower  Center  Upper  -------+---------+---------+---------+--
1        9.06   14.09  19.13                      (------*------)
                              -------+---------+---------+---------+--
                                   0.0       7.0      14.0      21.0
```

Figure 64: One way ANOVA statistics for Ireland Higher Level versus Massachusetts study (Realistic test results)

The ANOVA output of immediate interest, as outlined above, is the F-test statistic. As the associated P-value is 0.000, one can reject the null hypothesis and conclude that the means of the two samples are statistically different.

Minitab also generates confidence intervals (CIs) for the mean of both tests; the confidence intervals for this study do not demonstrate an overlapping of the intervals for the test samples (fig. 64). Additionally, the post-hoc testing performed using the Tukey test provides confidence intervals for the difference in the pair of means under evaluation. From this analysis, it can be concluded that there is a significant difference between the performance of ‘test 0’ and ‘test 1’ as the interval goes from 9.06 to 19.13 and zero is not in the interval. In this instance, where the ‘test 0’ has been subtracted from ‘test 1’ and the resultant CI contains positive values, one can equate that ‘test 1’ had significantly higher results. The centre point of the CI is 14.09 and is the estimated mean difference between the test groups.

In order to test that the ANOVA assumptions were not violated; a residual plot was created (fig. 65). The standard assumptions are as follows:

- The relationship between Y and X must be linear;
- The values are normally distributed; and
- The values of random error are independent.

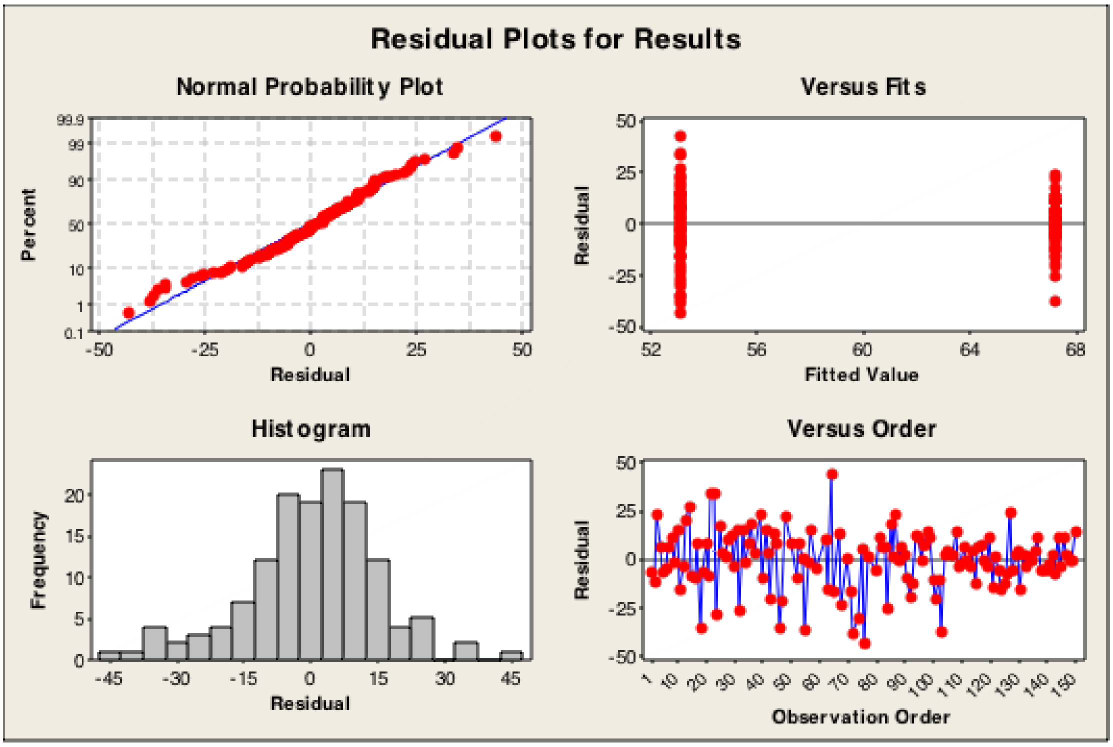


Figure 65: One way ANOVA Residual plot graphics for Ireland Higher Level versus Massachusetts study (Realistic test results)

Interpretation of the residual plots:

- The normal probability plot of residuals: Plots the residuals from each observation against the expected value of the residual had it come from a normal distribution. All plotted values appear in a straight line if the residuals are approximately normal (Mendenhall et al, 2009: 489). In this instance the values are in a reasonably straight line which suggests that there is no reason to state that the assumptions have been breached.
- The plot of residual versus fit: This graph plots the residual values against the expected value of the observation using the experimental design implemented. The plot is used to check for constant variance. The data appears to have a random pattern and as a result there is no reason to state that the assumptions have been breached.
- The histogram of residuals: The plot is used to check distribution. Viewed in conjunction with the normal plot, the histogram appears to support the normal distribution and as a result there is no reason to state that the assumptions have been breached.
- Residual versus overall plot: This plot is used to check for observational influence. The data appears to have a random pattern around the central line and as a result there is no reason to state that the assumptions have been breached.

The overall conclusion, based on the residual plots for the results, is that there is no evidence to suggest that standard assumptions of the regression have been violated.

8.3.2.5.4. Ireland Higher Level versus Massachusetts hypothesis test - Realistic test (Correlation study)

Correlations: Results, Region

```
Pearson correlation of Results and Region = 0.427
P-Value = 0.000
```

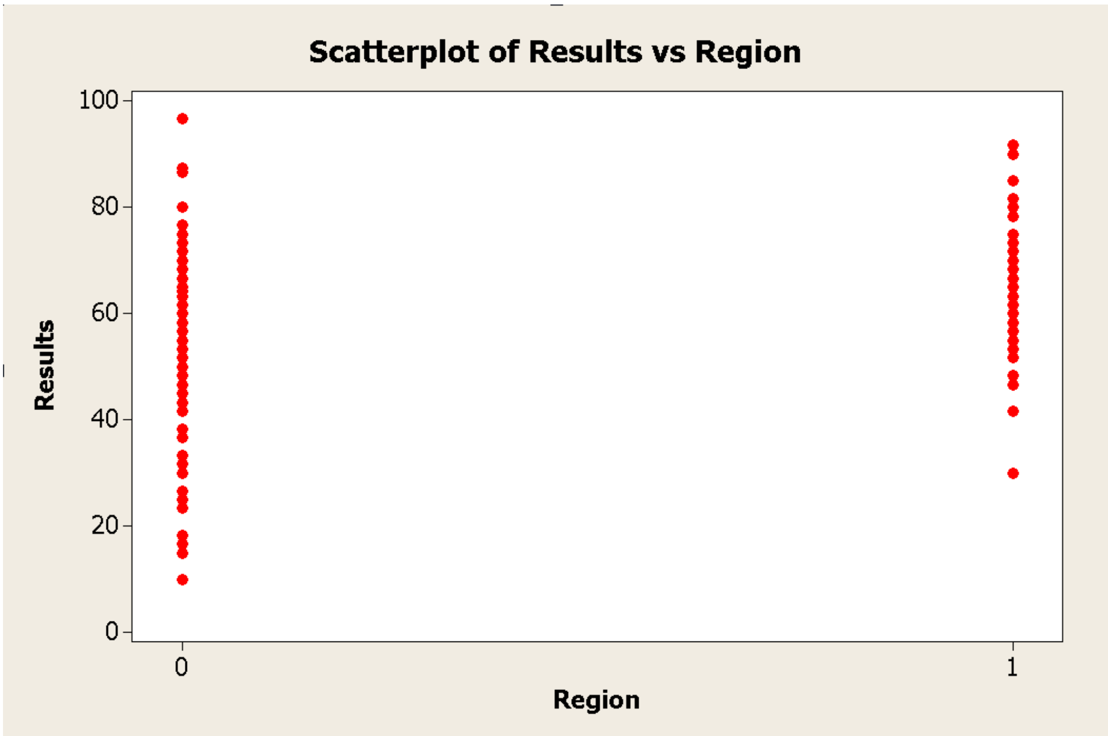


Figure 66: Correlation matrix plot graphics Ireland Higher Level versus Massachusetts study (Realistic test results)

There is sufficient evidence to support the presence of moderate correlation between the two variable tests, as demonstrated by the r-value of 0.427 and a p-value of 0.000 (fig. 66).

8.3.2.5.5. Implication of Results Ireland Higher Level versus Massachusetts study (Realistic test results)

The t-test analysis of the Irish Higher level results versus the Massachusetts results for the 'Realistic' test show a significant difference. The p-value is 0.000, as this is less than the alpha-value of 0.5 it suggests there is a significant difference in student performance between regions. Therefore we reject the null hypothesis '*That there is no difference in achievement levels in the 'Realistic' test between Irish Higher level students (test=0) and students from Massachusetts (test=1)'.* Irish Higher level students, test=0, (M=53.1, S.D.=18.4) score marginally higher than students from Massachusetts, test=1, (M=67.2, S.D.=10.8).

8.4 Gender Test Results

The following section considers the test results with respect to gender.

8.4.1. Gender (Female vs. Male for all tests)

The following statistical analysis considers the test results for gender: 'Male' (test=0) versus 'Female' (test=1). The difference is considered between male and female scores for all test results. The analysis seeks to either accept or reject the following:

- The Null Hypothesis: '*That there is no difference in achievement levels between male and female test performance*';
- The Alternative Hypothesis: '*There is a difference in achievement levels between male and female test performance*'.

The confidence level is again set at 95% to account for any difference that may arise by chance – the alpha level of 0.05 goes some way towards eliminating this risk.

8.4.1.1. Gender study: Descriptive statistics (t=0, t=1)

Descriptive Statistics: Results

Variable	Gender	N	N*	Mean	SE Mean	StDev	Minimum	Q1	Median	Q3
Results	0	180	16	63.22	1.82	24.40	0.00	51.67	66.67	81.46
	1	144	16	67.53	1.70	20.43	10.00	55.00	70.00	82.50

Variable	Gender	Maximum
Results	0	100.00
	1.	100.00

Figure 67: Descriptive Statistics of Gender study (t=0, t=1)

The graphical summary of the descriptive statistics for the gender study is available in Appendix XI:i.

8.4.1.2. Gender study: hypothesis test (Two sample t-test; t=0, t=1)

The approach taken to determine the difference between the mean responses for the ‘Traditional’ and ‘Realistic’ tests at the 5% level of significance is as follows:

- Assess the data sets for equal variance;
- Conduct a two-sample t-test, with or without equal variance, based on the variance findings; and
- Conclude from the t-test whether the difference is significant at the appropriate level.

The graphical information for the gender study is available in Appendix XI:ii. Minitab provides a test statistic and P-value for both the F-Test and Levene’s test. The P-values are 0.027 and 0.062 (Appendix XI:ii). The Levene value is greater than 0.05 which indicates an acceptance of equal variance. They are non-normal results as the p-value for the Anderson-darling test suggests in the descriptive statistics.

Two-Sample T-Test and CI: Results, Gender

```
Two-sample T for Results

Gender     N  Mean  StDev  SE Mean
0        180  63.2   24.4      1.8
1        144  67.5   20.4      1.7


Difference = mu (0) - mu (1)
Estimate for difference:  -4.31
95% CI for difference:  (-9.31, 0.69)
T-Test of difference = 0 (vs not =): T-Value = -1.70  P-Value = 0.091
DF = 322
Both use Pooled StDev = 22.7199
```

Figure 68: Gender study Two sample t-test statistics (t=0, t=1)

Based on the outcome of the two sample t-test, the author notes the estimate difference is -4.31, which would indicate that there is not a considerable difference between the performance responses of the female and male test results (fig. 68). Based on the p-value of 0.091, the author fails to reject the null hypothesis at the 5% level of significance and concludes that there is no significant difference between the results for female and male students. The individual and box-plot graphics for the gender study are available in Appendix XI:iii.

8.4.1.3. Gender study, hypothesis test-one way ANOVA (t=0, t=1)

One-way ANOVA: Results versus Gender

The ANOVA output of immediate interest (Appendix XI:iv) is the F-test statistic. As the associated P-value is 0.091, one can fail to reject the null hypothesis and conclude that the means of the two samples are not statistically different.

Minitab also generates confidence intervals (CIs) for the mean of both tests; the confidence intervals for this study do not demonstrate an overlapping of the intervals for the test samples (Appendix XI:iv). Additionally, the post-hoc testing performed using the Tukey test provides confidence intervals for the difference in the pair of means under evaluation. From this analysis, it can be concluded that there is no significant difference between the performance of ‘test 0’ and ‘test 1’ as the interval goes from – 0.69 to –9.31 and zero is in the interval. The centre point of the CI is 4.31 and is the estimated mean difference between the test groups.

In order to test that the ANOVA assumptions were not violated; a residual plot was created (fig. 69). The standard assumptions are as follows:

- The relationship between Y and X must be linear;
- The values are normally distributed; and

- The values of random error are independent.

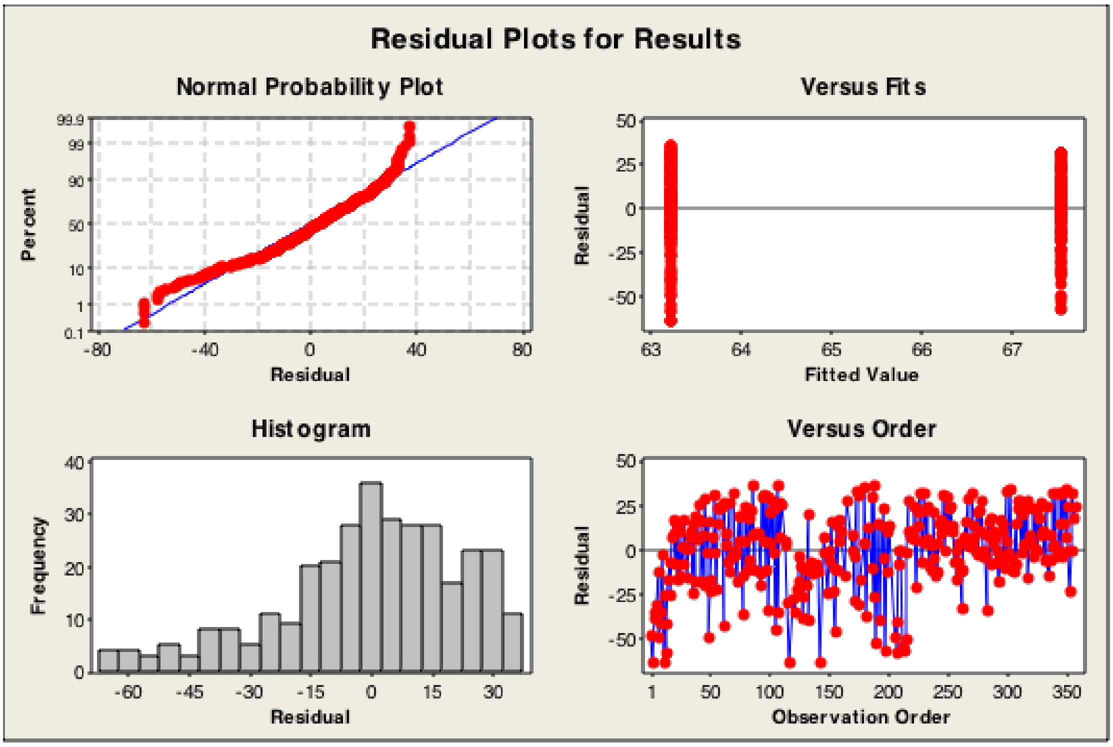


Figure 69: One way ANOVA Residual plot graphics of Gender study (t=0, t=1)

Interpretation of the residual plots:

- The normal probability plot of residuals: Plots the residuals from each observation against the expected value of the residual had it come from a normal distribution. All plotted values appear in a straight line if the residuals are approximately normal (Mendenhall et al, 2009: 489). In this instance the values are in a reasonably straight line which suggests that there is no reason to state that the assumptions have been breached.
- The plot of residual versus fit: This graph plots the residual values against the expected value of the observation using the experimental design implemented. The plot is used to check for constant variance. The data appears to have a random pattern and as a result there is no reason to state that the assumptions have been breached.
- The histogram of residuals: The plot is used to check distribution. Viewed in conjunction with the normal plot, the histogram appears to support the normal distribution and as a result there is no reason to state that the assumptions have been breached.

- Residual versus overall plot: This plot is used to check for observational influence. The data appears to have a random pattern around the central line and as a result there is no reason to state that the assumptions have been breached.

The overall conclusion, based on the residual plots for the results, is that there is no evidence to suggest that standard assumptions of the regression have been violated.

8.4.1.4. Gender: Correlation study (t=0, t=1)

There is sufficient evidence to support the lack of presence of linear correlation between the two variable tests, as demonstrated by the r-value of 0.094 and a P-value of 0.091 (Appendix XI:v).

8.4.1.5. Implication of Results: Gender study

The statistical analysis of the tests, provided by the t-test, indicates that there is no significant difference between test performance for male and females students. The t-test finds a p-value=0.091. As the p-value is greater than 0.05 it is shown that there is no significance difference, and the difference that exists may be due to chance. Therefore the author fails to reject the null hypothesis '*That there is no difference in achievement levels between male and female test performance'*. The difference that does exist favours female performance. Test=0, male test results, (M=63.2, S.D.=24.4) scored lower than test=1, female test results (M=67,5, S.D.=20.4). The difference in the mean performance between male and female students is 4.31, with female students scoring higher. The Pearson Correlation result is 0.094 which indicates an extremely weak, positive correlation between male and female test performance.

8.4.2. Gender (Female only)

The following statistical analysis considers the test results for female performance in both tests ('Realistic' test=0 versus 'Traditional' test=1). The analysis seeks to either accept or reject the following:

- The Null Hypothesis: *'That there is no difference in achievement levels, for female students, between the realistic and traditional tests';*
- The Alternative Hypothesis: *'There is a difference in achievement levels, for female students, between the realistic and traditional tests'.*

The confidence level is again set at 95% to account for any difference that may arise by chance – the alpha level of 0.05 goes some way towards eliminating this risk.

8.4.2.1. Female study Descriptive statistics (t=0, t=1)

Descriptive Statistics: Results

Variable	Test	N	N*	Mean	SE Mean	StDev	Minimum	Q1	Median	Q3
Results	0	71	9	77.24	2.20	18.57	16.67	66.67	82.50	91.67
	1	73	7	58.08	2.06	17.61	10.00	50.00	63.33	70.00

Variable	Test	Maximum
Results	0	100.00
	1	90.00

Figure 70: Descriptive statistics of Female study (t=0, t=1)

8.4.2.2. Female study hypothesis test: two sample t-test (t=0, t=1)

The approach taken to determine the difference between the mean responses for the 'Traditional' and 'Realistic' tests at the 5% level of significance is as follows:

- Assess the data sets for equal variance;

- Conduct a two-sample t-test, with or without equal variance, based on the variance findings; and
- Conclude from the t-test whether the difference is significant at the appropriate level.

Minitab provides a test statistic and P-value for both the F-Test and Levene's test. It is of note that both P-values of 0.656 and 0.652 are greater than 0.05 (Appendix XII:ii). Therefore, this result is not significant and there is sufficient evidence to conclude that the variances are equal.

Two-Sample T-Test and CI: Results, Test

```
Two-sample T for Results
Test   N  Mean  StDev  SE Mean
0     71  77.2   18.6      2.2
1     73  58.1   17.6      2.1
Difference = mu (0) - mu (1)
Estimate for difference:  19.16
95% CI for difference:  (13.20, 25.12)
T-Test of difference = 0 (vs not =): T-Value = 6.36  P-Value = 0.000
DF = 142
Both use Pooled StDev = 18.0867
```

Figure 71: Female study Two sample t-test statistics (t=0, t=1)

The individual and box-plot graphics for the female study are available in Appendix XII:iii. Based on the outcome of the two sample t-test, the author notes the estimate difference is 19.61, which would indicate that there is a significant difference between the performance responses of the female students in the 'Traditional' and 'Realistic' tests. Based on the p-value of 0.000, the author rejects the null hypothesis at the 5% level of significance and concludes that there is a difference between the results of the 'Traditional' and 'Realistic' tests for female students.

8.4.2.3. Female study Hypothesis test, One-way ANOVA (t=0, t=1)

The ANOVA output of immediate interest (Appendix XII:iv) is the F-test statistic. As the associated P-value is 0.000, one can reject the null hypothesis and conclude that the means of the two samples are statistically different.
Minitab also generates confidence intervals (CIs) for the mean of both tests; the confidence intervals for this study do not demonstrate an overlapping of the intervals for the test samples. Additionally, the post-hoc testing performed using the Tukey test provides confidence intervals for the difference in the pair of means under evaluation. From this analysis, it can be concluded that there is a significant difference between the performance of 'test 0' and 'test 1' as the interval goes from -25.12 to -13.20 and zero is not in the interval. In this instance, where the 'test 0' has been subtracted from 'test 1' and the resultant CI contains negative values, one can equate that 'test 1' had significantly lower results. The centre point of the CI is -19.16 and is the estimated mean difference between the test groups.

In order to test that the ANOVA assumptions were not violated; a residual plot was created (fig. 72). The standard assumptions are as follows:

- The relationship between Y and X must be linear;
- The values are normally distributed; and
- The values of random error are independent.

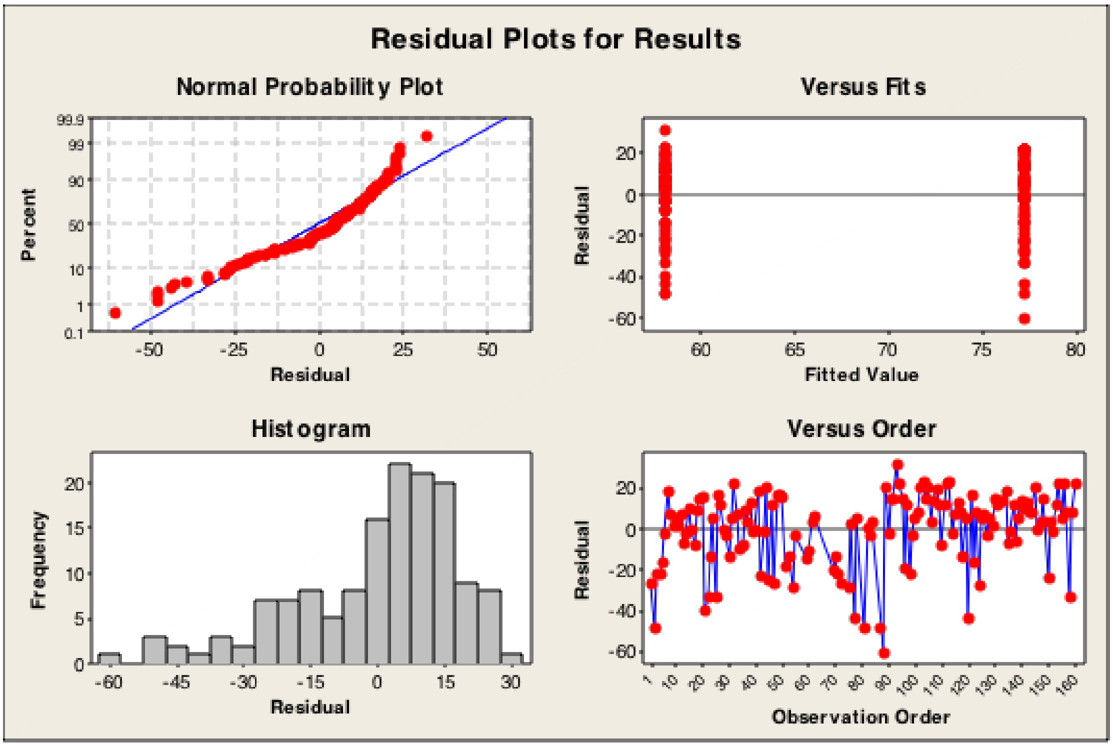


Figure 72: One way ANOVA Residual plot graphics of Female study (t=0, t=1)

Interpretation of the residual plots:

- The normal probability plot of residuals: Plots the residuals from each observation against the expected value of the residual had it come from a normal distribution. All plotted values appear in a straight line if the residuals are approximately normal (Mendenhall et al, 2009: 489). In this instance the values are in a reasonably straight line which suggests that there is no reason to state that the assumptions have been breached.
- The plot of residual versus fit: This graph plots the residual values against the expected value of the observation using the experimental design implemented. The plot is used to check for constant variance. The data appears to have a random pattern and as a result there is no reason to state that the assumptions have been breached.
- The histogram of residuals: The plot is used to check distribution. Viewed in conjunction with the normal plot, the histogram appears to support the normal distribution and as a result there is no reason to state that the assumptions have been breached.
- Residual versus overall plot: This plot is used to check for observational influence. The data appears to have a random pattern

around the central line and as a result there is no reason to state that the assumptions have been breached.

The overall conclusion, based on the residual plots for the results, is that there is no evidence to suggest that standard assumptions of the regression have been violated.

8.4.2.4. Female study Correlation Study (t=0, t=1)

There is sufficient evidence to support the presence of moderate negative linear correlation between the two variable tests, as demonstrated by the r-value of -0.471 and a P-value of 0.000 (Appendix XII:v).

8.4.2.5. Implications of Results (Female only)

The t-test results for female students show a significant difference between female test performance in the 'Traditional' test and the 'Realistic' test. The t-test provides a p-value=0.000, as this value is less than the alpha value of 0.05 it indicates that there is a significant difference (fig. 71). Therefore the null hypothesis 'that there is no difference in achievement levels, for female students, between the realistic and traditional tests' is rejected. Female students scored higher in the 'Traditional' test (test=0) than the 'Realistic' test (test=1). Test=0 (M=77.2, S.D.=18.6) scored higher than test=1 (M=58.1, S.D.=17.6). There is a difference between the mean of the two tests=19.16.

8.4.3. Gender (Male only)

The following statistical analysis considers the male test results for both tests ('Realistic' test=0 versus 'Traditional' test=1). The analysis seeks to either accept or reject the following:

- The Null Hypothesis: *'That there is no difference in achievement levels, for male students, between the realistic and traditional tests';*
- The Alternative Hypothesis: *'There is a difference in achievement levels, for male students, between the realistic and traditional tests'.*

The confidence level is again set at 95% to account for any difference that may arise by chance – the alpha level of 0.05 goes some way towards eliminating this risk.

8.4.3.1. Male study Descriptive statistics (t=0, t=1)

Variable	Test	N	N*	Mean	SE Mean	StDev	Minimum	Q1	Median	Q3
Results	0	86	12	74.18	2.33	21.65	0.00	63.33	79.17	91.67
	1	94	4	53.19	2.32	22.46	0.00	43.33	56.67	68.33

Variable	Test	Maximum
Results	0	100.00
	1	96.67

Figure 73: Descriptive statistics Male study (t=0, t=1)

8.4.3.2. Male study Hypothesis test, two sample t-test (t=0, t=1)

The approach taken to determine the difference between the mean responses for the 'Traditional' and 'Realistic' tests at the 5% level of significance is as follows:

- Assess the data sets for equal variance;

- Conduct a two-sample t-test, with or without equal variance, based on the variance findings; and
- Conclude from the t-test whether the difference is significant at the appropriate level.

Minitab provides a test statistic and P-value for both the F-Test and Levene's test. It is of note that both P-values of 0.731 and 0.576 are greater than 0.05 (Appendix XIII:ii). Therefore, this result is significant and there is sufficient evidence to conclude that the variances are equal.

Two-Sample T-Test and CI: Results, Test

```
Two-sample T for Results
Test   N  Mean  StDev  SE Mean
0     86  74.2   21.6      2.3
1     94  53.2   22.5      2.3
Difference = mu (0) - mu (1)
Estimate for difference:  20.98
95% CI for difference:  (14.48, 27.49)
T-Test of difference = 0 (vs not =): T-Value = 6.37  P-Value = 0.000
DF = 178
Both use Pooled StDev = 22.0791
```

Figure 74: Male study Two sample t-test statistics (t=0, t=1)

Appendix XIII:iii displays the individual and box-plot graphics for the male study. Based on the outcome of the two sample t-test, the author notes the estimate difference is 20.98, which would indicate that there is a significant difference between the performance responses of the male students in the 'Traditional' and 'Realistic' tests (fig. 74). Based on the p-value of 0.000, the author rejects the null hypothesis at the 5% level of significance and concludes that there is a difference between the results of the 'Traditional' and 'Realistic' tests for male students.

8.4.3.3. Male study Hypothesis test, One way ANOVA (t=0, t=1)

One-way ANOVA: Results versus Test

The ANOVA output of immediate interest (Appendix XIII:iv) is the F-test statistic. As the associated P-value is 0.000, one can reject the null hypothesis and conclude that the means of the two samples are statistically different.
Minitab also generates confidence intervals (CIs) for the mean of both tests; the confidence intervals for this study do not demonstrate an overlapping of the intervals for the test samples. Additionally, the post-hoc testing performed using the Tukey test provides confidence intervals for the difference in the pair of means under evaluation. From this analysis, it can be concluded that there is a significant difference between the performance of 'test 0' and 'test 1' as the interval goes from $-$ 27.49 to $-$14.48 and zero is not in the interval. In this instance, where the 'test 0' has been subtracted from 'test 1' and the resultant CI contains negative values, one can equate that 'test 1' had significantly lower results. The centre point of the CI is $-$ 20.28 and is the estimated mean difference between the test groups.

In order to test that the ANOVA assumptions were not violated; a residual plot was created (fig. 75). The standard assumptions are as follows:

- The relationship between Y and X must be linear;
- The values are normally distributed; and
- The values of random error are independent.

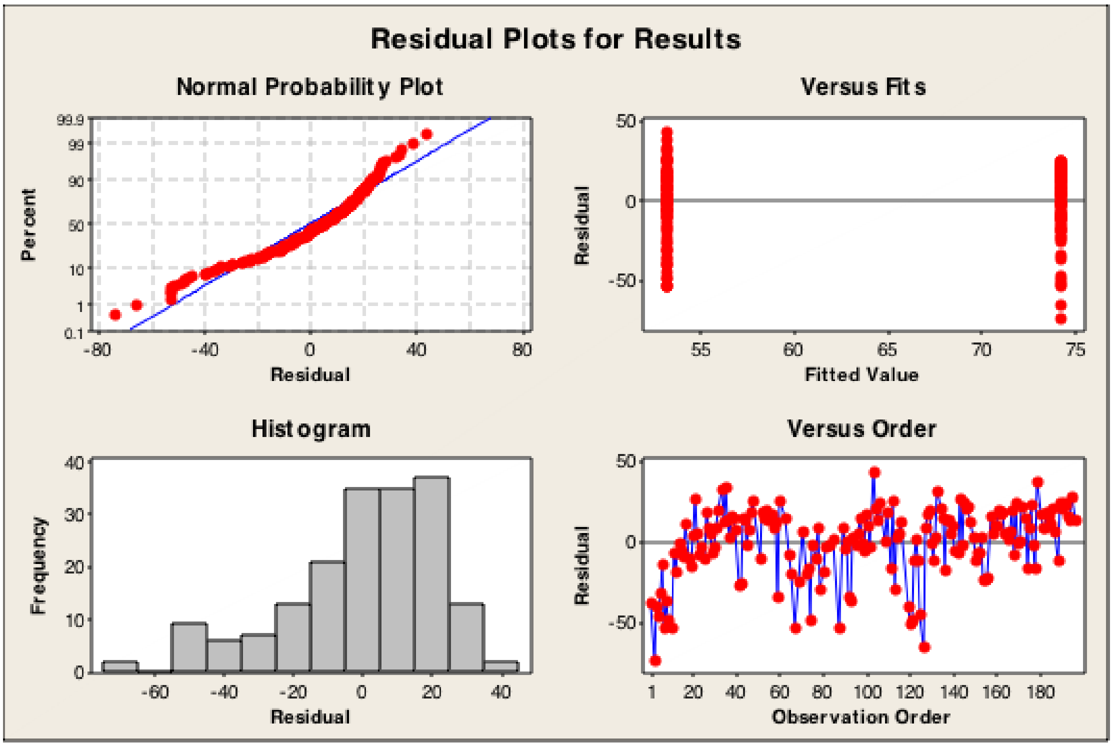


Figure 75: One way ANOVA Residual plot graphics of Male study (t=0, t=1)

Interpretation of the residual plots:

- The normal probability plot of residuals: Plots the residuals from each observation against the expected value of the residual had it come from a normal distribution. All plotted values appear in a straight line if the residuals are approximately normal (Mendenhall et al, 2009: 489). In this instance the values are in a reasonably straight line which suggests that there is no reason to state that the assumptions have been breached.
- The plot of residual versus fit: This graph plots the residual values against the expected value of the observation using the experimental design implemented. The plot is used to check for constant variance. The data appears to have a random pattern and as a result there is no reason to state that the assumptions have been breached.
- The histogram of residuals: The plot is used to check distribution. Viewed in conjunction with the normal plot, the histogram appears to support the normal distribution and as a result there is no reason to state that the assumptions have been breached.
- Residual versus overall plot: This plot is used to check for observational influence. The data appears to have a random pattern

around the central line and as a result there is no reason to state that the assumptions have been breached.

The overall conclusion, based on the residual plots for the results, is that there is no evidence to suggest that standard assumptions of the regression have been violated.

8.4.3.4. Male study Correlation study

Correlations: Results, Test

There is sufficient evidence to support presence of moderate negative linear correlation between the two variable tests (Appendix XIII:v), as demonstrated by the r-value of -0.431and a P-value of 0.000.

8.4.3.5. Implication of Results (Male students)

The t-test analysis of male performance between the 'Traditional' test (test=0) and the 'Realistic' test (test=1) shows a significant difference. The t-test provides a p-value=0.000. As this p-value is lower than the alpha value of 0.05 the hypothesis '*That there is no difference in achievement levels, for male students, between the realistic and traditional tests'* is rejected. Male students perform better in the 'Traditional' test, with a mean difference of 20.98. Test=0 (M=74.2, S.D.= 21.6) scored higher than Test=1 (M=53.2, S.D.=22.5).

8.4.3.6. Implication of overall results (Male versus Female students)

	Number	Trad (test=0)	Realistic (test=1)	p-value
Female	71	M=77.2, SD=18.6	M=58.1, SD=17.6	0.000
Male	86	M=74.3, SD=21.6	M=53.2, SD=22.5	0.000

Table 25: Male versus Female overall results

The above table shows performance for male and female students across both tests, 'Traditional' and 'Realistic'. The mean difference between male and female scores in test=0 (the 'Traditional' test) is 2.9 in favour of female students. The mean difference between male and female scores in test=1 (the 'Realistic' test) is 4.9, in favour of female students again. It is interesting to note that not only do female students perform marginally better than male students in the implemented tests, but that the margin is greater for the 'Realistic' tests. This is noteworthy as much research has indicated that female students perform better when context is provided in mathematics questions (Bolger & Kellaghan, 1990; Burton, 1994; Tims,1994).

8.5 Performance in the Traditional Test as an Indicator

The following section considers the test performance by students between the 'Realistic' and 'Traditional' tests when performance in the 'Traditional' test is used as an indicator.

8.5.1. Traditional Test Performance ≥80%

The following statistical analysis considers the test results for students that attained a score of greater, or equal to, 80% in the 'Traditional' test. It seeks to determine if there is a difference in test performance for this student group between the tests implemented ('Realistic' versus 'Traditional'). The author is interested in considering the overall group in this subsection as it contains the students that educators in Ireland would typically consider 'more mathematically able'. The analysis seeks to either accept or reject the following:

- The Null Hypothesis: '*That there is no difference in achievement level between the realistic and traditional tests, for students that obtained a score of ≥80% in test=0*';

- The Alternative Hypothesis: '*There is a difference in achievement levels between the realistic and traditional tests, for students that obtained a score of ≥80% in test=0*'.

The confidence level is again set at 95% to account for any difference that may arise by chance – the alpha level of 0.05 goes some way towards eliminating this risk.

8.5.1.1. Traditional test performance ≥80% study Descriptive statistics (t=0, t=1)

Descriptive Statistics: Results

Variable	Test	N	N*	Mean	SE Mean	StDev	Minimum	Q1	Median	Q3
Results	0	80	0	90.542	0.674	6.029	80.000	85.207	91.250	95.623
	1	80	0	62.43	1.77	15.80	15.00	55.00	63.33	71.67

Variable	Test	Maximum
Results	0	100.000
	1.	96.67

Figure 76: Descriptive statistics traditional ≥ 80% study (t=0, t=1)

8.5.1.2. Traditional performance ≥80% Hypothesis test, two sample t-test (t=0, t=1)

The approach taken to determine the difference between the mean responses for the 'Traditional' and 'Realistic' tests at the 5% level of significance is as follows:

- Assess the data sets for equal variance;
- Conduct a two-sample t-test, with or without equal variance, based on the variance findings; and
- Conclude from the t-test whether the difference is significant at the appropriate level.

Minitab provides a test statistic and P-value for both the F-Test and Levene's test. It is of note that both P-values of 0.000 and 0.000 are less than 0.05 (Appendix XIV:ii). Therefore, this result is significant and there is sufficient evidence to conclude that the variances are not equal.

Two-Sample T-Test and CI: Results, Test

```
Two-sample T for Results
Test   N   Mean  StDev  SE Mean
0     80  90.54   6.03     0.67
1     80   62.4   15.8      1.8
Difference = mu (0) - mu (1)
Estimate for difference:  28.11
95% CI for difference:  (24.36, 31.87)
T-Test of difference = 0 (vs not =): T-Value = 14.87  P-Value = 0.000
DF = 101
```

Figure 77: Trad ≥ 80% Two sample t-test statistics (t=0, t=1)

The Individual and box-plot graphics for the traditional test ≥80% are available in Appendix XIV:iii. Based on the outcome of the two sample t-test, the author notes the estimate difference is 28.11, which would indicate that there is a considerable difference between the performance responses of the 'Traditional' and 'Realistic' tests, for students who scored **≥**80% in the 'Traditional' test (fig. 77). Based on the p-value of 0.000, the author rejects the null hypothesis at the 5% level of significance and concludes that there is a difference between the results of the 'Traditional' and 'Realistic' tests for this cohort of students.

8.5.1.3. Traditional performance ≥80% Hypothesis test, One way ANOVA (t=0, t=1)

One-way ANOVA: Results versus Test

The ANOVA output of immediate interest (Appendix XIV:iv) is the F-test statistic. As the associated P-value is 0.000, one can reject the null hypothesis and conclude that the means of the two samples are statistically different.
Minitab also generates confidence intervals (CIs) for the mean of both tests; the confidence intervals for this study do not demonstrate an overlapping of the intervals for the test samples (Appendix XIV:iv). Additionally, the post-hoc testing performed using the Tukey test provides confidence intervals for the difference in the pair of means under evaluation. From this analysis, it can be

concluded that there is a significant difference between the performance of 'test 0' and 'test 1' as the interval goes from –31.85 to – 24.38 and zero is not in the interval. In this instance, where the 'test 0' has been subtracted from 'test 1' and the resultant CI contains negative values, one can equate that 'test 1' had significantly lower results. The centre point of the CI is – 28.11 and is the estimated mean difference between the test groups.

In order to test that the ANOVA assumptions were not violated; a residual plot was created (fig. 78). The standard assumptions are as follows:

- The relationship between Y and X must be linear;
- The values are normally distributed; and
- The values of random error are independent.

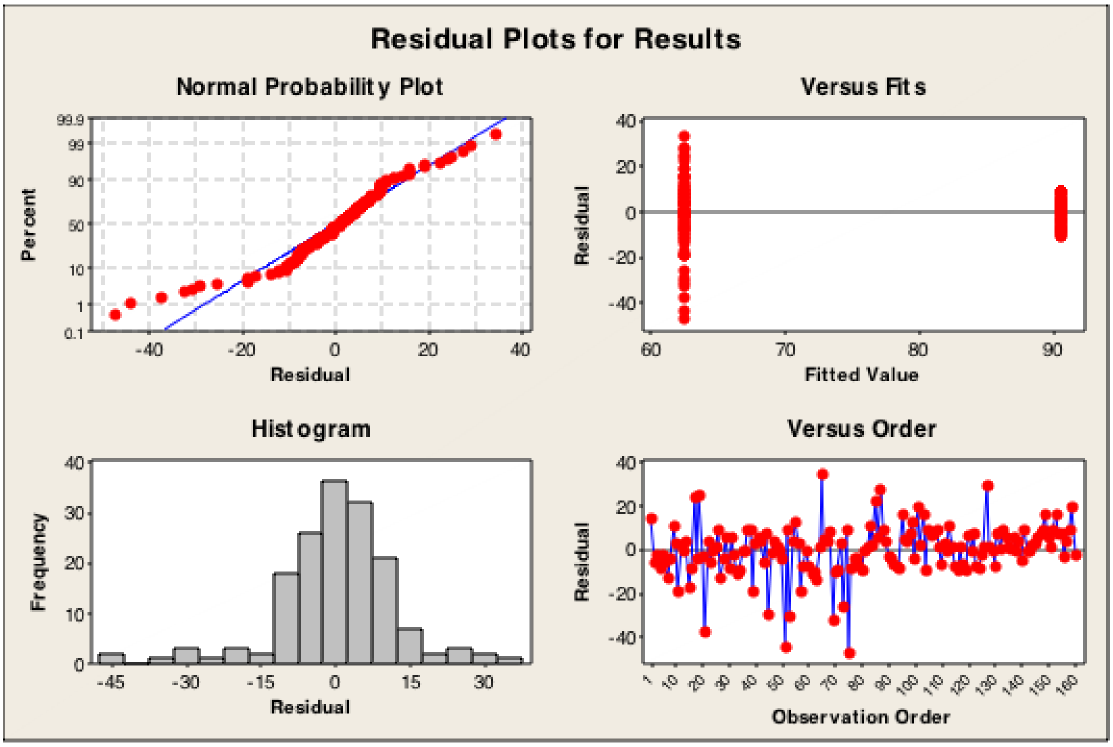


Figure 78: One way ANOVA Residual plot graphics of traditional. ≥ 80% study (t=0, t=1)

Interpretation of the residual plots:

- The normal probability plot of residuals: Plots the residuals from each observation against the expected value of the residual had it come from a normal distribution. All plotted values appear in a straight line if the residuals are approximately normal (Mendenhall et al, 2009: 489). In this instance the values are in a reasonably straight line which suggests that there is no reason to state that the assumptions have been breached.
- The plot of residual versus fit: This graph plots the residual values against the expected value of the observation using the experimental design implemented. The plot is used to check for constant variance. The data appears to have a random pattern and as a result there is no reason to state that the assumptions have been breached.
- The histogram of residuals: The plot is used to check distribution. Viewed in conjunction with the normal plot, the histogram appears to support the normal distribution and as a result there is no reason to state that the assumptions have been breached.
- Residual versus overall plot: This plot is used to check for observational influence. The data appears to have a random pattern around the central line and as a result there is no reason to state that the assumptions have been breached.

The overall conclusion, based on the residual plots for the results, is that there is no evidence to suggest that standard assumptions of the regression have been violated.

8.5.1.4. Traditional performance ≥80%

Correlation study

Correlations: Results, Test

There is sufficient evidence to support the presence of strong weak linear correlation between the two variable tests, as demonstrated by the r-value of -0.764 and a P-value of 0.000 (Appendix XIV:v).

8.5.1.5. Implication of Results (for students scoring ≥80% in test=0)

The t-test shows a significant difference between scores in the ‘Traditional’ and ‘Realistic’ tests for students who scored ≥80% in test=0. This differences is indicated by a t-test p-value=0.000. As the p-value is less than the set alpha value of 0.05 this indicates a significant difference between test performance (fig. 77). Therefore, the null hypothesis *‘that there is no difference in achievement level between the realistic and traditional tests, for students that obtained a score of ≥80% in test =0’* is rejected. The ‘Traditional’ test, test=0, (M=90.54, S.D.=6.03) scores higher than the ‘Realistic’ test, test=1, (M=62.4, S.D.=15.8). The difference between the mean is 28.11. This is a significant gap in performance for students that are typically considered to be ‘more mathematically able’. The greater standard deviation for test=1 can somewhat be explained due to the fact that there are parameters put on the scores for test=0 (all scores ≥ 80%) but none for test=1. The Pearson Correlation result is -0.764 indicating a strong, negative correlation between students scoring more than 80% in the traditional test and their result in the realistic test.

8.5.2. Traditional performance 60%≤x<80%

The following statistical analysis considers the results for both tests (‘Realistic’ versus ‘Traditional’) for students who scored between 60% and 80% in the ‘Traditional’ test (test=0). The analysis seeks to either accept or reject the following:

- The Null Hypothesis: ‘*That there is no difference in achievement levels between the realistic and traditional tests, for students who scored between 60% and 80% in test=0*’;

- The Alternative Hypothesis: ‘*There is a difference in achievement levels between the realistic and traditional tests, for students who scored between 60% and 80% in test=0*’.

The confidence level is again set at 95% to account for any difference that may arise by chance – the alpha level of 0.05 goes some way towards eliminating this risk.

8.5.2.1. Traditional performance 60%≤x<80% Descriptive statistics (t=0, t=1)

Descriptive Statistics: Results

Variable	Test	N	N*	Mean	SE Mean	StDev	Minimum	Q1	Median	Q3
Results	0	47	0	71.366	0.847	5.809	60.830	65.830	72.500	76.670
	1	47	0	56.33	2.76	18.89	5.00	46.67	61.67	70.00

Variable	Test	Maximum
Results	0	79.170
	1	81.67

Figure 79: Descriptive statistics traditional 60%≤x<80% study (t=0, t=1)

8.5.2.2. Traditional performance 60%≤x<80% Hypothesis test, two sample t-test (t=0, t=1)

The approach taken to determine the difference between the mean responses for the 'Traditional' and 'Realistic' tests at the 5% level of significance is as follows:

- Assess the data sets for equal variance;
- Conduct a two-sample t-test, with or without equal variance, based on the variance findings; and
- Conclude from the t-test whether the difference is significant at the appropriate level.

Minitab provides a test statistic and P-value for both the F-Test and Levene's test. It is of note that both P-values of 0.000 and 0.000 are less than 0.05 (Appendix XV:ii). Therefore, this result is significant and there is sufficient evidence to conclude that the variances are not equal.

Two-Sample T-Test and CI: Results, Test

```
Two-sample T for Results
Test    N   Mean  StDev  SE Mean
0      47  71.37   5.81     0.85
1      47   56.3   18.9      2.8

Difference = mu (0) - mu (1)
Estimate for difference:  15.04
95% CI for difference:  (9.25, 20.82)
T-Test of difference = 0 (vs not =): T-Value = 5.21  P-Value = 0.000
DF = 54
```

Figure 80: Trad 60%≤x<80% Two sample t-test statistics (t=0, t=1)

Appendix XV:iii displays the Individual and box-plot graphics for the traditional test from 60% to 80%. Based on the outcome of the two sample t-test, the author notes the estimate difference is 15.04, which would indicate that there is a considerable difference between the performance responses of the 'Traditional' and 'Realistic' tests, for students who scored between 60% and 80% in the 'Traditional' test (fig. 80). Based on the p-value of 0.000, the author rejects the null hypothesis at the 5% level of significance and concludes that there is a difference between the results of the ‘Traditional’ and ‘Realistic’ tests for this cohort of students.

8.5.2.3. Traditional performance 60%≤x<80% Hypothesis test, One way ANOVA (t=0, t=1)

One-way ANOVA: Results versus Test

The ANOVA output of immediate interest (Appendix XV:iv) is the F-test statistic. As the associated P-value is 0.000, one can reject the null hypothesis and conclude that the means of the two samples are statistically different.
Minitab also generates confidence intervals (CIs) for the mean of both tests; the confidence intervals for this study do not demonstrate an overlapping of the intervals for the test samples (Appendix XV:iv). Additionally, the post-hoc testing performed using the Tukey test provides confidence intervals for the difference in the pair of means under evaluation. From this analysis, it can be concluded that there is a significant difference between the performance of ‘test 0’ and ‘test 1’ as the interval goes from $-$ 20.76 to $-$ 9.31 and zero is not in the interval. In this instance, where the ‘test 0’ has been subtracted from ‘test 1’ and the resultant CI contains negative values, one can equate that ‘test 1’ had significantly lower results. The centre point of the CI is $-$ 15.04 and is the estimated mean difference between the test groups.

In order to test that the ANOVA assumptions were not violated; a residual plot was created (fig. 81). The standard assumptions are as follows:

- The relationship between Y and X must be linear;
- The values are normally distributed; and
- The values of random error are independent.

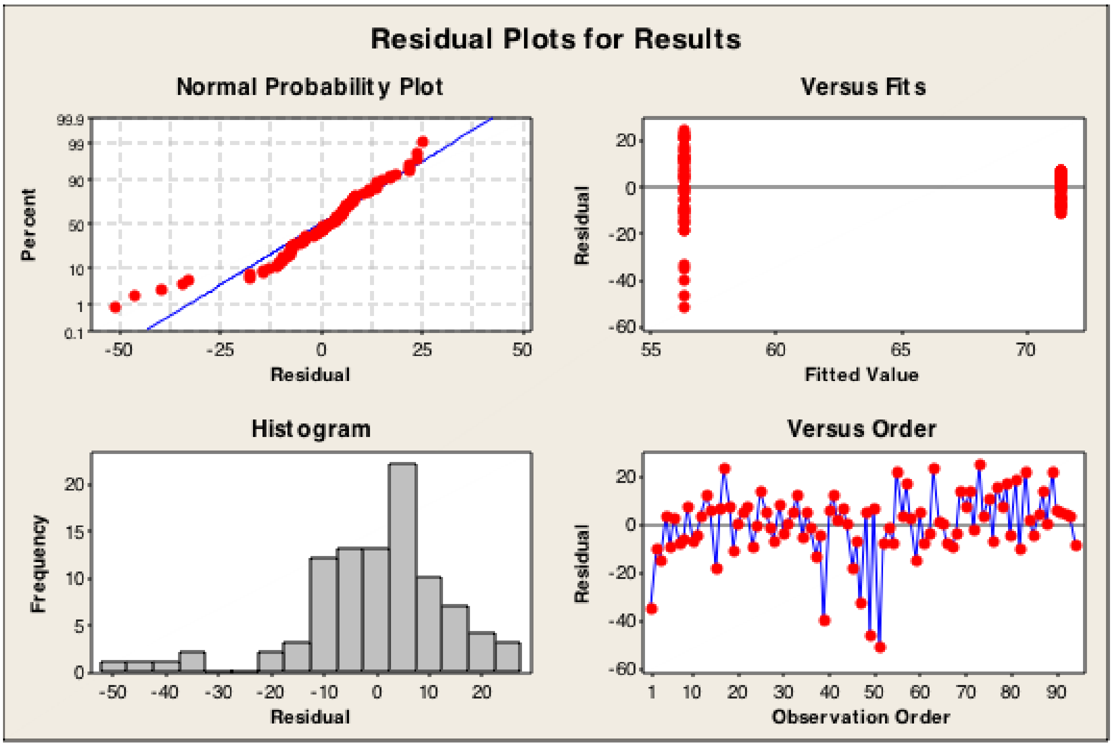


Figure 81: One way ANOVA Residual plot graphics of traditional 60%≤x<80% study (t=0, t=1)

Interpretation of the residual plots:

- The normal probability plot of residuals: Plots the residuals from each observation against the expected value of the residual had it come from a normal distribution. All plotted values appear in a straight line if the residuals are approximately normal (Mendenhall et al, 2009: 489). In this instance the values are in a reasonably straight line which suggests that there is no reason to state that the assumptions have been breached.
- The plot of residual versus fit: This graph plots the residual values against the expected value of the observation using the experimental design implemented. The plot is used to check for constant variance. The data appears to have a random pattern and as a result there is no reason to state that the assumptions have been breached.
- The histogram of residuals: The plot is used to check distribution. Viewed in conjunction with the normal plot, the histogram appears to support the normal distribution and as a result there is no reason to state that the assumptions have been breached.

- Residual versus overall plot: This plot is used to check for observational influence. The data appears to have a random pattern around the central line and as a result there is no reason to state that the assumptions have been breached.

The overall conclusion, based on the residual plots for the results, is that there is no evidence to suggest that standard assumptions of the regression have been violated.

8.5.2.4. Traditional performance 60%≤x<80% Correlation study

Correlations: Results, Test

There is sufficient evidence to support the presence of a slightly weak linear correlation between the two variable tests, as demonstrated by the r-value of -0.478 and a P-value of 0.000 (Appendix XV:v).

8.5.2.5. Implication of results (60%≤x<80% in test=0)

The t-test analysis shows that there is a difference in achievement between the 'Traditional' and 'Realistic' tests for students who scored 60%≤x<80% in test =0 ('Traditional"). The t-test provides a p-value =0.000 and due to the fact that this is less than the set alpha level of 0.05 this indicates a significant difference. Therefore, the hypothesis '*that there is no difference in achievement levels between the realistic and traditional tests, for students who score 60%≤x<80% in test=0*' is rejected. There is an estimate for difference between the mean of test=0 and test=1 of 15.04 for the 47 students who scored in this range in the 'Traditional' test. Test=0, 'Traditional', (M=71.37, S.D.=5.81) scored higher than test=1, 'Realistic' (M-56.3, S.D.=18.9). Again there is a considerably larger standard deviation (S.D.) for test=1, the 'Realistic' test but part of this is due to the fact that parameters were set for test=0 ($60\% \leq x < 80\%$) but not for test=1. The Pearson Correlation result of -0.478 indicates a moderately weak correlation between test performance of between 60% and 80% in the traditional test, and test performance in the realistic test.

8.5.3. Traditional Test performance 0%≤x<60%

The following statistical analysis considers the test results for both tests (‘Realistic’ versus ‘Traditional’), for students who scored less than 60% in the ‘Traditional’ test (test=0). The analysis seeks to either accept or reject the following:

- The Null Hypothesis: ‘*That there is no difference in achievement levels between the realistic and traditional tests, for students who score less than 60% in test=0*’;

- The Alternative Hypothesis: ‘*There is a difference in achievement levels between the realistic and traditional tests, for students who score less than 60% in test=0*’.

The confidence level is again set at 95% to account for any difference that may arise by chance – the alpha level of 0.05 goes some way towards eliminating this risk.

8.5.3.1. Traditional performance 0%≤x<60% Descriptive statistics (t=0, t=1)

Descriptive Statistics: Results

Variable	Test	N	N*	Mean	SE Mean	StDev	Minimum	Q1	Median	Q3
Results	0	30	21	42.19	2.98	16.32	0.00	28.75	49.59	55.21
	1	40	11	39.96	3.63	22.96	0.00	19.58	42.50	59.17

Variable	Test	Maximum
Results	0	58.33
	1	76.67

Figure 82: Descriptive statistics traditional 0%≤x<60% study (t=0, t=1)

8.5.3.2. Traditional performance 0%≤x<60% Hypothesis test, two sample t-test (t=0, t=1)

The approach taken to determine the difference between the mean responses for the 'Traditional' and 'Realistic' tests at the 5% level of significance is as follows:

- Assess the data sets for equal variance;
- Conduct a two-sample t-test, with or without equal variance, based on the variance findings; and
- Conclude from the t-test whether the difference is significant at the appropriate level.

Minitab provides a test statistic and P-value for both the F-Test and Levene's test. The P-values of 0.059 and 0.017 (Appendix XVI:ii). The Levene value of 0.017 suggests that the variance is not equal.

Two-Sample T-Test and CI: Results, Test

```
Two-sample T for Results
Test   N  Mean  StDev  SE Mean
0     30  42.2   16.3      3.0
1     40  40.0   23.0      3.6

Difference = mu (0) - mu (1)
Estimate for difference:  2.24
95% CI for difference:  (-7.14, 11.61)
T-Test of difference = 0 (vs not =): T-Value = 0.48  P-Value = 0.636
DF = 67
```

Figure 83: Trad <60% Two sample t-test statistics (t=0, t=1)

Appendix XVI:iii displays the Individual and box-plot graphics for the traditional test less than 60%. Based on the outcome of the two sample t-test, the author notes the estimate difference is 2.24, which would indicate that there is not a significant difference between the performance responses of the 'Traditional' and 'Realistic' tests, for students who scored less than 60% in the 'Traditional' test (fig. 83). Based on the p-value of 0.636, the author fails to reject the null hypothesis at the 5% level of significance and concludes that there is no significant difference between the results of the ‘Traditional’ and ‘Realistic’ tests for this cohort of students.

8.5.3.3. Traditional performance 0%≤x<60% Hypothesis test, One way ANOVA (t=0, t=1)

One-way ANOVA: Results versus Test

The ANOVA output of immediate interest (Appendix XVI:iv) is the F-test statistic. As the associated P-value is 0.636, one can fail to reject the null hypothesis and conclude that the means of the two samples are not statistically different.

Minitab also generates confidence intervals (CIs) for the mean of both tests; the confidence intervals for this study do not demonstrate an overlapping of the intervals for the test samples (Appendix XVI:iv). Additionally, the post-hoc testing performed using the Tukey test provides confidence intervals for the difference in the pair of means under evaluation. From this analysis, it can be concluded that there is no significant difference between the performance of ‘test 0’ and ‘test 1’ as the interval goes from $-$ 12.07 to 7.60 and zero is in the interval. The centre point of the CI is $-$ 2.24 and is the estimated mean difference between the test groups.

In order to test that the ANOVA assumptions were not violated; a residual plot was created (fig. 84). The standard assumptions are as follows:

- The relationship between Y and X must be linear;
- The values are normally distributed; and
- The values of random error are independent.

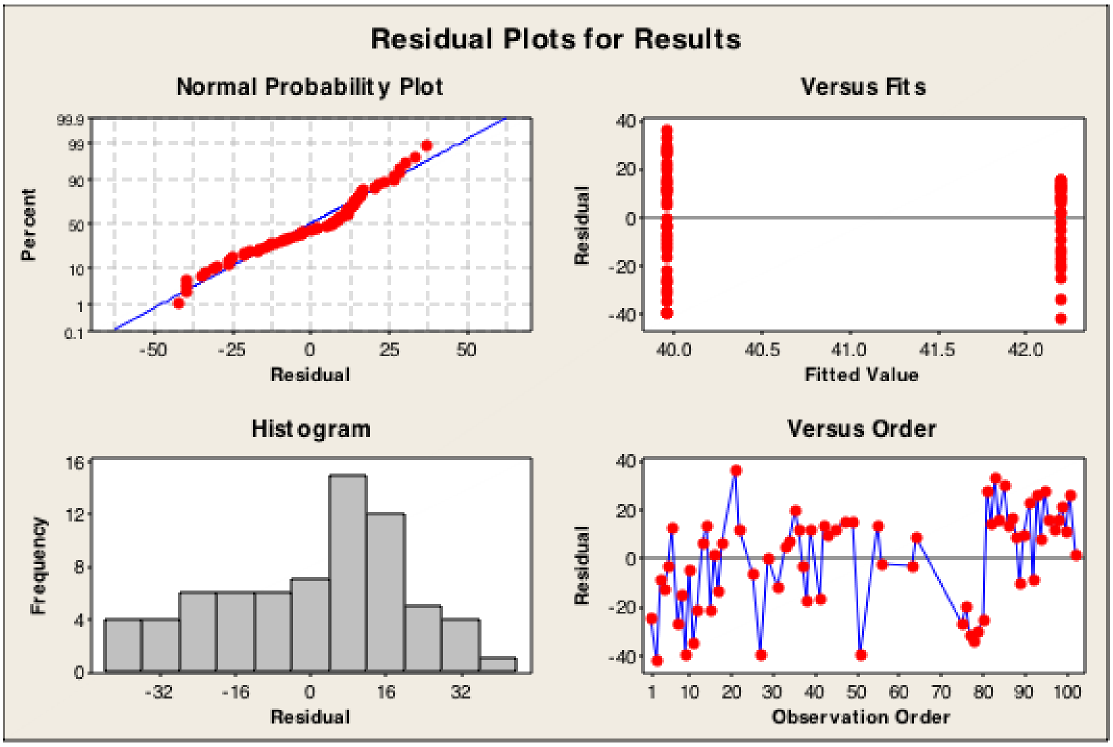


Figure 84: One way ANOVA Residual plot graphics of traditional 0%≤x<60% study (t=0, t=1)

Interpretation of the residual plots:

- The normal probability plot of residuals: Plots the residuals from each observation against the expected value of the residual had it come from a normal distribution. All plotted values appear in a straight line if the residuals are approximately normal (Mendenhall et al, 2009: 489). In this instance the values are in a reasonably straight line which suggests that there is no reason to state that the assumptions have been breached.
- The plot of residual versus fit: This graph plots the residual values against the expected value of the observation using the experimental design implemented. The plot is used to check for constant variance. The data appears to have a random pattern and as a result there is no reason to state that the assumptions have been breached.
- The histogram of residuals: The plot is used to check distribution. Viewed in conjunction with the normal plot, the histogram appears to support the normal distribution and as a result there is no reason to state that the assumptions have been breached.

- Residual versus overall plot: This plot is used to check for observational influence. The data appears to have a random pattern around the central line and as a result there is no reason to state that the assumptions have been breached.

The overall conclusion, based on the residual plots for the results, is that there is no evidence to suggest that standard assumptions of the regression have been violated.

8.5.3.4. Traditional performance 0%≤x<60% Correlation study

Correlations: Results, Test

There is sufficient evidence to support the lack of presence of linear correlation between the two variable tests, as demonstrated by the r-value of -0.055 and a P-value of 0.651 (Appendix XVI:v).

8.5.3.5. Implication of Results (Traditional performance 0%≤x<60

The t-test analysis of test results for students who scored less than 60% in test=0 ('Traditional') shows that there is no difference between test performance in the 'Traditional' and 'Realistic' tests. The t-test produces a p-value of 0.636 (fig. 83). As this p-value is greater than the set alpha level of 0.05 it indicates that any difference that may exist between tests is coincidental. Therefore, the author fails to reject the hypothesis '*that there is no difference in achievement levels between the realistic and traditional tests, for students who score less than 60% in test=0'*. There is a small mean difference between test=0 and test=1 of 2.24. This difference favours the 'Traditional' test.

Test=0, 'Traditional, (M=42.2, S.D. 16.3) scores marginally higher than test=1, 'Realistic' (M=40, S.D.=23.0). Again the larger standard deviation (S.D.) for test=1 may be due to the fact that parameters were set for test=0 (<60%) but not for test=1. The Pearson Correlation result is -0.055 which indicates a weak, negative correlation between test performance for students who scored less than 60% in the traditional test.

8.5.4. An analysis of the test results between ability groupings.

As outlined above, the author analysed between test performance for different groups based on achievement in the 'Traditional' test. The author subdivided the overall research sample as follows:

- Students who achieved a score of ≥80% in the 'Traditional' test, test=0 (N=80);
- Students who achieved a score of 60%≤x<80% in the 'Traditional' test, test=0 (N=47); and
- Students who achieved a score of <60% in the 'Traditional' test, test=0 (N=30).

The author decided on the above divisions based on the premise that students who perform well in the Junior Certificate examination (on which the 'Traditional' test is directly based) are considered to be mathematically able by Irish standards. The author was interested in considering the link between traditional mathematical ability (a demonstration of the ability to reproduce mathematical information in a familiar manner) and the ability to demonstrate mathematical understanding (by solving unfamiliar mathematical problems that demand thought and reflection). The author designed the two tests to be implemented based on these concepts with the 'Traditional' test designed to test mathematical knowledge and the ‘Realistic’ test designed to test mathematical transfer ability and understanding.

The following table displays the results from the implemented tests for each of the ability sub-groups:

	Number	**Trad (test=0)**	**Realistic (test=1)**	**p-value**
≥80% in Test=0	N=80	M=90.54, S.D.=6.03	M=62.4, S.D.=15.8	0.000
60% ≤ x < 80% in Test=0	N=47	M=71.36, S.D.=5.81	M=56.3, S.D.=18.9	0.000
<60% in Test=0	N=30	M=42.2, S.D.=16.3	M=40.0, S.D.=23.0	0.636

Table 31: Ability group results

It is interesting to note that while there is a significant difference between test performance for the first two groups: ‘≥80% in Test=0’ and ‘60%≤x<80% in Test=0’, indicated by a p-value=0.000, there is no significant difference in test performance for the third group (<60% in Test=0) indicated with a p-value=0.635. It is also worth noting that the difference between mean test performance for the group ‘≥80% in Test=0’ is larger (28.14) than the mean difference between tests for the second group ‘60%≤x<80% in Test=0’ (15.06). This would suggest that there is less of a relationship between reproducing knowledge effectively and demonstrating mathematical understanding than one may expect. Indeed despite a mean difference of 19.18 in test=0 for the first two groups, there is a mean difference of just 6.1 between the same two groups for test=1. The author is interested in the possibilities suggested by the above results, including a suggestion that the skills needed to perform successfully in the two tests are very separate, and in some cases almost unrelated.

It is worth recapping at this stage that the two tests were designed as follows:

- **The Traditional Test**: is based on the Junior Certificate examination paper 1. The questions are directly sourced from a past Junior Certificate examination paper and focus on the topics of algebra and arithmetic. The questions are based on the ordinary level Junior Certificate examination and therefore reflect what is covered in the Ordinary level Junior Certificate syllabus. It is important to note that all students that sit the Higher Level junior certificate examination would have covered both the Ordinary level course, in addition to the Higher level course. The students would be very familiar with the style of question used in the Traditional test as the syllabus directly prepares students for this examination. As mentioned in the literature review many Irish teachers focus on examinable skills and as a results most (if not all) of the students involved in the test would be familiar to a classroom environment in which the answering of Junior Certificate style questions would be commonplace. The questions involved are closed-ended questions and have only one correct answer. Again this is the basis of all Junior Certificate teaching and learning.

- **The Realistic Test**: is based on a realistic, problem-solving scenario. The questions involve a significant amount of reading which may be problematic for some students. (Students involved in this research were allowed to ask for help with reading of the questions if necessary. It is also worth noting that Irish students with particularly severe learning difficulties would have had the assistance of an S.N.A., a 'special needs assistant'). The questions required the students to think about a realistic scenario in which socio-cultural issues , including the consideration of ethical issues, have a role in the decision making process. Minor surplus information was provided in some of the questions. Demonstration of reflection and justification was asked for. The questions asked were open-ended and could have more than one correct answer.

8.6 Level of Junior Certificate Course studied by Irish students

The following section considers the Irish test results for students by level studied at the time of testing: higher level or ordinary level.

8.6.1. Higher Level Junior Certificate Course (all tests)

The following statistical analysis considers the Irish test results for students following the higher level course over both tests ('Realistic' versus 'Traditional'). The analysis seeks to either accept or reject the following:

- The Null Hypothesis: '*That there is no difference in achievement levels between the realistic and traditional tests for Irish higher level students*';

- The Alternative Hypothesis: '*There is a difference in achievement levels between the realistic and traditional tests for Irish higher level students*'.

The confidence level is again set at 95% to account for any difference that may arise by chance – the alpha level of 0.05 goes some way towards eliminating this risk.

8.6.2. Higher Level Junior Certificate Course Descriptive statistics (t=0, t=1)

Descriptive Statistics: Results

Variable	Test	N	N*	Mean	SE Mean	StDev	Minimum	Q1	Median	Q3
Results	0	139	11	80.04	1.27	14.97	33.33	71.67	82.50	91.67
	1	139	11	60.31	1.40	16.54	10.00	51.67	63.33	70.00

Variable	Test	Maximum
Results	0	100.00
	1	96.67

Figure 85: Descriptive statistics of higher level junior certification study (t=0, t=1)

8.6.2.1. Higher Level Junior Certificate Course Hypothesis test, two sample t-test (t=0, t=1)

The approach taken to determine the difference between the mean responses for the 'Traditional' and 'Realistic' tests at the 5% level of significance is as follows:

- Assess the data sets for equal variance;
- Conduct a two-sample t-test, with or without equal variance, based on the variance findings; and
- Conclude from the t-test whether the difference is significant at the appropriate level.

Minitab provides a test statistic and P-value for both the F-Test and Levene's test. It is of note that both P-values of 0.023 and 0.032 are less than 0.05 (Appendix XVII:ii). Therefore, this result is significant and there is sufficient evidence to conclude that the variances are not equal.

Two-Sample T-Test and CI: Results, Test

```
Two-sample T for Results
Test    N  Mean  StDev  SE Mean
0      68  80.4   13.9      1.7
1      68  53.1   18.4      2.2

Difference = mu (0) - mu (1)
Estimate for difference:  27.30
95% CI for difference:  (21.77, 32.84)
T-Test of difference = 0 (vs not =): T-Value = 9.76  P-Value = 0.000
DF = 124
```

Figure 86: Higher Level JC course Two sample t-test statistics (t=0, t=1)

Appendix XVII:iii illustrates the Individual and box-plot graphics for the higher level course. Based on the outcome of the two sample t-test, the author notes the estimate difference is 27.3, which would indicate that there is a considerable difference between the performance responses of the 'Traditional' and 'Realistic' tests, for higher level, Junior Certificate students (fig. 86). Based on the p-value of 0.000, the author rejects the null hypothesis at the 5% level of significance and concludes that there is a difference between the results of the 'Traditional' and 'Realistic' tests for this cohort of students.

8.6.2.2. Higher Level Junior Certificate Course Hypothesis test, One way ANOVA (t=0, t=1)

One-way ANOVA: Results versus Test

The ANOVA output of immediate interest (Appendix XVII:iv) is the F-test statistic. As the associated P-value is 0.000, one can reject the null hypothesis and conclude that the means of the two samples are statistically different.

Minitab also generates confidence intervals (CIs) for the mean of both tests; the confidence intervals for this study do not demonstrate an overlapping of the intervals for the test samples (Appendix XVII:iv). Additionally, the post-hoc testing performed using the Tukey test provides confidence intervals for the difference in the pair of means under evaluation. From this analysis, it can be concluded that there is a significant difference between the performance of 'test 0' and 'test 1' as the interval goes from $-$ 32.84 to $-$ 21.77 and zero is not in the interval. In this instance, where the 'test 0' has been subtracted from 'test 1' and the resultant CI contains negative values, one can equate that 'test 1' had significantly lower results. The centre point of the CI is $-$ 27.30 and is the estimated mean difference between the test groups.

In order to test that the ANOVA assumptions were not violated; a residual plot was created (fig. 87). The standard assumptions are as follows:

- The relationship between Y and X must be linear;
- The values are normally distributed; and
- The values of random error are independent.

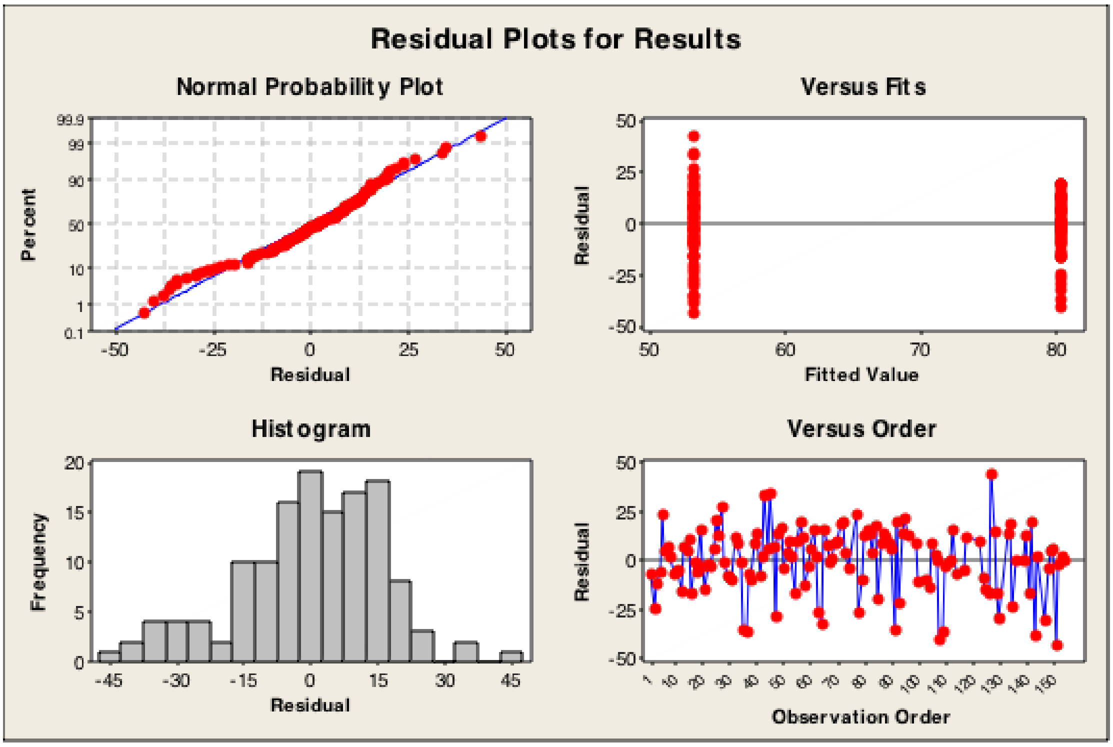


Figure 87: One way ANOVA Residual plot graphics of higher level junior certification study (t=0, t=1)

Interpretation of the residual plots:

- The normal probability plot of residuals: Plots the residuals from each observation against the expected value of the residual had it come from a normal distribution. All plotted values appear in a straight line if the residuals are approximately normal (Mendenhall et al, 2009: 489). In this instance the values are in a reasonably straight line which suggests that there is no reason to state that the assumptions have been breached.
- The plot of residual versus fit: This graph plots the residual values against the expected value of the observation using the experimental design implemented. The plot is used to check for constant variance.

The data appears to have a random pattern and as a result there is no reason to state that the assumptions have been breached.

- The histogram of residuals: The plot is used to check distribution. Viewed in conjunction with the normal plot, the histogram appears to support the normal distribution and as a result there is no reason to state that the assumptions have been breached.
- Residual versus overall plot: This plot is used to check for observational influence. The data appears to have a random pattern around the central line and as a result there is no reason to state that the assumptions have been breached.

The overall conclusion, based on the residual plots for the results, is that there is no evidence to suggest that standard assumptions of the regression have been violated.

8.6.3.

63. Higher Level Junior Certificate Course Correlation study

Correlations: Results, Test

There is sufficient evidence to support the presence of a slightly weak linear correlation between the two variable tests, as demonstrated by the r-value of -0.531 and a P-value of 0.000 (Appendix XVII:v).

8.6.3.1. Implication of Results (Higher Level Junior Certificate Course)

The t-test demonstrates that there is a difference between test achievement in the 'Traditional' test (test=0) and the 'Realistic' test (test=1) for Irish higher level students. This is indicated with a p-value=0.000, which is less than the set alpha level of 0.05 indicated a significant difference between the tests (fig. 86). Therefore the author rejects the hypothesis '*that there is no difference in achievement levels between the realistic and traditional tests for*

Irish higher level students'. Test=0 (M=80.4, S.D.=13.9) scores higher than test=1 (M=53.1, S.D.=18.4). This shows a difference in the mean scores between tests of 27.3 in favour of the 'Traditional' test (test=0). It is interesting to note that while the mean score in the 'Traditional' test for higher level students is at a level that may be expected (due to the fact that more mathematically students take the Higher level mathematics course), the mean score for the 'Realistic' test is at a very low level (53.1) for students that are considered to be 'more mathematically able'. The Pearson Correlation result is -0.531 indicating a moderately weak, negative correlation between test performance for Higher level, Irish students.

8.6.4. Ordinary Level Junior Certificate Course (all tests)

The following statistical analysis considers the Irish test results for students studying the ordinary level course over both tests ('Realistic' versus 'Traditional'). The analysis seeks to either accept or reject the following:

- The Null Hypothesis: *'That there is no difference in achievement levels between the realistic and traditional tests for ordinary level students'*;

- The Alternative Hypothesis: *'There is a difference in achievement levels between the realistic and traditional tests for ordinary level students'*.

The confidence level is again set at 95% to account for any difference that may arise by chance – the alpha level of 0.05 goes some way towards eliminating this risk.

8.6.4.1. Ordinary Level Junior Certificate Course Descriptive statistics (t=0, t=1)

Descriptive Statistics: Results

Variable	Test	N	N*	Mean	SE Mean	StDev	Minimum	Q1	Median	Q3
Results	0	18	10	41.02	5.46	23.18	0.00	22.08	43.34	61.45
	1	28	0	30.60	3.97	20.99	0.00	10.83	32.50	51.25

Variable	Test	Maximum
Results	0	83.33
	1	68.33

Figure 88: Descriptive statistics of ordinary level junior certification study (t=0, t=1)

8.6.4.2. Ordinary Level Junior Certificate Course Hypothesis test, two sample t-test (t=0, t=1)

The approach taken to determine the difference between the mean responses for the 'Traditional' and 'Realistic' tests at the 5% level of significance is as follows:

- Assess the data sets for equal variance;
- Conduct a two-sample t-test, with or without equal variance, based on the variance findings; and
- Conclude from the t-test whether the difference is significant at the appropriate level.

Minitab provides a test statistic and P-value for both the F-Test and Levene's test. It is of note that both P-values of 0.628 and 0.508 are greater than 0.05 (Appendix XVIII:ii). Therefore, this result is not significant and there is sufficient evidence to conclude that the variances are equal.

Two-Sample T-Test and CI: Results, Test

```
Two-sample T for Results
Test    N  Mean  StDev  SE Mean
0      18  41.0   23.2      5.5
1      28  30.6   21.0      4.0
Difference = mu (0) - mu (1)
Estimate for difference:  10.42
95% CI for difference:  (-2.89, 23.73)
T-Test of difference = 0 (vs not =): T-Value = 1.58  P-Value = 0.122
DF = 44
Both use Pooled StDev = 21.8626
```

Figure 89: Ordinary Level JC course Two sample t-test statistics (t=0, t=1)

Appendix XVIII:iii displays the Individual and box-plot graphics for the ordinary level course. Based on the outcome of the two sample t-test, the author notes the estimate difference is 10.42, which would indicate that there is some difference between the performance responses of the 'Traditional' and 'Realistic' tests, for ordinary level, Junior Certificate students (fig. 89). Based on the p-value of 0.122, the author fails to reject the null hypothesis at the 5% level of significance and concludes that there is no significant difference between the results of the 'Traditional' and 'Realistic' tests for ordinary level, Irish students.

8.6.4.3. Ordinary Level Junior Certificate Course Hypothesis test, One way ANOVA (t=0, t=1)

One-way ANOVA: Results versus Test

The ANOVA output of immediate interest (Appendix XVIII:iv) is the F-test statistic. As the associated P-value is 0.122, one can fail reject the null hypothesis and conclude that the means of the two samples are not statistically different.

Minitab also generates confidence intervals (CIs) for the mean of both tests; the confidence intervals for this study do not demonstrate an overlapping of the intervals for the test samples (Appendix XVIII:iv). Additionally, the post-hoc testing performed using the Tukey test provides confidence intervals for the difference in the pair of means under evaluation. From this analysis, it can be concluded that there is no significant difference between the performance of 'test 0' and 'test 1' as the interval goes from $-$ 23.73 to 2.89 and zero is in the interval. The centre point of the CI is $-$ 10.42 and is the estimated mean difference between the test groups.

In order to test that the ANOVA assumptions were not violated; a residual plot was created (fig. 90). The standard assumptions are as follows:

- The relationship between Y and X must be linear;
- The values are normally distributed; and
- The values of random error are independent.

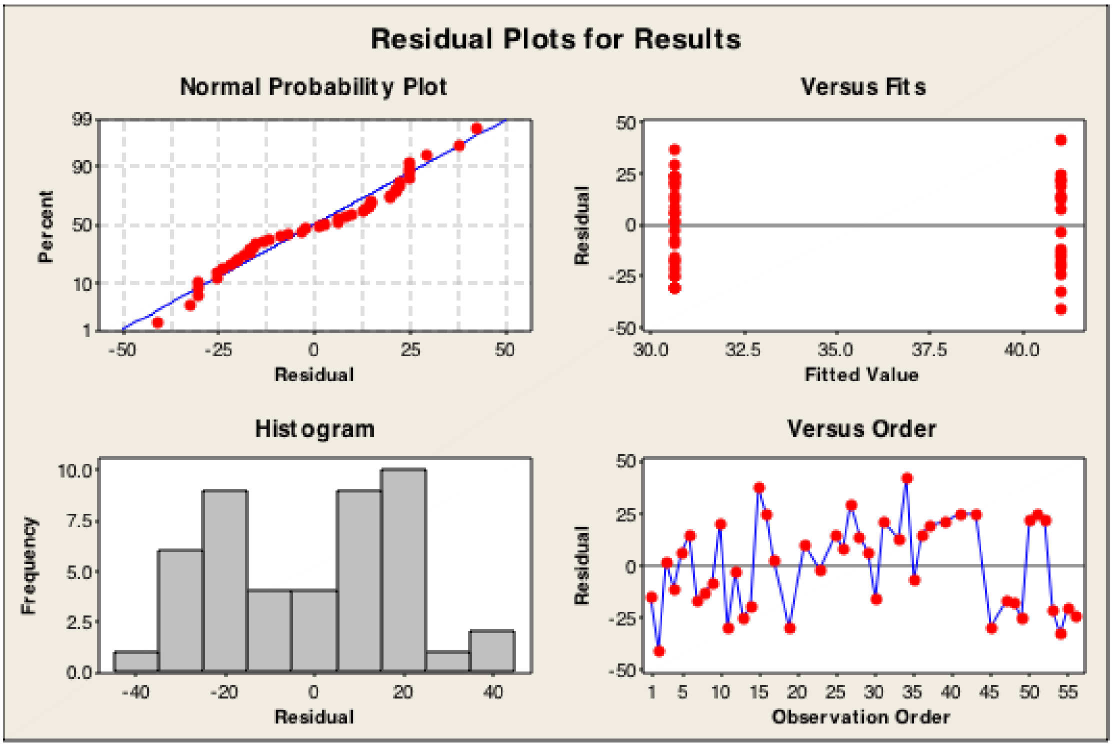


Figure 90: One way ANOVA Residual plot graphics of ordinary level junior certification study (t=0, t=1)

Interpretation of the residual plots:

- The normal probability plot of residuals: Plots the residuals from each observation against the expected value of the residual had it come from a normal distribution. All plotted values appear in a straight line if the residuals are approximately normal (Mendenhall et al, 2009: 489). In this instance the values are in a reasonably straight line which suggests that there is no reason to state that the assumptions have been breached.
- The plot of residual versus fit: This graph plots the residual values against the expected value of the observation using the experimental design implemented. The plot is used to check for constant variance. The data appears to have a random pattern and as a result there is no reason to state that the assumptions have been breached.

- The histogram of residuals: The plot is used to check distribution. Viewed in conjunction with the normal plot, the histogram appears to support the normal distribution and as a result there is no reason to state that the assumptions have been breached.
- Residual versus overall plot: This plot is used to check for observational influence. The data appears to have a random pattern around the central line and as a result there is no reason to state that the assumptions have been breached.

The overall conclusion, based on the residual plots for the results, is that there is no evidence to suggest that standard assumptions of the regression have been violated.

8.6.4.4. Ordinary Level Junior Certificate Course Correlation study

Correlations: Results, Test

There is sufficient evidence to support the lack of presence of linear correlation between the two variable tests, as demonstrated by the r-value of -0.231 and a P-value of 0.122 (Appendix XVIII:v).

8.6.4.5. Implication of Results (Ordinary Level Junior Certificate Course)

The t-test analysis for Ordinary level Irish students across the two tests ('Traditional' and 'Realistic') indicates that there is no significant difference between test performances. The t-test produces a p-value of 0.122. As this p-value is great than the defined alpha-level of 0.05 the test indicates no difference (fig. 89). Therefore, the author fails to reject the hypothesis '*that there is no difference in achievement levels between the realistic and traditional tests for ordinary level students'.* The mean difference between test performances is 10.42, in favour of the 'Traditional' test. Test=0 (M=41.0, S.D.=23.2) scores higher than test=1 (M=30.6, S.D.=21). It is very interesting that there is no significant difference between test performance between the 'Traditional' and 'Realistic' tests for Ordinary level Irish students as there is a significant difference between tests for Higher level students (as shown earlier). The Pearson Correlation result is -0.231 indicating a weak, negative correlation in test performance for Irish, Ordinary level students.

8.6.5. An analysis of the test results between Higher and Ordinary level students

The following table shows the results for Irish higher and ordinary level students for both tests ('Traditional' and 'Realistic').

	Number	**Trad (test=0)**	**Realistic (test=1)**	**p-value**
Higher Level	68	M=80.4, SD=13.9	M=53.1, SD=18.4	0.000
Ordinary Level	18	M=41.0, SD=23.2	M=30.6, SD=21.0	0.122

Table 26: Test results for higher and ordinary level students

As expected students who were following the higher level Junior Certificate course scored better than those who were studying the Ordinary Level course in both the 'Traditional' and 'Realistic' test. There is a mean difference for the 'Traditional' test of 39.4 in favour of students following the Higher level course. The mean difference for the 'Realistic' test is 22.5 which is noticeable lower. It should also be noted that, as discussed earlier, while there is a difference in test performance for Higher level students between test=0 and test=1 (p=0.000), there is no significant difference for Ordinary level students (p=0.122). The implication appears to be that students who perform well in the 'Traditional' test are less likely to score at a similar level in the 'Realistic' test than Ordinary level students. Those students who perform at the lower end of the ‘Traditional’ test and are more likely to perform at a similar level in the ‘Realistic’ test.

8.7 Summary of Test Findings

The analysis of the test findings shows that there is a significant difference between test performances in the ‘Traditional’ test versus the ‘Realistic’ test. The two tests implemented for all students (n=157) show a mean result of 75.6% for the ‘Traditional’ test and a mean of 55.3% for the ‘Realistic’ test. The mean difference is substantial, 20.3%. One would assume that Irish students perform better in the ‘Traditional’ test due to the fact that it is based on the Junior Certificate examination style to which they are accustomed. This assumption holds true with Irish students (n=86) obtaining a mean of 72.2% in the ‘Traditional’ test and 46.5% in the ‘Realistic’ test. However, unexpectedly the students from Massachusetts performed at a higher rate than the Irish students in the Junior Certificate questions posed in the ‘Traditional’ test with a mean score of 79.7%. Students from Massachusetts also outperformed Irish students in the ‘Realistic’ test with a mean score of 67.2%. There was no significant difference in test performance due to gender. However, the slight difference that exists favours female students with girls scoring a mean of 77.2% in the ‘Traditional’ test and 58.1% in the ‘Realistic’ test, versus males scoring 74.2% in the ‘Traditional’ test and 53.2% in the ‘Realistic’ test.

9.0 Chapter 9: The Conclusion

9.1 Introduction

The purpose of this work is to establish if Irish students have the ability to mathematise and transfer the mathematical knowledge learned in the classroom to unfamiliar, problem-solving situations. The author selected data collection techniques, including a systematic, structured observation, testing and semi-structured interviews, in order to gain an insight into Irish mathematics education in general, and the research question in particular.

9.2 Findings from the Structured Observation

The systematic, structured observations implemented in the classrooms of the Irish research participants provide interesting data, in that the activities in the classes observed are very similar. None of the mathematics lessons observed involved active-learning, group work or the use of information technology. All lessons observed involved high levels of teacher explanation, teacher question and answer, student question and answer, book work, board work and teacher instruction. It is interesting to note that the activities occurring in Irish mathematics lessons are behaviourist and traditional in nature with teaching to the examination very much the focus.

9.2.1. Teaching in Ireland

Interestingly, the systematic, structured observation of the class groups involved suggested that teaching practices in Ireland are largely behaviourist and relativist in style. While past literature may have suggested this to be the case, the complete lack of relativist teaching methods in the classrooms observed was surprising. All seven teachers observed in the structured observations taught in very much the same way, despite the differences in teaching experience, gender and age. Group-work, active learning, discovery learning and other relativist practices were not observed. In the subsequent interviews the teachers commented that the observed class is indicative of what they would usually do and how a class would normally unfold. One would imagine that when under pressure, as is possibly the case when one is being observed, human nature would dictate that one would teach in a manner which one considers one's best performance. This would suggest that Irish teachers' comfort zone involves a significant amount of teacher explanation, teacher questioning, book work, and perhaps most noticeably board work involving the teacher writing on the board and explaining as they write.

Despite information technology resources being freely available in all schools involved in the research, the teachers involved did not appear to use them. No technological resources were used in the observed lessons. Only two teachers made any reference to using I.T. (information technology) resources when interviewed. Despite significant financial funding for the acquisition of I.T. resources in schools the research suggests that I.T. has not, as yet, made a significant impact on mathematics teaching and learning in Ireland. I.T. training has also been provided for mathematics teachers in conjunction with the in-service training available for the 'Project Maths' curriculum. Specific mathematics and I.T. courses have been made available for all second-level mathematics teachers in the evening time. In addition to this, the 'Project Maths' workshops that all second-level mathematics teachers are required to attend (during the school day) provide training in teaching mathematics with the use of I.T. resources as an aid. All seven teachers observed had attended three 'Project Maths' workshops at the time of observation and as a result would have some training in I.T. resources as a teaching and learning tool.

All schools involved in the study also had significant I.T. resources including, but not limited to, teacher lap-tops (with financial aid towards purchase provided by the school), teacher and student computers, and interactive white-boards. The lack of incorporation of these resources in the mathematics lessons observed, and the impression gained by the author during the interviews, would suggest that the majority of the teachers observed rarely, if ever, use I.T. as a teaching and learning resource.

Irish teachers may be resistant to changing the tried and tested behaviourist teaching methods of old, which they are familiar with and have developed to a proficient degree. This may be a significant problem is attempting to rectify the poor performance of Irish mathematics students in international assessments. Generations of teaching, learning and assessing mathematics in one particular way creates a resistance to the incorporation of new techniques and teaching methods in the classroom. It should be noted that, for the most part, this resistance is not due to teachers trying to be difficult, but rather a difficulty and indeed a nervousness on the teachers' part in attempting to incorporate new techniques. A fear of failure is a very real worry for Irish mathematics teachers with regard to implementing new teaching and learning techniques. The interviews provided the author with an insight into the trepidation felt by the teachers interviewed with regard to incorporating the new teaching methods required for successful implementation of the 'Project Maths' curriculum. With this level of trepidation it is important that teachers are provided with sufficient in-service training and ongoing assistance where required. One must wonder if the roll-out of 'Project Maths' just two years after implementation in the pilot schools, and with teachers having attending two 'Project Maths' workshops at the time of initial implementation of the curriculum in September 2010, is sufficient, both time and preparation wise, for the vast changes required in teaching and learning techniques.

9.3 Findings from the Semi-structured Interview

The purpose of the semi-structured interviews was to collect qualitative data that would give context to the quantitative data produced by the implementation of tests and the structured observations. The findings gave context to the individual situation for each class group and verified that the activities observed in the mathematics lesson on the day of the structured observation were typical of a mathematics class with that group. The interviews provided the teachers involved with the research the opportunity to provide any additional information they deem pertinent.

The general findings from the semi-structured interview include:

1. Positive and enthusiastic attitudes displayed by all teachers interviewed to mathematics and mathematics teaching;
2. A behaviourist approach to teaching and learning is the common theme when the teachers interviewed describe teaching and learning in their mathematics lessons;
3. All teachers interviewed displayed an open-mind with regard to their involvement in the author's research;
4. An appreciation of young people and their qualities;
5. A security in the familiarity of the course;
6. Trepidation with regard to the implementation of the new 'Project Maths' curriculum and the impact this will have on their teaching techniques; and
7. A frustration with the restrictions placed on teaching and learning by the terminal examination.

9.4 Overall test findings

The overall test findings show that students in both Ireland and Massachusetts performed better on the Traditional test than on the Realistic test. The mean score for the Traditional test is 75.6% compared to 55.3% for the Realistic test, with a mean difference of 20.23. It is interesting to note that the mathematics required in the Realistic test were considerably less difficult in terms of mathematical content than the content in the Traditional test. The mean difference is exceptional when one considers the content level and mathematical knowledge required to successfully answer the questions in each test. Therefore the poorer performance in the Realistic test is possibly due to the following factors:

- Lack of familiarity with the question format;
- Difficulty in recognising the mathematical information required when the questions are presented in a different format;
- Inability to mathematise (that is the ability to transfer a realistic situation into a mathematical situation);
- Difficulty when applying mathematical knowledge in unfamiliar situations;
- Experience of learning for knowledge acquisition rather than understanding;
- Inexperience when provided with context – this possibly confuses some of the students as they now have other factors to consider. Research shows that female students value context as it makes the mathematical situation relevant to their lives and to their reference points – male students tend to perform well in context free situations and do not display the same need to be connected to the familiar with the provision of context (Tims, 1994; Bolger and Kellaghan, 1990). One should ask the question how so do male students perform so well in the PISA assessment series?;

- Confusion when faced with open-ended questions. Irish students are familiar with closed-ended questions only, and do not encounter open-ended questions in either the curriculum or the state examinations. As a result it is unusual for Irish teachers to provide students with open-ended mathematics questions as they are not related to the curriculum;

- The vagaries of real-life are unexpected and confusing. Irish students are trained to see mathematics as an exact process used only in exact situations. Anything that appears to have numerous valid possibilities may be confusing as it is against everything they have been trained to expect;

- Difficulty in assessing what is being asked, and what information is unnecessary, when dealing with authentic questions. Irish students never encounter mathematics questions where surplus information is required – in the Irish mathematics tradition one uses all information provided. As a result, the style of the Realistic test where the numerical data provided does not necessarily relate directly to the questions asked, is unfamiliar to Irish students; and

- For some students the higher word content used in the Realistic test is possibly an issue. This is not a usual aspect of Irish mathematics examination questions where the word content, and therefore the amount of reading needed, is normally kept to a minimum. Irish students are familiar with a higher proportion of numerical data, and relatively little word content. The high-word content, and amount of reading involved, in solving the questions in the Realistic test are probably a particular issue for students with reading issues, including students with dyslexia. No precise questions were asked in the interviews relating to this, but at all stages of testing students were told that help was available if there were any issues with reading. A small number of Irish students had SNAs (special needs assistants) available for individual assistance with reading when completing the tests. Despite the availability of literary assistance to all students, the unfamiliar nature of the mathematics questions in the Realistic test in terms of the amount of reading required may still have been an issue for some students.

The suggestion appears to be that students, regardless of nationality, gender or ability, perform better in the Traditional test. As explained in more detail later in this chapter this tends to be a far less significant difference as mathematical ability reduces (students who scored higher than 80% in the Traditional test have a between test mean difference of 28.11, whereas students who scored less than 60% in the Traditional test have a mean test difference of 2.24). The higher performance in the Traditional test across the board appears to suggest that it is easier to teach and learn mathematics for knowledge acquisition rather than for understanding. In the Irish scenario it would appear that behaviourist and absolutist teaching methods encourage the skills needed for successful performance in traditional tests which have the following characteristics:

- Context-free;
- Closed-ended questions;
- Easily identifiable content;
- Posed in a familiar format;
- Requiring little to no real-life experience; and
- Providing only the necessary information for successful completion of the question (no surplus information is provided).

The same teaching methods ill-prepare students for the authentic, open-ended questions required for successful performance in the Realistic test. In-class practice of the following techniques are essential if students are to develop the necessary skills to transfer the mathematics learned in the classroom to authentic, real-life scenarios:

- mathematisation techniques;
- modeling;

- investigative learning;
- group-work; and
- discussion.

9.4.1. No noticeable gender difference

Interestingly, there was no evidence of any significant gender difference in test performance. In terms of overall test performance there is no significant gender difference between male and female students. Female students achieved a mean score (across both tests) of 67.5%, compared to a mean score for male students of 63.2%. The mean test difference of 4.31, favouring female students, is not a significant difference but is notable as PISA test results consistently show a significant difference in test performance in favour of male students across the vast majority of OECD countries. The following table shows Irish gender performance in three of the PISA assessments (Eivers et al, 2007 and Shiel et al, 2009):

PISA	Female	Male	Difference
2003	495	510	15 (in favour of males)
2006	495.8	507.8	12 (in favour of males)
2009	483	491	8 (in favour of males)

Table 27: Irish gender performance in three of the PISA assessments (Eivers et al, 2007 and Shiel et al, 2009):

The above table considers Irish gender performance in PISA 2003, 2006 and 2009. Shiel et al (2001: viii) note that in PISA 2000 male students performed significantly better than females by about one-sixth of a standard deviation. As is notable from the table above, male Irish students consistently outperform female students in mathematics performance in the PISA assessment series. While the difference is not significant in the case of the years mentioned in the table above, it is still interesting and cannot be dismissed that male students outperform female students in each PISA cycle. This is notable in its contrast to the results found by the author (as mentioned above) where female students outperformed male test performance across both tests.

In terms of female only performance, test performance was higher in the Traditional test, with a mean score of 77.2%. Female test performance in the Realistic test was 58.1%. The difference between tests of 19.16 in favour of the Traditional test is only marginally lower, and not dissimilar, to the mean difference between tests for all students (male and female) of 20.23. Male test performance in the Traditional test is better than test performance in the Realistic test with mean values of 74.2% and 53.2%. The mean difference of 20.98 is marginally higher than that between tests of all students (20.23).

9.4.2. Massachusetts' students

Students in Massachusetts are taught mathematics in a different way to those in Ireland. Interviews with the Massachusetts' head of department suggest that all of the following components are incorporated in mathematics lessons in the school involved in the research:

- Group work;
- Individual work;
- Continuous assessment;
- Class tests;
- Discovery learning;
- Project work;
- Problem-solving involving open-ended questions;
- Teaching and learning involving the use of a text-book;
- Teacher explanation while standing at a white-board;

- Computer-work and the use of other information technology resources; and
- Reinforcement through questions and answers.

As is evident from the teaching and learning methods, and resources, identified as those commonly used in the mathematics classroom in Massachusetts, the level of variety is much greater than that in the Irish classroom. The learning theory underpinning mathematics activity in the Massachusetts classroom appears to include both absolutist and relativist practices. This is in stark contrast to the Irish classroom where mathematics activity appears to be firmly steeped in the behaviourist and absolutist theories of learning. In all seven mathematics lessons observed in Ireland there was no group work, active learning, exploratory learning, project work or activities involving computers or other I.C.T. (information and communication technology) resources.

Interestingly, the Massachusetts cohort not only out-performed their Irish associates in the Realistic test but they also scored at a surprisingly high level in the Traditional test. This is particularly interesting, and unexpected, as Irish students are specifically geared towards this style of examination, and the Traditional test used in the testing process is derived directly from past Junior Certificate examinations. As a result the author did not anticipate a particularly strong performance for the Massachusetts contingent as the test is based directly on the Irish curriculum and examines content that the Irish students involved in the research would have studied by the time of testing. The format of the Traditional test would also be familiar to Irish students as most Irish mathematics tests in second-level schooling would be directly based on this format. The teachers in Massachusetts were not consulted as to content studied prior to testing so it is surprising that students from Massachusetts matched Irish students in terms of performance on the Traditional test when all these factors are taken into account. As the least able students from the school in Massachusetts involved in the study did not participate (due to the fact they had not commenced an algebra course at the time of data collection), when comparing Traditional test performance between Ireland and Massachusetts the author considered Irish students following the higher level course. The Irish mean score (for higher level students) in the Traditional test is 80.4% and for students from Massachusetts the mean score is 79.7%. The mean difference is marginal at 0.75.

Students from Massachusetts also performed well in the Realistic test and outperformed Irish students significantly. Again the author considered Irish higher level students for comparison purposes. The Irish higher level mean result is 53.1% versus a mean result of 67.2% for students from Massachusetts. The mean difference is 14.1, in favour of students from Massachusetts. If teaching and learning methods in Massachusetts are considerably different to those implemented in Ireland, and it would appear they are, then this appears to have a positive impact on teaching for understanding as demonstrated by the superior test performance of Massachusetts' students in the Realistic test. As mentioned above, this positive impact on test performance in the Realistic test does not have a negative impact on test performance in the Traditional test, and indeed if one takes into account that the Massachusetts cohort were not directly catered for in the content choice, or in particular the assessment style, one would imagine that the traditional mathematics taught and valued in Ireland are not down-played in Massachusetts. The combination of traditional and authentic mathematics taught in Massachusetts has the benefit of catering for both teaching for knowledge acquisition, and teaching for understanding.

9.4.3. High-performing students

The most significant result from the research is perhaps the variance in test difference between higher performing and lower performing mathematics students. Students who perform well in the Traditional test (thus demonstrating an ability to reproduce and display mathematical knowledge) tend to perform significantly worse in the Realistic test (which requires the application of mathematical knowledge and reflection, demonstration understanding). There is a particularly large gap for high-achieving, Irish students. Teaching in Ireland prepares mathematically able students for the Traditional test but does not appear to influence overall mathematical understanding if the Realistic test results in this study are indicative of a general trend. There is a significantly greater discrepancy in test performance for high-achieving students than for lower performing students. Students who performed at the lower end of the scale in the Traditional test had a much smaller mean difference between tests.

Irish higher level students performed better in both tests than ordinary level students (as would be expected). The interesting data arose from the significantly greater difference between test performances for higher level students than for their ordinary level colleagues. Irish higher level students scored a mean result of 80.4% in the Traditional test and a mean score of 53.1% in the Realistic test. This gave a mean difference of 27.3 in favour of the Traditional test. While the author anticipated some difference in between test performance, and past performance in International assessments would suggest that Irish students may perform better in the Traditional test, the vast mean difference was still surprising. The implemented t-test shows a significant difference in between test performance for the higher level group. Ordinary level Irish students scored a mean result of 41.0% in the Traditional test and 30.6% in the Realistic test. This resulted in a mean difference of 10.42. The implemented t-test also shows no significant difference in between test performance for the Ordinary level group. This possibly arises from the fact that in some instances Ordinary level students performed better in the Realistic test than in Traditional test. The mean difference between higher and ordinary level students in the Traditional test is 39.4, but for the Realistic test it is considerably lower at 22.5. The higher level students, by virtue of the fact that they are following the higher level course, have mastered the methods used to succeed (for the most part) in the higher level assessments. This would suggest that the teaching and learning methods used in teaching mathematics for knowledge acquisition are successful for this particular cohort of students. While natural mathematical ability no doubt plays some role in determining mathematical success, the possibility exists that the style of context-free mathematics that Irish mathematics education currently embraces is neglecting a cohort of mathematics students that require a different learning style. The discrepancy in the mean test difference for ordinary and higher level students supports the possibility that this may be the case. Authentic scenarios, real-life reference and the provision of information to provide context may be a necessity for some learners.

In order to expand on this concept of a greater difference in between test performance for more mathematically able students (based on the Irish concept of mathematical ability) the author decided to group all students by performance in the Traditional test. This also includes the students from Massachusetts in the testing with regard to ability. The author used student performance in the Traditional test as an indicator of mathematically ability. The reasoning behind this is that Irish classes are streamed into higher and ordinary level classes, and ability groupings within these levels, based on traditional mathematics test results. The author considered three groups from the research sample:

- Students who scored a result greater or equal to 80% in the traditional test;
- Students who obtained a score between 60% and 80% (60%≤x<80%) in the traditional test; and
- Students who scored less than 60% in the traditional test.

The between test performance gap was significantly greater for students in the first group (≥80%) than students in the third group (<60%). Students in the group who scored ≥80% achieved a mean result of 90.54% in the Traditional test compared to 62.4%. This gives a significant mean difference of 28.11, in favour of the Traditional test. Students in the second group (based on a Traditional test result of between 60% and 80%) scored a mean result of 71.37% in the Traditional test compared to 56.3% in the Realistic test. The mean difference in this instance is considerably lower than for those students in the first group (≥80% in the Traditional test) at 15.04, favouring the Traditional test. The third group, involving students who scored less than 60% in the

Traditional test, scored a mean result of 42.2% in the Traditional test compared to 40% in the Realistic test. The mean difference for this group is negligible at 2.24 (favouring the Traditional test).

The following table demonstrates the mean difference for the various ability groups considered.

Group	Mean difference	Significant Difference	Better test performance
≥80% in Traditional Test	28.11	Yes	Traditional
60%≤mean<80% in Traditional Test	15.04	Yes	Traditional
<60% in Traditional Test	2.24	No	Traditional
Irish Higher level	27.3	Yes	Traditional
Irish Ordinary level	10.42	No	Traditional

Table 28: Mean difference for the various ability groups

High achieving students demonstrate an ability to learn mathematical knowledge effectively, recognise when they are being asked to showcase this, and reproduce the knowledge when required. Despite the acquisition of these skills, and a demonstration in the Traditional test of their ability to use them, poor test performance in the Realistic test indicated that there is little mathematical understanding and an inability to recognise what is being asked (in a practical sense and mathematically). The poor performance by high achieving mathematics students in the Realistic test demonstrates inadequate problem-solving skills when faced with an authentic task. It is important to reiterate that the mathematical skills required to solve the questions in the Realistic test were no more difficult, and possibly easier, than those in the Traditional test. Therefore as mentioned earlier in this chapter, the perceived difficulty of the Realistic test is possibly due to the following factors:

- Lack of familiarity with authentic problem-solving questions;

- Difficulty in recognising what is being asked on both a practical and mathematical level;
- Inexperience with open-ended questions;
- Confusion when provided with information that provides context but may not be required (in a mathematical sense) to find a solution;
- Discomfort with the high word count and the level of reading required; and
- Lack of familiarity with the question style.

The discrepancy in test performance between the Traditional test and the Realistic test reduces as student performance in the Traditional test reduces. It is interesting to note that there is a noticeably smaller gap in between test performance for both ordinary level students, and particularly for all students who scored less than 60% in the Traditional test. One can also note, from analysis of test performance for those students who scored between 60% and 80%, that mean difference between tests reduces as student performance in the Traditional test decreases. It is interesting to note the narrowing of the performance gap as scores in the Traditional test lower. There is also no significant difference between tests for both ordinary level students and those students who scored less than 60% in the Traditional test.

Is the suggestion that teaching can improve performance when it comes to knowledge acquisition but that it has less of an impact when application skills and teaching for understanding is involved? Or, in the Irish case in particular, is the implication that behaviourist methods of teaching iron out difficulties students may have when it comes to reproducing knowledge but these absolutist methods fail to prepare students for authentic, real-life problem-solving scenarios? Perhaps teaching and learning influenced by absolutist learning theories favour students that learn mathematics more easily and this results in higher performance in the traditional style assessment format, but fails to prepare these same students for realistic scenarios. Or as suggested above, the absolutist model currently followed in Ireland may suit some learners but neglect others, especially those who require contextual information.

9.5 Irish mathematics performance 2003-2009

In the course of the research the author developed an awareness of how mathematical performance in the Irish examination system is not in line with Irish performance in international mathematics assessments such as PISA. As discussed in Chapter 3, Irish results declined significantly in the period from 2003 to 2009 (incorporating the assessments PISA 2003, 2006 and 2009). In contrast to this, examination performance in the mathematics Junior Certificate examination did not change significantly over the same period and participation rates at higher level increased (for comparison purposes the author specifically looks at Junior Certificate examination results for the same years as the PISA assessments – 2003, 2006, 2009).

In the Junior Certificate examination in 2003, 43.12% of students of students sat the higher level paper, 46.86% sat the ordinary level paper and 12.53% the foundation level paper. The percentage of A-grades at higher, ordinary and foundation level respectively was 17.2%, 9.2% and 15.4%. In the 2006 Junior Certificate mathematics examination the participation rates at higher and ordinary level had increased, while at foundation level they had reduced. All of these participation changes would indicate improvement with regard to mathematical ability – if the number of students sitting the examination at the upper two levels is increasing while the proportion of students sitting the examination at the most basic level reduces, one would imagine mathematical performance is improving. The increase in participation rates from 2003 to 2006 was as follows:

- Higher Level: 1.88% of an increase to 42.29%. The number of A-grades at this level increased also by 0.8% to 18%.

- Ordinary Level: 0.22% of an increase in participation to 47.08%. The number of A-grades at this level also increased from 9.2% to 13.3%, an increase of 4.2%.

- Foundation Level: A reduction in participation of 2.1% to 10.43%. The number of A-grades at this level increased from 15.4% to 17.1%, an increase of 1.7%.

The changes that occurred from the 2003 examination to the 2006 Junior Certificate examination are all positive in terms of mathematical performance. As outlined above, participation rates increased at the two highest levels and decreased at the most basic level. Furthermore, the number of A-grades at each level also increased, which would suggest that the number of students excelling at each level is also improving. In contrast to this, Irish performance in PISA declined over the same period, from a mean score of 502.8 in 2003 to 501.5 in 2006. The number of students scoring at the highest levels (level 5 and 6) in the PISA 2006 assessment also reduced from 11.3% in 2003 to 10.2% in 2006.
The participation rates of Irish students sitting the higher level Junior Certificate mathematics examination continued to increase for higher and ordinary level and reduce for the foundation level paper in the period from 2006 to 2009 as follows:

- Higher Level: Participation levels increased from 42.49% to 43.12%, an increase of 0.63%. The number of A-grades reduced slightly from 18% to 16.7%.

- Ordinary Level: Participation levels increased by 0.35% to 47.4%. The number of A-grades reduced from 13.3% to 11.7%.

- Foundation Level: The number of students sitting the foundation level mathematics examination reduced from 10.43% to 9.48%, a reduction of 0.95%. The number of A-grades at this level increased from 17.1% to 19%.

In the period from 2006 to 2009 participation rates continued to suggest improved mathematical performance in Ireland, with an increase in the proportion of students sitting the higher and ordinary level papers and a reduction in those sitting the foundation level paper. The number of A-grades at higher and ordinary level reduced slightly. This reduction in grades at the highest level could be due to the increased participation rate – students that previously may have attained an A at ordinary level were now sitting the higher level examination and possibly obtaining a relatively lower grade.

It is interesting that while participation rates continued to improve in this period, 2006 to 2009, and the number of A-grades did not change significantly, Irish mathematical performance in PISA deteriorated. The Irish mean score in the mathematics component of PISA 2006 is 501.5. In the PISA 2009 assessment this had reduced to 487.1, a reduction of 14.1 points. Even more poignantly, Ireland had moved from a position of 16th place among the OECD countries in PISA 2006 to 26th position in PISA 2009. In 2006 the Irish score did not differ significantly from the OECD average, however, in 2009 Ireland's score was now ranked as significantly below the OECD average. All of these results from PISA are disturbing. Despite this Junior Certificate performance for the same period remained relatively unchanged.

The following table shows the pattern of results for both the Junior Certificate mathematics examination and the PISA assessments from 2003 to 2009:

Year	PISA mean score	PISA rank (among OECD countries)	Level 5&6 in PISA	H.L. participation Junior Cert. (% of A's)	O.L. participation Junior Cert. (% of A's)	F.L. participation Junior Cert. (% of A's)
2009	487.1	26th	6.7%	43.12% (16.7%)	47.4% (11.7%)	9.48% (19%)
2006	501.5	16th	10.2%	42.49% (18%)	47.08% (13.3%)	10.43% (17.1%)
2003	502.8	20th	11.3%	40.61% (17.2%)	46.86% (9.2%)	12.53% (15.4%)

Table 29: Irish mathematics performance in both PISA and the Junior Certificate examination

2003, 2006, 2009 (www.examinations.ie, Eivers et al, 2007 and Shiel et al, 2010).

It is worth questioning why Irish mathematical performance continues to succeed on its own terms, in the Junior Certificate examination, when it is clearly deteriorating at an international level. The most likely answer is that the teaching and learning in the Irish mathematics classroom is specifically geared towards one particular examination style, the Junior Certificate, with little regard for the qualities that PISA considers important: reflection, analysis and problem-solving skills. The behaviourist teaching methods used in the Irish classroom are effective in preparing students to succeed in the Junior Certificate examination which consists of predictable questions requiring recognition, memorisation and reproduction skills. These skills are not particularly valued in terms of international assessment where the emphasis has moved towards preparedness for the real world. It is in this regard that Irish mathematics education is struggling the most.

9.6 Curriculum objectives

A primary objective of the incoming 'Project Maths' curriculum is to '*allow students to appreciate how mathematics relates to real-life and work*' and to '*make mathematics more meaningful for students and relatable to their own life* *experience'* (www.ncca.ie). It is interesting to note that the relatability of mathematics to real-life has been a recurring theme in the objectives of the various mathematics curricula in the Irish education system. The 1973 objectives of the Intermediate Certificate mathematics course included '*an understanding and association of mathematics and their role in everyday life'* (Report of the Irish National Committee, 1976: 18). The Junior Certificate objectives from the curriculum introduced in 2000 (for examination in 2003) state that students should have the ability to apply their mathematical knowledge and '*they should be able to use mathematics (and perhaps also to recognise uses beyond their own scope and employ) – hence seeing that it is a powerful tool with many areas of applicability'*. The 2000 curriculum objectives also state that students should develop the ability to analyse information, including information presented in unfamiliar contexts (NCCA, 2002:9).

The author finds it interesting that the objective with regard to relatability to real-life experience remains relatively consistent through the syllabus changes. Despite this, Irish students continue to struggle with this aspect in assessments. Employers also complain that while Irish graduates have significant mathematical knowledge, their ability to apply this knowledge is of a low standard and is a consistent failure in graduates they employ. Despite the enthusiastic objectives of the 'Project Maths' curriculum, which include making mathematics more relatable to students' personal experience and more meaningful for students, it remains to be seen if the new curriculum will improve students' application of mathematics. While the author believes that this objective is admirable, she is very aware that this has been a consistent objective, in some shape or form, since 1973 and unfortunately has not had any considerable impact on the ability of Irish students to demonstrate any significant ability to utilise their mathematical knowledge in unfamiliar scenarios as demonstrated by the worsening performance of students in international assessments (including TIMSS and PISA), and indeed by the results from the tests in this research.

9.7 Limitations of the study

There were several limitations to this study, not least the proposition of great statements based on the relatively low numbers studied. The author accepts that this study is indicative of current trends in mathematics education among the cohorts studied but does not represent Irish mathematics education in its entirety. The two tests, the Traditional and the Realistic, were relatively subjective in how the marks were allocated to the different questions (see Appendix VII:ii; VIII:ii; IX:ii; X:ii). This weakens the validity and the reliability of the tests somewhat, and thus the comparability of the scores of the two tests. Another limitation of the quantitative aspect of the study is the fact that the Massachusetts sample was not matched to the Irish sample on any relevant variable which meant that analysis of covariance could not be carried out.

9.8 Conclusion:

This research suggests that Irish students struggle to mathematise and use their mathematical knowledge in unfamiliar situations. The behaviourist style of the teaching and learning implemented in Irish mathematics lessons is of considerable note as it does not give ample opportunity to develop the necessary skills for problem-solving in authentic situations. While the new ‘Project Maths’ curriculum aims to address this issue the author is of the opinion that the Junior and Leaving Certificate assessment style remains an issue and runs the risk of repeating the pattern of the current assessment with regard to question predictability. Curriculum change, as with the introduction of ‘Project Maths’, is essential but the single most important factor for change rests with the teachers and their teaching and learning belief system and techniques. While in-service training provides some assistance in this regard the author is of the opinion that one cannot simply roll out a new curriculum and hope that it will adjust classroom practices – significant work must focus on changing the teaching and assessment habits of generations of Irish teaching and learning experiences.

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Appendices

Appendix I - Junior Certificate Mathematics Examination Results

Year (Level)	Gender	A	B	C	D	E	F	G
2009(H)	Female (51%)	16.5	31.9	29.1	18.8	3.2	0.4	0.0
2009(H)	Male (49%)	16.9	31.4	29.2	18.4	3.5	0.6	0.0
2009(O)	Female (48.22%)	13.4	35.1	28.3	17.1	4.5	1.5	0.1
2009(O)	Male (51.78%)	10.2	31.9	30.7	18.7	6.3	2.1	0.1
2009(F)	Female (43.14%)	17.4	34.2	29.1	15.6	3.2	0.4	0.0
2009(F)	Male (56.86%)	20.2	31.5	27.8	16.0	3.7	0.6	0.2
2008(H)	Female (51.58%)	16.2	32.9	32.1	16.5	2.0	0.2	0.0
2008(H)	Male (48.42%)	17.2	30.1	31.1	18.3	2.7	0.5	0.0
2008(O)	Female (48.5%)	13.8	37.6	27.5	15.2	4.3	1.5	0.1
2008(O)	Male (51.5%)	10.9	34.4	29.4	17.5	5.4	2.2	0.2
2008(F)	Female (42.84%)	17.0	39.0	28.8	12.3	2.4	0.5	0.0
2008(F)	Male (57.16%)	19.4	36.3	26.8	13.4	3.3	0.7	0.0
2007(H)	Female (51.93%)	18.1	30.9	27.8	18.4	4.1	0.6	0.0
2007(H)	Male (48.07%)	17.1	28.8	28.3	20.0	4.7	1.0	0.1
2007(O)	Female (49.27%)	10.4	34.7	30.5	18.9	4.4	1.1	0.1
2007(O)	Male (50.73%)	8.3	30.4	32.3	21.4	5.8	1.7	0.1
2007(F)	Female (42.13%)	16.0	33.4	30.7	16.4	3.1	0.4	0.0
2007(O)	Male (57.87%)	16.3	32.0	30.6	16.7	3.5	0.8	0.1
2006(H)	Female (51.41%)	17.9	32.6	29.1	16.7	3.0	0.5	0.0
2006(H)	Male (48.59%)	18.1	31.4	28.3	17.8	3.7	0.7	0.1
2006(O)	Female (48.68%)	14.9	38.2	27.3	14.9	3.6	1.0	0.1
2006(O)	Male (51.32%)	11.8	35.3	28.5	17.2	5.2	1.9	0.1
2006(F)	Female (43.36%)	15.6	35.6	31.1	15.1	2.0	0.5	0.0
2006(F)	Male (56.64%)	18.2	39.4	27.3	12.7	2.1	0.3	0.0
2005(H)	Female (52.2%)	14.1	31.8	31.4	19	3.3	0.4	0.0

Year (Level)	Gender	A	B	C	D	E	F	G
2005(H)	Male (47.8%)	14.9	28.6	30.3	21	4.4	0.8	0.0
2005(O)	Female (48.73%)	14.1	35	28	16.7	4.6	1.4	0.1
2005(O)	Male (51.27%)	9.5	30.2	29.4	20.7	7.1	2.8	0.3
2005(F)	Female (41.59%)	18.6	31.9	26.9	17.3	4.2	1.1	0.0
2005(F)	Male (58.41%)	17.6	31.2	27.9	17.9	4.5	0.9	0.0
2004(H)	Female (52.59)	16.88	30.15	28.86	18.84	4.34	0.88	0.04
2004(H)	Male (47.41)	15.24	26.36	28.9	21.86	6.07	1.44	0.14
2004(O)	Female (48.97)	11.8	36.62	29.52	16.35	4.22	1.36	0.1
2004(O)	Male (51.03)	8.43	32.01	31.95	18.95	6.06	1.61	0.21
2004(F)	Female (43.12)	16.17	40.65	28.95	12.43	1.66	0.14	0.0
2004(F)	Male (56.88)	16.53	40.24	29.21	11.75	1.84	0.43	0.0
2003(H)	Female (52.17%)	17.74	35.24	28.29	15.68	2.56	0.44	0.03
2003(H)	Male (47.83%)	16.66	31.73	28.98	18.38	3.62	0.61	0.03
2003(O)	Female (49.67%)	10.84	33.47	30.47	18.95	4.88	1.33	0.06
2003(O)	Male (50.33%)	7.58	28.57	32.12	22.68	6.77	2.18	0.09
2003(F)	Female (41.32%)	13.71	37.77	31.26	13.62	3.24	0.40	0.00
2003(F)	Male (58.68%)	16.61	37.83	28.29	13.66	3.09	0.40	0.05

Appendix II- TIMSS 2003 Achievement Testing for 8th Grade

Countries	**Years of Schooling**	**Average Age at Time of Testing**	**Average Scale Score**	**Human Development Index**
Singapore	8	14.3	605 (3.6)	0.884
Korea, Rep. of	8	14.6	589 (2.2)	0.879
Hong Kong, SAR	8	14.4	586 (3.3)	0.889
Chinese Taipei	8	14.2	585 (4.6)	-
Japan	8	14.4	570 (2.1)	0.932
Belgium (Flemish)	8	14.1	537 (2.8)	0.937
Netherlands	8	14.3	536 (3.8)	0.938
Estonia	8	15.2	531 (3.0)	0.833
Hungary	8	14.5	529 (3.2)	0.837
Malaysia	8	14.3	508 (4.1)	0.79
Lativa	8	15.0	508 (3.2)	0.811
Russian Federation	7 or 8	14.2	508 (3.7)	0.779
Slovak Republic	8	14.3	508 (3.3)	0.836
Australia	8 or 9	13.9	505 (4.6)	0.939
United States	8	14.2	504 (3.3)	0.937
Lithuania	8	14.9	502 (2.5)	0.824
Sweden	8	14.9	499 (2.6)	0.941
Scotland	9	13.7	498 (3.7)	0.930
Israel	8	14.0	496 (3.4)	0.905
New Zealand	8.5-9.5	14.1	494 (5.3)	0.917
Slovenia	7 or 8	13.8	493 (2.2)	0.881
Italy	8	13.9	484 (3.2)	0.916
Armenia	8	14.9	478 (3.0)	0.729
Serbia	8	14.9	477 (2.6)	-

Countries	Years of Schooling	Average Age at Time of Testing	Average Scale Score	Human Development Index
Bulgaria	8	14.9	476 (4.3)	0.795
Romania	8	15.0	475 (4.8)	0.773
INTERNATIONAL AVERAGE	8	14.5	467 (0.5)	-
Norway	7	13.8	461 (2.5)	0.944
Moldova, Rep. of	8	14.9	460 (4.0)	0.700
Cyprus	8	13.8	459 (1.7)	0.891
Macedonia, Rep. of	8	14.6	435 (3.5)	0.784
Lebanon	8	14.6	433 (3.1)	0.752
Jordan	8	13.9	424 (4.1)	0.743
Iran, Islamic Rep. of	8	14.4	411 (2.4)	0.719
Indonesia	8	14.5	411 (4.8)	0.682
Tunisia	8	14.8	410 (2.2)	0.740
Egypt	8	14.4	406 (3.5)	0.648
Bahrain	8	14.1	401 (1.7)	0.839
Palestinian Nat’l Auth.	8	14.1	390 (3.1)	0.731
Chile	8	14.2	387 (3.3)	0.831
Morocco	8	15.2	387 (2.5)	0.606
Philippines	8	14.8	378 (5.2)	0.751
Botswana	8	15.1	366 (2.6)	0.614
Saudi Arablia	8	14.1	332 (4.6)	0.769
Ghana	8	15.5	276 (4.7)	0.567
South Africa	8	15.1	264 (5.5)	0.684
England	9	14.3	498 (4.7)	0.930

Countries	Years of Schooling	Average Age at Time of Testing	Average Scale Score	Human Development Index
Benchmarking Participants				
Basque Country, Spain	8	14.1	487 (2.7)	-
Indiana State, U.S.	8	14.5	508 (5.2)	-
Ontario Province, Canada	8	13.8	521 (3.1)	-
Quebec Province, Canada	8	14.2	543 (3.0)	-

Appendix III- TIMSS 2007 Achievement Testing for 8th Grade

Country	Years of Schooling	Average Age at Time of Testing	Average Scale Score	Human Development Index
Chinese Taipei	8	14.2	598(4.5)	0.932
Korea, Rep. of	8	14.3	597(2.7)	0.921
Singapore	8	14.4	593(3.8)	0.922
Hong Kong SAR	8	14.4	572(5.8)	0.937
Japan	8	14.5	570(2.4)	0.953
Hungary	8	14.6	517(3.5)	0.874
England	9	14.2	513(4.8)	0.946
Russian Federation	7 OR 8	14.6	512(4.1)	0.802
United States	8	14.3	508(2.8)	0.951
Lithuania	8	14.9	506(2.3)	0.862
Czech Republic	8	14.4	504(2.4)	0.891
Slovenia	7 or 8	13.8	501(2.1)	0.917
TIMSS Scale Avg.	-	-	500	-
Armenia	8	14.9	499(3.5)	0.775
Australia	8	13.9	496(3.9)	0.962
Sweden	8	14.8	491(2.3)	0.956
Malta	9	14.0	488(1.2)	0.878
Scotland	9	13.7	487(3.7)	0.946
Serbia	8	14.9	486(3.3)	0.810
Italy	8	13.9	480(3.0)	0.941
Malaysia	8	14.3	474(5.0)	0.811
Norway	8	13.8	469(2.0)	0.968
Cyprus	8	13.8	465(1.6)	0.903

Country	Years of Schooling	Average Age at Time of Testing	Average Scale Score	Human Development Index
Bulgaria	8	14.9	464(5.0)	0.824
Israel	8	14.0	463(3.9)	0.932
Ukraine	8	14.2	462(3.6)	0.788
Romania	8	15.0	461(4.1)	0.813
Bosnia and Herzegovina	8 or 9	14.7	456(2.7)	0.803
Lebanon	8	14.4	449(4.0)	0.772
Thailand	8	14.3	441(5.0)	0.781
Turkey	8	14.0	432(4.8)	0.775
Jordan	8	14.0	427(4.1)	0.773
Tunisia	8	14.5	420(2.4)	0.766
Georgia	8	14.2	410(5.9)	0.754
Iran, Islamic Rep. of	8	14.2	403(4.1)	0.759
Bahrain	8	14.1	398(1.6)	0.866
Indonesia	8	14.3	397(3.8)	0.728
Syrian Arab Rep.	8	13.9	395(3.8)	0.724
Egypt	8	14.1	391(3.6)	0.708
Algeria	8	14.5	387(2.1)	0.733
Colombia	8	14.5	380(3.6)	0.791
Oman	8	14.3	372(3.4)	0.814
Palestinian Nat’l Auth.	8	14.0	367(3.5)	0.731
Botswana	8	14.9	364(2.3)	0.654
Kuwait	8	14.4	354(2.3)	0.891
El Salvador	8	15.0	340(2.8)	0.735

Country	Years of Schooling	Average Age at Time of Testing	Average Scale Score	Human Development Index
Saudi Arabia	8	14.4	329(2.9)	0.812
Ghana	8	15.8	309(4.4)	0.553
Qatar	8	13.9	307(1.4)	0.875
Morocco	8	14.8	381(3.0)	0.646
Benchmarking Participants				
Massachusetts, U.S.	8	14.2	547(4.6)	-
Minnesota, U.S.	8	14.3	532(4.4)	-
Quebec, Canada	8	14.2	528(3.5)	-
Ontario, Canada	8	13.8	517(3.5)	-
British Colombia, Canada	8	13.9	509(3.0)	-
Basque Country, Spain	8	14.1	499(3.0)	-
Dubai, UAE	8	14.2	461(2.4)	-

Appendix IV- TIMSS 1995, 1999, 2003, 2007.

Country	Scale Score 1995	Scale Score 1999	Scale Score 2003	Scale Score 2007	2003 to 2007 diff.	1999 to 2007 diff.	1995 to 2007 diff.
Chinese Taipei	-	585	585	598	13	13	-
Korea, Rep. of	581	587	589	597	8	10	17
Singapore	609	604	605	593	-13	-12	-16
Hong Kong SAR	569	582	586	572	-14	-10	4
Japan	581	579	570	570	0	-9	-11
Hungary	527	532	529	517	-12	-15	-10
England	498	496	498	513	15	17	16
Russian Federation	524	526	508	512	4	-14	-12
United States	492	502	504	508	4	7	16
Lithuania	472	482	502	506	4	24	34
Czech Republic	546	520	-	504	-	-16	-42
Slovenia	494	-	493	501	9	-	7
Armenia	-	-	478	499	21	-	-
Australia	509	-	505	496	-8	-	-13
Sweden	540	-	499	491	-8	-	-48
Scotland	493	-	498	487	-10	-	-6
Serbia	-	-	477	486	9	-	-
Italy	-	479	484	480	-4	0	-
Malaysia	-	519	508	474	-34	-45	-
Norway	498	-	461	469	8	-	-29
Cyprus	468	476	459	465	6	-11	-2
Bulgaria	527	511	476	464	-13	-47	-63
Israel	-	466	496	463	-32	-3	-

Country	**Scale Score 1995**	**Scale Score 1999**	**Scale Score 2003**	**Scale Score 2007**	**2003 to 2007 diff.**	**1999 to 2007 diff.**	**1995 to 2007 diff.**
Romania	474	472	475	461	-14	-11	-12
Lebanon	-	-	433	449	16	-	-
Thailand	-	467	-	441	-	-26	-
Jordan	-	428	424	427	3	-1	-
Tunisia	-	448	410	420	10	-28	-
Indonesia	-	403	411	405	-5	2	-
Iran, Islamic Rep. of	418	422	411	403	-8	-19	-15
Bahrain	-	-	401	398	-3	-	-
Egypt	-	-	406	391	-16	-	-
Colombia	332	-	-	380	-	-	47
Palestinian Nat'l Auth.	-	-	390	367	-23	-	-
Botswana	-	-	366	364	-3	-	-
Ghana	-	-	276	309	34	-	-
Benchmarking Participants							
Massachusetts, U.S.	-	513	-	547	-	34	-
Minnesota, U.S.	518	-	-	532	-	-	14
Quebec, Canada	56	566	543	528	-15	-38	-28
Ontario, Canada	501	517	521	517	-4	1	17
British Columbia, Canada	-	522	-	509	-	-12	-
Basque Country, Spain	-	-	2003	2007	11	-	-

Appendix V – PISA 2003: Participating countries

OECD Countries	**Non-OECD Countries**
Australia	Argentina
Austria	Brazil
Belgium	Chile
Canada	China
Czech Republic	Hong Kong
Denmark	Indonesia
Finland	Russian Federation
France	Slovenia
Germany	Tunisia
Greece	Thailand
Hungary	
Iceland	
Ireland	
Italy	
Japan	
Korea	
Luxembourg	
Mexico	
New Zealand	
The Netherlands	
Norway	
Poland	
Portugal	
Rep. of Korea	

OECD Countries	Non-OECD Countries
Slovakia	
Spain	
Sweden	
Switzerland	
Turkey	
United States	
United Kingdom	

Appendix VI - Structured observation schedule

Type of Activity	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Role																				
Arrival/settling/packing																				
Discipline																				
H.W.																				
Active Learning																				
Book-work																				
Board-Work																				
Individual Work																				
Group Work																				
Teacher Explanation																				
Student Q																				
Teacher Q																				
Student Ans																				
Positive Reinforcement																				
OHP																				
Interactive white board																				
Student computer work																				
Real-life reference																				
Non-maths activity																				
Teacher going around																				
Teacher instruction																				
Student Discussion																				

Type of Activity	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Role																				
Arrival/settling/packing																				
Discipline																				
H.W.																				
Active Learning																				
Book-work																				
Board-Work																				
Individual Work																				
Group Work																				
Teacher Explanation																				
Student Q																				
Teacher Q																				
Student Ans																				
Positive Reinforcement																				
OHP																				
Interactive white board																				
Student computer work																				
Real-life reference																				
Non-maths activity																				
Teacher going around																				
Teacher instruction																				
Student Discussion																				

Type of Activity	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
Role																				
Arrival/settling/packing																				
Discipline																				
H.W.																				
Active Learning																				
Book-work																				
Board-Work																				
Individual Work																				
Group Work																				
Teacher Explanation																				
Student Q																				
Teacher Q																				
Student Ans																				
Positive Reinforcement																				
OHP																				
Interactive white board																				
Student computer work																				
Real-life reference																				
Non-maths activity																				
Teacher going around																				
Teacher instruction																				
Student Discussion																				

Type of Activity	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
Role																				
Arrival/settling/packing																				
Discipline																				
H.W.																				
Active Learning																				
Book-work																				
Board-Work																				
Individual Work																				
Group Work																				
Teacher Explanation																				
Student Q																				
Teacher Q																				
Student Ans																				
Positive Reinforcement																				
OHP																				
Interactive white board																				
Student computer work																				
Real-life reference																				
Non-maths activity																				
Teacher going around																				
Teacher instruction																				
Student Discussion																				

Appendix VII - Realistic Test implemented in Ireland

Appendix VII:i: Realistic test implemented in Ireland

Appendices:

Name:

School:

Date:

Please answer each question below to the best of your ability. All questions should be answered in the space provided. There are no exact answers so your opinion is important! Thank you.

A medical breakthrough has enabled scientists to start working on treatments for a rare and dangerous cancer, which when untreated kills patients within just a few months. After extensive research and clinical trials, three drug companies have come up with their version of a drug to treat the cancer. The table below shows the average number of years patients survive after diagnosis for each of the three drugs. It also shows how much each drug costs per patient per year.

Question 1: It's your job to decide which of the three drugs patients should be given this year.

- Your budget for this year is **€1,000,000**, which you must not exceed, and there are currently **2,150** patients that need to be treated.

	Drug A	**Drug B**	**Drug C**
Average survival time in years	2.8	3.5	4.2
Cost per patient per year	€350	€400	€470

- If you can only buy one type of drug **which one would you choose** to increase the average time that people with this type of cancer will live (known as survival time)? You can not spend more than the given budget. Please show all your workings.

Question 2: Just as you are going to communicate your decision to doctors, some new research results are published. It turns out that the three drugs affect men and women differently. The differences are shown in the table below. Out of the 2150 patients 800 are men and 1350 are women.

Men (800)				**Women (1350)**			
	Drug A	**Drug B**	**Drug C**		**Drug A**	**Drug B**	**Drug C**
Average survival time in years	**3.9**	**3.5**	**4.0**	**Average survival time in years**	**2.1**	**3.5**	**4.3**
Cost per patient per year	**€350**	**€400**	**€470**	**Cost per patient per year**	**€350**	**€400**	**€470**

(i) Work out which combination of drugs (drug X for men and drug Y for women) gives the **greatest average survival time** calculated over *all* patients.

(ii) Can you use this to come up with a final choice of drug for both groups? Give reasons for your answer.

(iii) What other considerations might you make when allocating the drugs?

Question 3:

A year has passed when you receive the news that the drug companies that produce drugs B and C have lowered their prices to match that of drug A. The average cost per patient per year is now the same for all three drugs. This would suggest that everyone should be treated with drug C, since this maximises the overall survival time.

However, the drug companies have also been forced to publish the results of a study into the side effects of their drugs. These can be severe, causing patients pain and even confining them to bed. The results of the study show how patients have rated their quality of life on the drugs on a scale from 0 to 1, where 1 means best possible quality of life and 0 means worst possible quality of life. The average rating for each drug is given in the table below.

	Drug A	**Drug B**	**Drug C**
Average survival time in years	**2.8**	**3.5**	**4.2**
Average quality of life rating	**0.5**	**0.8**	**0.1**
Cost per patient per year	**€350**	**€350**	**€350**

(i) Using the table above compare the benefits and disadvantages of each drug. List the benefits and disadvantages of each. (Show any maths you have used to come to your conclusion).

(ii) What is your final choice of drug? Remember you cannot go over the given budget. What reasons do you have for your choice.

Additional workings (if necessary):

Appendix VII:ii: Marking scheme for Realistic test implemented in Ireland

Appendices:

Name:

School:

Date:

Please answer each question below to the best of your ability. All questions should be answered in the space provided. There are no exact answers so your opinion is important! Thank you.

A medical breakthrough has enabled scientists to start working on treatments for a rare and dangerous cancer, which when untreated kills patients within just a few months. After extensive research and clinical trials, three drug companies have come up with their version of a drug to treat the cancer. The table below shows the average number of years patients survive after diagnosis for each of the three drugs. It also shows how much each drug costs per patient per year.

Question 1: It's your job to decide which of the three drugs patients should be given this year.

- Your budget for this year is **€1,000,000**, which you must not exceed, and there are currently **2,150** patients that need to be treated.

	Drug A	**Drug B**	**Drug C**
Average survival time in years	2.8	3.5	4.2
Cost per patient per year	€350	€400	€470

- If you can only buy one type of drug **which one would you choose** to increase the average time that people with this type of cancer will live (known as survival time)? You can not spend more than the given budget. Please show all your workings.

Q. 1 Solution:
Drug C gives the longest average survival time.
For 2,150 patients it costs 2150 x 470 = €1,010,500. However, this is over budget.
Drug B costs 2,150 x 400=€860,000. Drug B is the best choice as you can afford it and it gives a longer survival time than Drug A.

5 marks, attempt 2 for correctly working out the cost of any one drug.
5 marks, attempt 2 for a valid justification.
Total 10 marks.

Question 2: Just as you are going to communicate your decision to doctors, some new research results are published. It turns out that the three drugs affect men and women differently. The differences are shown in the table below. Out of the 2150 patients 800 are men and 1350 are women.

Men (800)				Women (1350)			
	Drug A	Drug B	Drug C		Drug A	Drug B	Drug C
Average survival time in years	3.9	3.5	4.0	Average survival time in years	2.1	3.5	4.3
Cost per patient per year	€350	€400	€470	Cost per patient per year	€350	€400	€470

(i) Work out which combination of drugs (drug X for men and drug Y for women) gives **the greatest average** survival **time** calculated over *all* patients.

(ii) Can you use this to come up with a final choice of drug for both groups? Give reasons for your answer.

<u>Q. 2 Solution:</u>

(i) The greatest average survival time:
C to men and B to women: ((800x4.0)+(1,350x3.5))/2150=3.69 years
A to men, C to women: ((800x3.9)+(4.3x1,350))/2150=4.14 years
Therefore the greatest average survival time that is within budget consists of giving Drug A to men and C to women.
5 marks, attempt 2 for calculating the average survival time for any one group or combination of groups correctly.
5 marks, attempt 2 for selecting an option within budget with a valid justification

(ii) Consider cost with justification
A to men and C to women (800x350)+(1,350x470)=€914,500
5 marks, attempt 2 Consideration of life expectancy with cost taken into account. Justification provided.

(iii) What other considerations might you make when allocating the drugs?

<u>Solution:</u>
(ii)
Ethical considerations, equality, severity of cancer etc. etc. etc.
5 marks for any reasonable suggestion.

Total for Q.2
20 marks

Question 3:

A year has passed when you receive the news that the drug companies that produce drugs B and C have lowered their prices to match that of drug A. The average cost per patient per year is now the same for all three drugs. This would suggest that everyone should be treated with drug C, since this maximises the overall survival time.

However, the drug companies have also been forced to publish the results of a study into the side effects of their drugs. These can be severe, causing patients pain and even confining them to bed. The results of the study show how patients have rated their quality of life on the drugs on a scale from 0 to 1, where 1 means best possible quality of life and 0 means worst possible quality of life. The average rating for each drug is given in the table below.

	Drug A	Drug B	Drug C
Average survival time in years	**2.8**	**3.5**	**4.2**
Average quality of life rating	**0.5**	**0.8**	**0.1**
Cost per patient per year	**€350**	**€350**	**€350**

(i) Using the table above compare the benefits and disadvantages of each drug. List the benefits and disadvantages of each. (Show any maths you have used to come to your conclusion).

Question 3:
<u>Solution:</u>
(i)
Drug A: Poor survival time, average quality of life
Drug B: Average survival time, best quality of life
Drug C: Best survival time, poor quality of life

5 marks, attempt 2 Full marks for a full list of advantages/disadvantages.

(ii) What is your final choice of drug? Remember you cannot go over the given budget. What reasons do you have for your choice.

Additional workings (if necessary):

Solution:

(ii)
You could decide to multiply the survival time by the quality of life giving the following:
Drug A: 1.4
Drug B: 2.8
Drug C: 0.42
Using this formula Drug B performs best.

5 marks, attempt 2 for any reasonable solution with justification.

Appendix VIII - Realistic Test implemented in Massachusetts

Appendix VIII:i: Realistic test implemented in MA

Name:

School:

Date:

Please answer each question below to the best of your ability. All questions should be answered in the space provided. There are no exact answers so your opinion is important! Thank you.

A medical breakthrough has enabled scientists to start working on treatments for a rare and dangerous cancer, which when untreated kills patients within just a few months. After extensive research and clinical trials, three drug companies have come up with their version of a drug to treat the cancer. The table below shows the average number of years patients survive after diagnosis for each of the three drugs. It also shows how much each drug costs per patient per year.

Question 1: It's your job to decide which of the three drugs patients should be given this year.

- Your budget for this year is **$1,000,000**, which you must not exceed, and there are currently **2,150** patients that need to be treated.

	Drug A	**Drug B**	**Drug C**
Average survival time in years	**2.8**	**3.5**	**4.2**
Cost per patient per year	**$350**	**$400**	**$470**

- If you can only buy one type of drug **which one would you choose** to increase the average time that people with this type of cancer will live (known as survival time)? You can not spend more than the given budget. Please show all your workings.

Question 2: Just as you are going to communicate your decision to doctors, some new research results are published. It turns out that the three drugs affect men and women differently. The differences are shown in the table below. Out of the 2150 patients 800 are men and 1350 are women.

Men (800)				**Women (1350)**			
	Drug A	**Drug B**	**Drug C**		**Drug A**	**Drug B**	**Drug C**
Average survival time in years	**3.9**	**3.5**	**4.0**	**Average survival time in years**	**2.1**	**3.5**	**4.3**
Cost per patient per year	**$350**	**$400**	**$470**	**Cost per patient per year**	**$350**	**$400**	**$470**

(i) Work out which combination of drugs (drug X for men and drug Y for women) gives **the greatest average survival time** calculated over *all* patients.

(ii) Can you use this to come up with a final choice of drug for both groups? Give reasons for your answer.

(iii) What other considerations might you make when allocating the drugs?

Question 3:

A year has passed when you receive the news that the drug companies that produce drugs B and C have lowered their prices to match that of drug A. The average cost per patient per year is now the same for all three drugs. This would suggest that everyone should be treated with drug C, since this maximises the overall survival time.

However, the drug companies have also been forced to publish the results of a study into the side effects of their drugs. These can be severe, causing patients pain and even confining them to bed. The results of the study show how patients have rated their quality of life on the drugs on a scale from 0 to 1, where 1 means best possible quality of life and 0 means worst possible quality of life. The average rating for each drug is given in the table below.

	Drug A	**Drug B**	**Drug C**
Average survival time in years	**2.8**	**3.5**	**4.2**
Average quality of life rating	**0.5**	**0.8**	**0.1**
Cost per patient per year	**$350**	**$350**	**$350**

(i) Using the table above compare the benefits and disadvantages of each drug. List the benefits and disadvantages of each. (Show any maths you have used to come to your conclusion).

(ii) What is your final choice of drug? Remember you cannot go over the given budget. What reasons do you have for your choice.

Additional workings (if necessary)

Appendix VIII:ii: Marking scheme for Realistic test implemented in MA

Name:

School:

Date:

Please answer each question below to the best of your ability. All questions should be answered in the space provided. There are no exact answers so your opinion is important! Thank you.

A medical breakthrough has enabled scientists to start working on treatments for a rare and dangerous cancer, which when untreated kills patients within just a few months. After extensive research and clinical trials, three drug companies have come up with their version of a drug to treat the cancer. The table below shows the average number of years patients survive after diagnosis for each of the three drugs. It also shows how much each drug costs per patient per year.

Question 1: It's your job to decide which of the three drugs patients should be given this year.

- Your budget for this year is **$1,000,000**, which you must not exceed, and there are currently **2,150** patients that need to be treated.

	Drug A	Drug B	Drug C
Average survival time in years	2.8	3.5	4.2
Cost per patient per year	$350	$400	$470

- If you can only buy one type of drug **which one would you choose** to increase the average time that people with this type of cancer will live (known as survival time)? You can not spend more than the given budget. Please show all your workings.

Q. 1 Solution:
Drug C gives the longest average survival time.
For 2,150 patients it costs 2150 x 470 = $1,010,500. However, this is over budget.
Drug B costs 2,150 x 400=$860,000. Drug B is the best choice as you can afford it and it gives a longer survival time than Drug A.

5 marks, attempt 2 for correctly working out the cost of any one drug.
5 marks, attempt 2 for a valid justification.
Total 10 marks.

Question 2: Just as you are going to communicate your decision to doctors, some new research results are published. It turns out that the three drugs affect men and women differently. The differences are shown in the table below. Out of the 2150 patients 800 are men and 1350 are women.

Men (800)				Women (1350)			
	Drug A	Drug B	Drug C		Drug A	Drug B	Drug C
Average survival time in years	3.9	3.5	4.0	Average survival time in years	2.1	3.5	4.3
Cost per patient per year	$350	$400	$470	Cost per patient per year	$350	$400	$470

(i) Work out which combination of drugs (drug X for men and drug Y for women) gives **the greatest average survival time** calculated over *all* patients.

(ii) Can you use this to come up with a final choice of drug for both groups? Give reasons for your answer.

<u>Q. 2 Solution:</u>

(i) The greatest average survival time:
C to men and B to women: ((800x4.0)+(1,350x3.5))/2150=3.69 years
A to men, C to women: ((800x3.9)+(4.3x1,350))/2150=4.14 years
Therefore the greatest average survival time that is within budget consists of giving Drug A to men and C to women.
5 marks, attempt 2 for calculating the average survival time for any one group or combination of groups correctly.
5 marks, attempt 2 for selecting an option within budget with a valid justification

(ii) Consider cost with justification
A to men and C to women (800x350)+(1,350x470)=$914,500
5 marks, attempt 2 Consideration of life expectancy with cost taken into account. Justification provided.

(iii) What other considerations might you make when allocating the drugs?

<u>Solution:</u>
(ii)
Ethical considerations, equality, severity of cancer etc. etc. etc.
5 marks for any reasonable suggestion.

Total for Q.2
20 marks

Question 3:

A year has passed when you receive the news that the drug companies that produce drugs B and C have lowered their prices to match that of drug A. The average cost per patient per year is now the same for all three drugs. This would suggest that everyone should be treated with drug C, since this maximises the overall survival time.

However, the drug companies have also been forced to publish the results of a study into the side effects of their drugs. These can be severe, causing patients pain and even confining them to bed. The results of the study show how patients have rated their quality of life on the drugs on a scale from 0 to 1, where 1 means best possible quality of life and 0 means worst possible quality of life. The average rating for each drug is given in the table below.

	Drug A	**Drug B**	**Drug C**
Average survival time in years	**2.8**	**3.5**	**4.2**
Average quality of life rating	**0.5**	**0.8**	**0.1**
Cost per patient per year	**$350**	**$350**	**$350**

(i) Using the table above compare the benefits and disadvantages of each drug. List the benefits and disadvantages of each. (Show any maths you have used to come to your conclusion).

Question 3:
<u>Solution:</u>
(i)
Drug A: Poor survival time, average quality of life
Drug B: Average survival time, best quality of life
Drug C: Best survival time, poor quality of life

5 marks, attempt 2 Full marks for a full list of advantages/disadvantages.

(ii) What is your final choice of drug? Remember you cannot go over the given budget. What reasons do you have for your choice.

Additional workings (if necessary):

Solution:

(ii)
You could decide to multiply the survival time by the quality of life giving the following:
Drug A: 1.4
Drug B: 2.8
Drug C: 0.42
Using this formula Drug B performs best.

5 marks, attempt 2 for any reasonable solution with justification.

Appendix IX - Traditional test implemented in Ireland

Appendix IX:i: Traditional test implemented in Ireland

Name:

School:

Date:

Please answer each question to the best of your ability. All questions should be answered in the space provided. There are no exact answers so your opinion is important! Thank you.

1 (a) Find the total cost of the following bill:

6 litres of milk at €1.05 a litre 3 loaves of bread at €1.20 a loaf 5 apples at 65 cent each.

(b) Vat at 21% is added to a bill of €750.

Calculate the total bill.

(c) €7450 is invested at 2.6% per annum.

What is the amount of the investment at the end of one year?

(d) John's weekly wage is €730.

He pays income tax at the rate of 20% on the first €440 of his wage and income tax at the rate of 42% on the remainder of his wage. John has a weekly tax credit of €65.

(i) Find the tax on the first €440 of his wage, calculated at the rate of 20%.

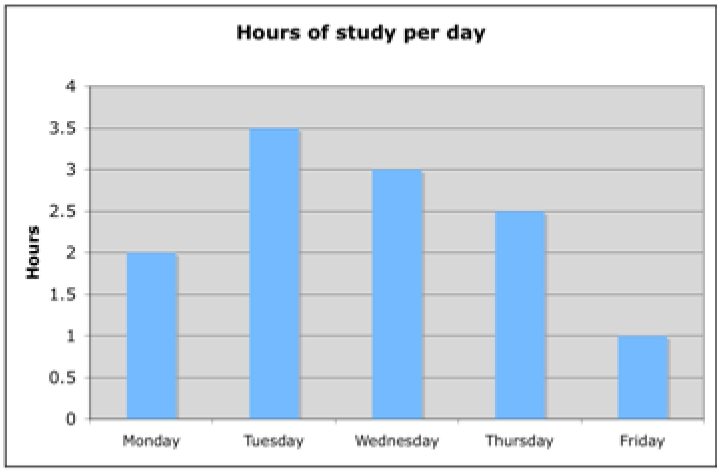
(ii) Find the tax on the remainder of his wage, calculated at the rate of 42%.

(iii) Hence calculate John's gross tax.

(iv) Calculate John's take home pay.

2 (a) Find the average of the numbers 1, 4, 3, 4, 1, 4, 12, 4, 15, 4.

(b) The bar chart shows the number of hours that Anne spent studying from Monday to Friday of a particular school week.



(i) How many hours study did Anne do on the Monday of that week?

(ii) On what day of that week did Anne do the least study?

(iii) Express the hours of study done by Anne on Wednesday as a

percentage of her total hours of study for that week.

3 (a) The cost of five books and one magazine is €32.

The cost of eight books and three magazines is €54.

Let €x be the cost of a book and let €y be the cost of a magazine.

(i) Write down two equations, each in x and y, to represent the above information.

First equation:

(ii) Solve these equations to find the cost of a book and the cost of a magazine.

Appendix IX:ii: Marking scheme for Traditional test implemented in Ireland

Name:

School:

Date:

Please answer each question to the best of your ability. All questions should be answered in the space provided. There are no exact answers so your opinion is important! Thank you.

1 (a) Find the total cost of the following bill:

6 litres of milk at €1.05 a litre

3 loaves of bread at €1.20 a loaf

5 apples at 65 cent each.

(b) Vat at 21% is added to a bill of €750.

Calculate the total bill.

<u>Solution:</u>
Question 1 (a)
(6x1.05)+(3x1.20)+(5x0.65)=€13.50
10 marks, attempt 3.

Question 1(b)
750x1.21=€907.50
10 marks, attempt 3.

(c) €7450 is invested at 2.6% per annum.

What is the amount of the investment at the end of one year?

Solution:

Question 1 (c)

(7450x2.6%)+7450=€7643.70

10 marks, attempt 3.

(d) John's weekly wage is €730.

He pays income tax at the rate of 20% on the first €440 of his wage and income tax at the rate of 42% on the remainder of his wage. John has a weekly tax credit of €65.

(i) Find the tax on the first €440 of his wage, calculated at the rate of 20%.

(ii) Find the tax on the remainder of his wage, calculated at the rate of 42%.

Solution:

Question 1 (d)

(i) 440x.2=€88

10 marks, attempt 3.

(ii) 730-440=290x.42=€121.80

10 marks, attempt 3.

(iii) Hence calculate John's gross tax.

(iv) Calculate John's take home pay.

Solution

Question 1 (d)

(iii) 88+121.80=€209.80

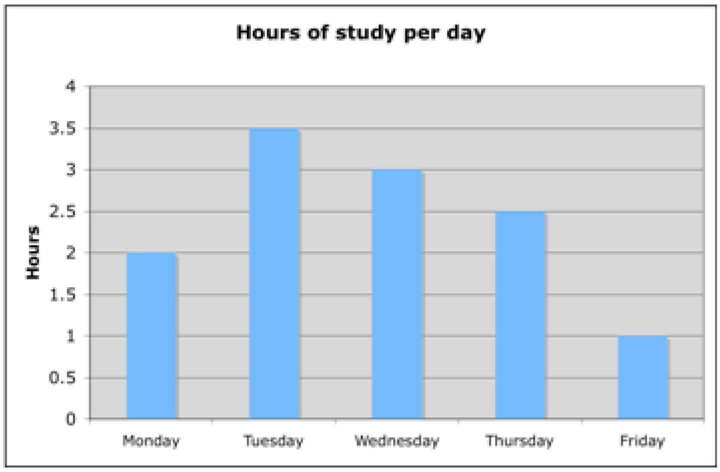
5 marks, attempt 2

(iv) 209.80-65=144.80 then 730-144.80=€585.20

15 marks, attempt 5.

2 (a) Find the average of the numbers 1, 4, 3, 4, 1, 4, 12, 4, 15, 4.

(b) The bar chart shows the number of hours that Anne spent studying from Monday to Friday of a particular school week.



(i) How many hours study did Anne do on the Monday of that week?

Solution

Question 2 (a)

(1+4+3+4+1+4+12+4+15+4)/10=5.2

10 marks, attempt 3.

Question 2 (b) (i)

2 hours

5 marks

(ii) On what day of that week did Anne do the least study?

Solution:
Question 2 (b) (ii)
Friday.
5 marks
Question 2 (b) (iii)
Total hours studied during the week: 2+3.5+3+2.5+1=12
3÷12x100%=25%
15 marks, attempt 5.

(iii) Express the hours of study done by Anne on Wednesday as a

percentage of her total hours of study for that week.

3 (a) The cost of five books and one magazine is €32.

The cost of eight books and three magazines is €54.

Let €x be the cost of a book and let €y be the cost of a magazine.

(i) Write down two equations, each in x and y, to represent the above information.

First equation:

(ii) Solve these equations to find the cost of a book and the cost of a magazine.

Solution:
Question 3 (a) (i)
$5x+y=32$
5 marks, attempt 2
Question 3 (b) (ii)
$8x+3y=54$
5 marks, attempt 2
Solve both equations simultaneously to get $x=6$ and $y=2$.
10 marks, attempt 3.

Appendix X - Traditional test implemented in Massachusetts

Appendix X:i – Traditional test implemented in MA

Name:

School:

Date:

Please answer each question to the best of your ability. All questions should be answered in the space provided. There are no exact answers so your opinion is important! Thank you.

1 (a) Find the total cost of the following bill:

6 litres of milk at \$1.05 a litre 3 loaves of bread at €1.20 a loaf 5 apples at 65 cent each.

(b) Vat at 21% is added to a bill of \$750.

Calculate the total bill.

(c) $7450 is invested at 2.6% per annum.

What is the amount of the investment at the end of one year?

(d) John's weekly wage is \$730.

He pays income tax at the rate of 20% on the first \$440 of his wage and income tax at the rate of 42% on the remainder of his wage.

(i) Find the tax on the first \$440 of his wage, calculated at the rate of 20%.

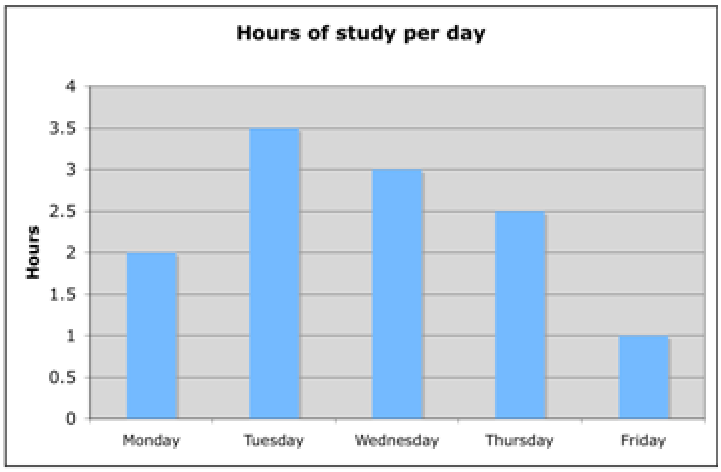
(ii) Find the tax on the remainder of his wage, calculated at the rate of 42%.

(iii) Hence calculate John's gross tax.

(iv) Calculate John's take home pay.

2 (a) Find the average of the numbers 1, 4, 3, 4, 1, 4, 12, 4, 15, 4.

(c) The bar chart shows the number of hours that Anne spent studying from Monday to Friday of a particular school week.



(i) How many hours study did Anne do on the Monday of that week?

(ii) On what day of that week did Anne do the least study?

(iii) Express the hours of study done by Anne on Wednesday as a

percentage of her total hours of study for that week.

3 (a) The cost of five books and one magazine is $32.

The cost of eight books and three magazines is $54.

Let $x be the cost of a book and let $y be the cost of a magazine.

(ii) Write down two equations, each in x and y, to represent the above information.

First equation:

(ii) Solve these equations to find the cost of a book and the cost of a magazine.

Appendix X:ii – Marking scheme for Traditional test implemented in MA

Name:

School:

Date:

Please answer each question to the best of your ability. All questions should be answered in the space provided. There are no exact answers so your opinion is important! Thank you.

1 (a) Find the total cost of the following bill:

6 litres of milk at $1.05 a litre 3 loaves of bread at €1.20 a loaf 5 apples at 65 cent each.

(b) Vat at 21% is added to a bill of $750.

Calculate the total bill.

Solution:
Question 1 (a)
(6x1.05)+(3x1.20)+(5x0.65)=$13.50
10 marks, attempt 3.

Question 1(b)
750x1.21=$907.50
10 marks, attempt 3.

(c) $7450 is invested at 2.6% per annum.

What is the amount of the investment at the end of one year?

Solution:
Question 1 (c)
(7450x2.6%)+7450=$7643.70
10 marks, attempt 3.

(d) John's weekly wage is \$730.

He pays income tax at the rate of 20% on the first \$440 of his wage and income tax at the rate of 42% on the remainder of his wage.

(i) Find the tax on the first \$440 of his wage, calculated at the rate of 20%.

(ii) Find the tax on the remainder of his wage, calculated at the rate of 42%.

Solution:

Question 1 (d)

(i) 440x.2=\$88

10 marks, attempt 3.

(ii) 730-440=290x.42=\$121.80

10 marks, attempt 3.

(iii) Hence calculate John's gross tax.

(iv) Calculate John's take home pay.

Solution

Question 1 (d)

(i) 88+121.80=$209.80

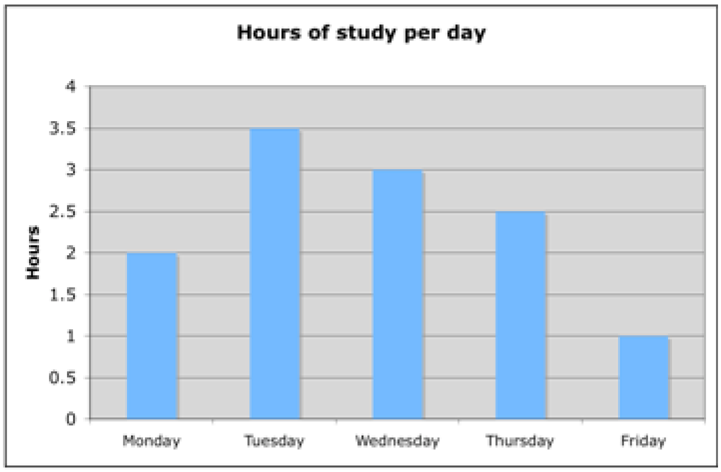
5 marks, attempt 2

(ii) 730-209.80=$520.20

5 marks, attempt 2

2 (a) Find the average of the numbers 1, 4, 3, 4, 1, 4, 12, 4, 15, 4.

(c) The bar chart shows the number of hours that Anne spent studying from Monday to Friday of a particular school week.



(i) How many hours study did Anne do on the Monday of that week?

<u>Solution</u>
Question 2 (a)
(1+4+3+4+1+4+12+4+15+4)/10=5.2
10 marks, attempt 3.
Question 2 (b) (i)
2 hours
5 marks

(ii) On what day of that week did Anne do the least study?

<u>Solution:</u>
Question 2 (b) (ii)
Friday.
5 marks
Question 2 (b) (iii)
Total hours studied during the week: 2+3.5+3+2.5+1=12
3÷12x100%=25%
15 marks, attempt 5.

(iii) Express the hours of study done by Anne on Wednesday as a percentage of her total hours of study for that week.

3 (a) The cost of five books and one magazine is $32.

The cost of eight books and three magazines is $54.

Let $x be the cost of a book and let $y be the cost of a magazine.

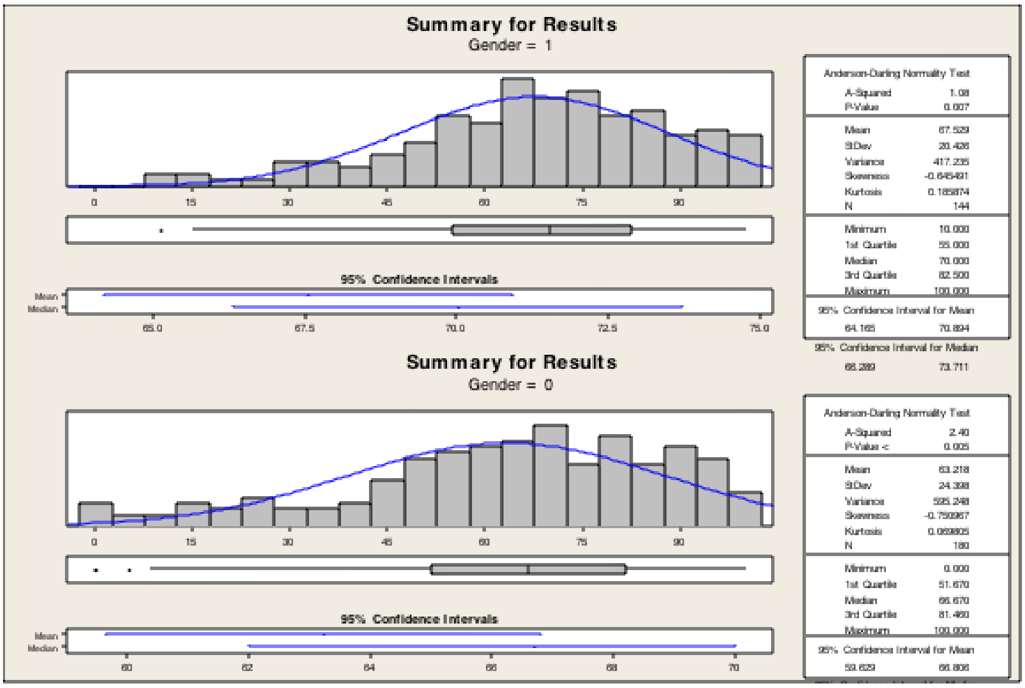
(ii) Write down two equations, each in x and y, to represent the above information.

First equation:

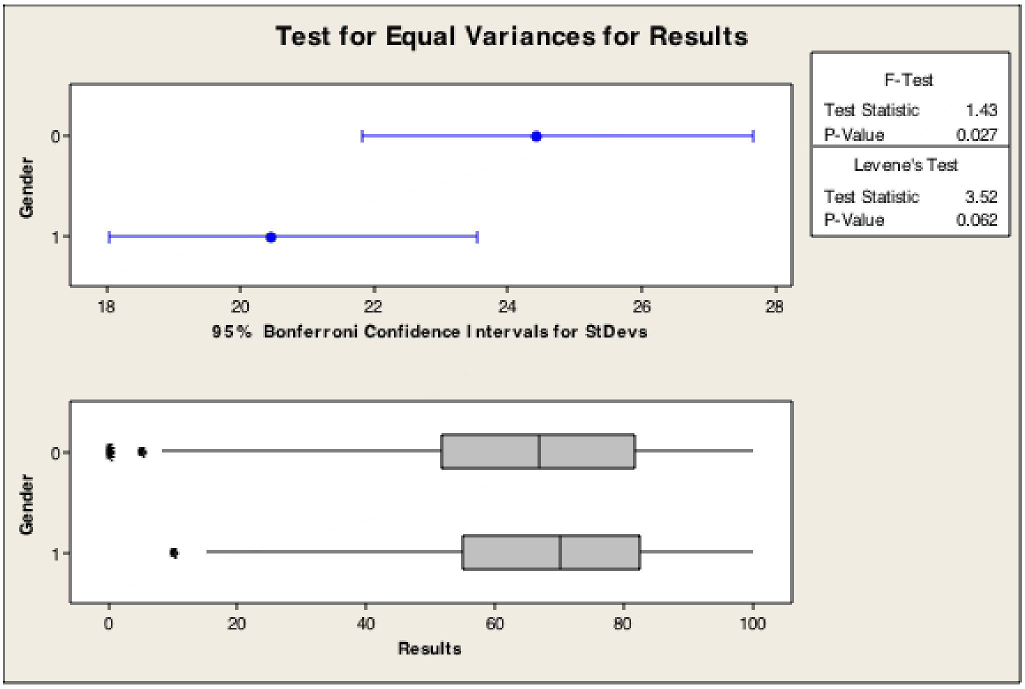
(ii) Solve these equations to find the cost of a book and the cost of a magazine.

Solution:
Question 3 (a) (i)
5x+y=32
5 marks, attempt 2
Question 3 (b) (ii)
8x+3y=54
5 marks, attempt 2
Solve both equations simultaneously to get x=6 and y=2.
10 marks, attempt 3.

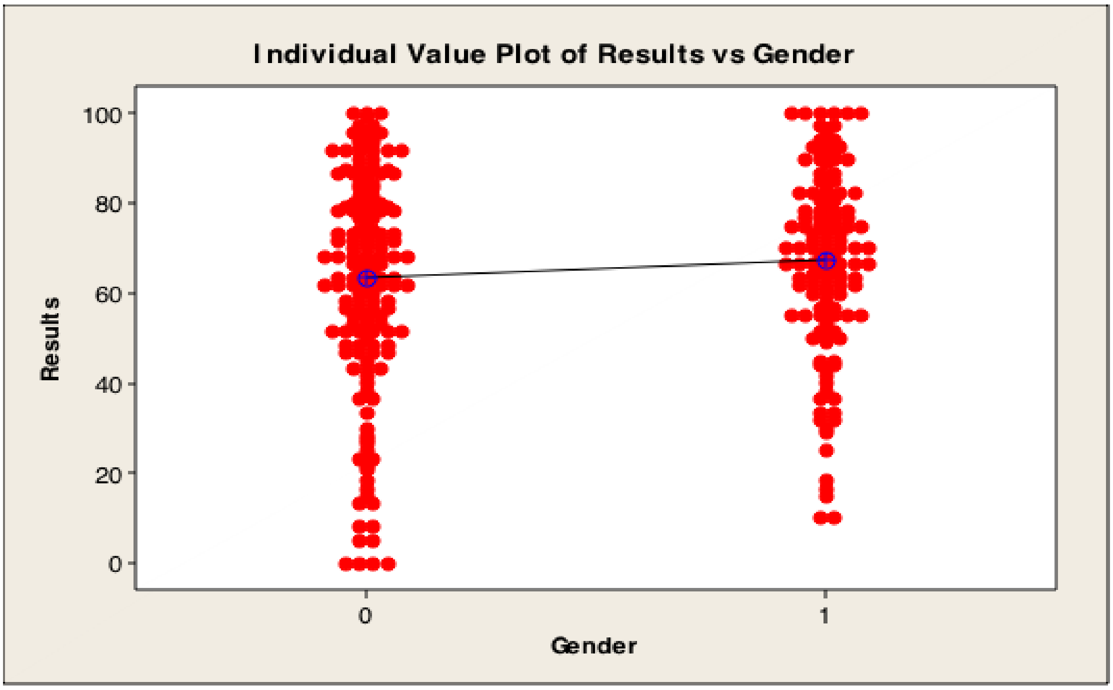
Appendix XI – Gender Test Results

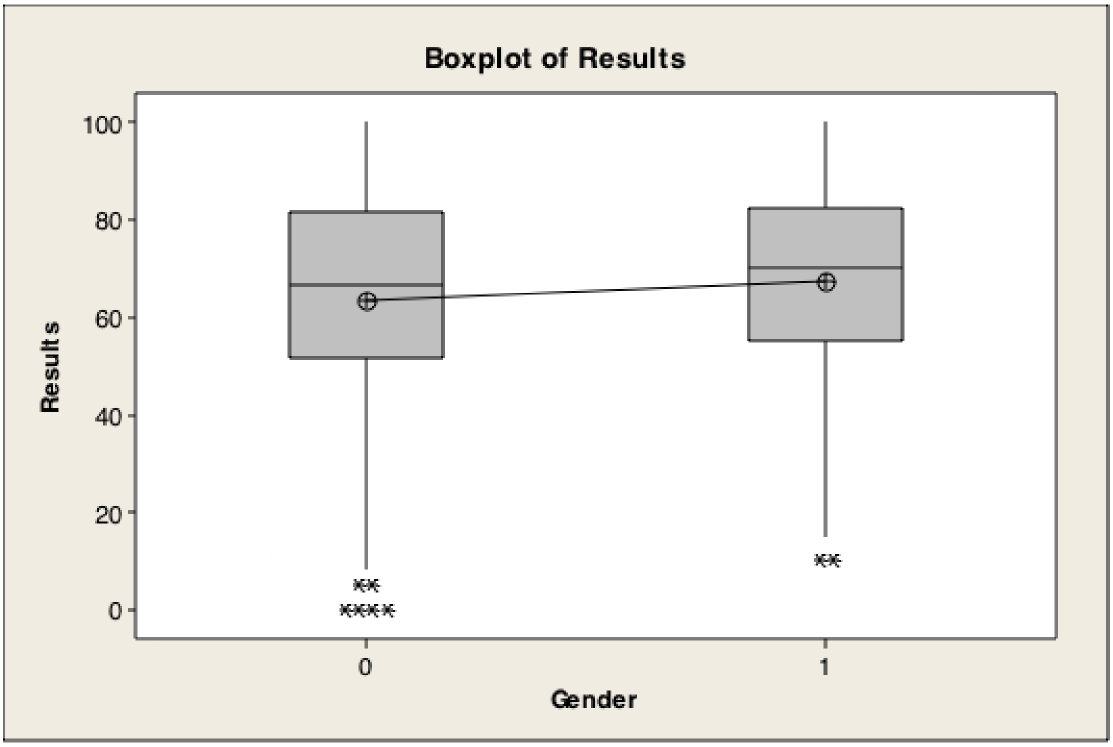


Appendix XI:i Graphical Summary of descriptive statistics of Gender study (t=0, t=1)



Appendix XI:ii: Gender study Graphical summary of test for equal variance (t=0, t=1)





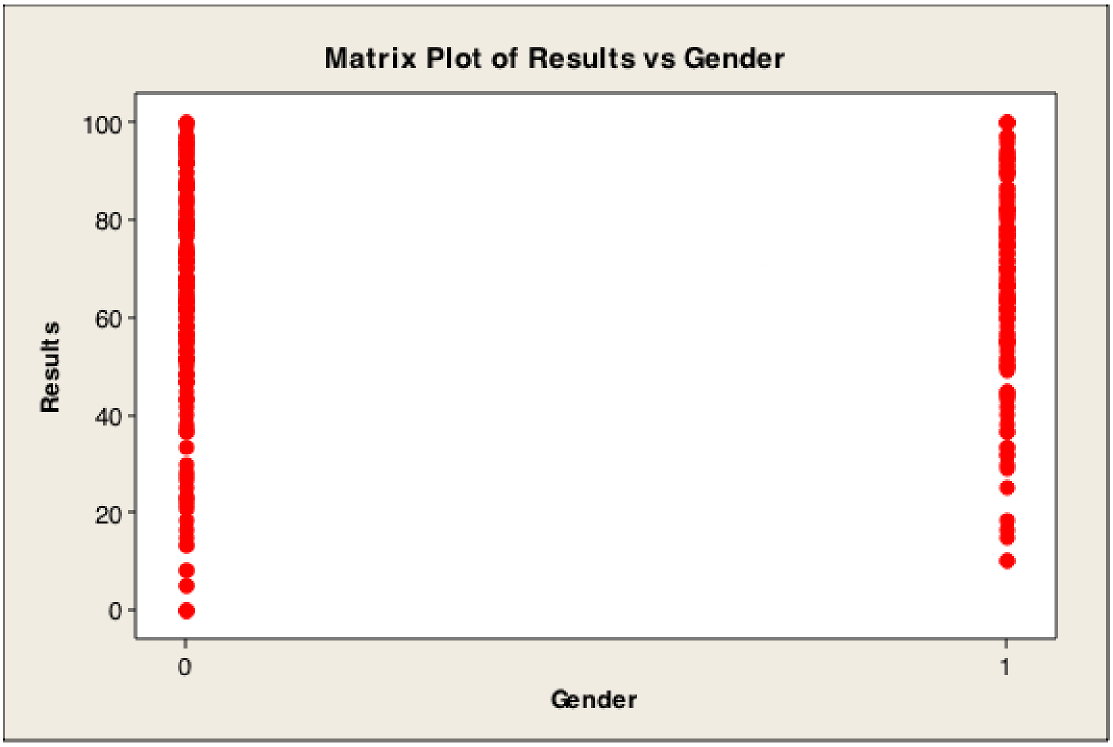
Appendix XI:iii: Two sample t-test Individual and box-plot graphics of Gender study (t=0, t=1)

```
Source   DF      SS    MS     F      P
Gender    1    1487  1487  2.88  0.091
Error   322  166214   516
Total   323  167701

S = 22.72   R-Sq = 0.89%   R-Sq(adj) = 0.58%


                          Individual 95% CIs For Mean Based on Pooled
StDev
Level    N   Mean  StDev     +---------+---------+---------+---------
0      180  63.22  24.40     (----------*----------)
1      144  67.53  20.43                 (------------*------------)
                             +---------+---------+---------+---------
                          60.0      63.0      66.0      69.0

Pooled StDev = 22.72


Tukey 95% Simultaneous Confidence Intervals
All Pairwise Comparisons among Levels of Gender

Individual confidence level = 95.00%


Gender = 0 subtracted from:

Gender  Lower  Center  Upper  ---+---------+---------+---------+------
1       -0.69    4.31   9.31             (------------*-----------)
                              ---+---------+---------+---------+------
                              -4.0       0.0       4.0       8.0
```

Appendix XI:iv: One way ANOVA statistics of Gender study (t=0, t=1)

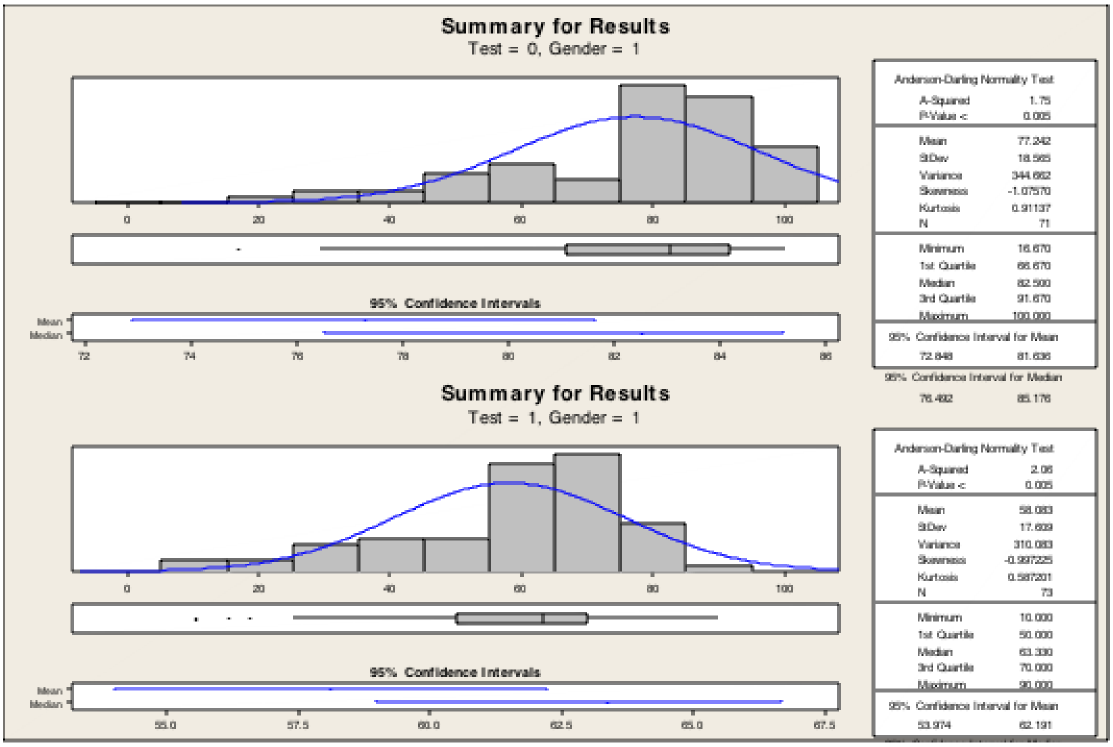
Correlations: Results, Gender

```
Pearson correlation of Results and Gender = 0.094
P-Value = 0.091
```

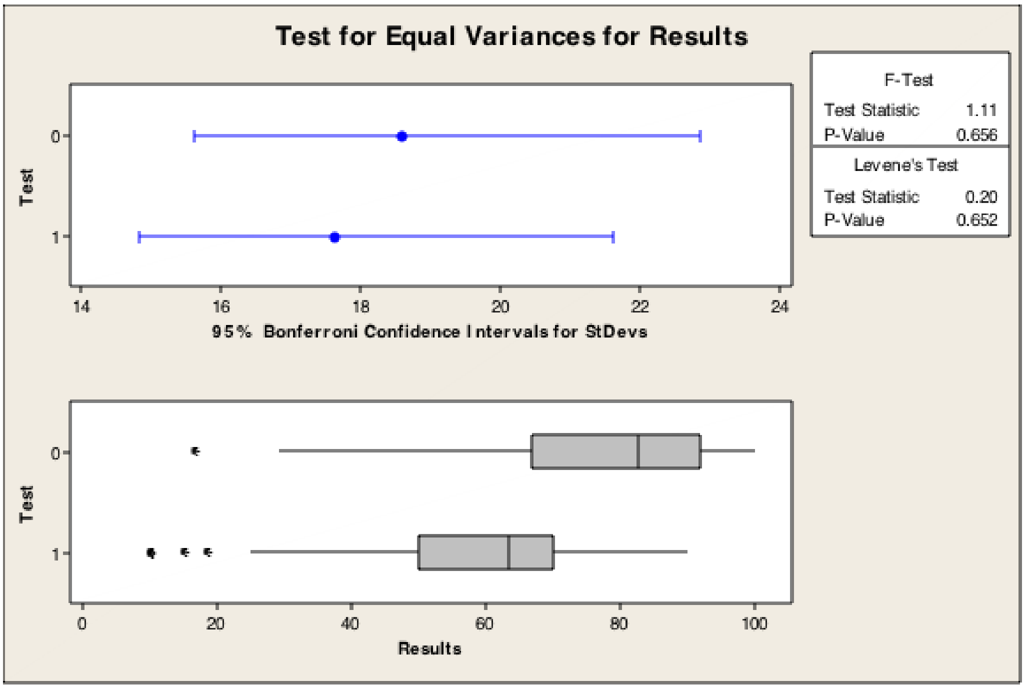


Appendix XI:v: Correlation statistics and matrix plot graphics of Gender study (t=0, t=1)

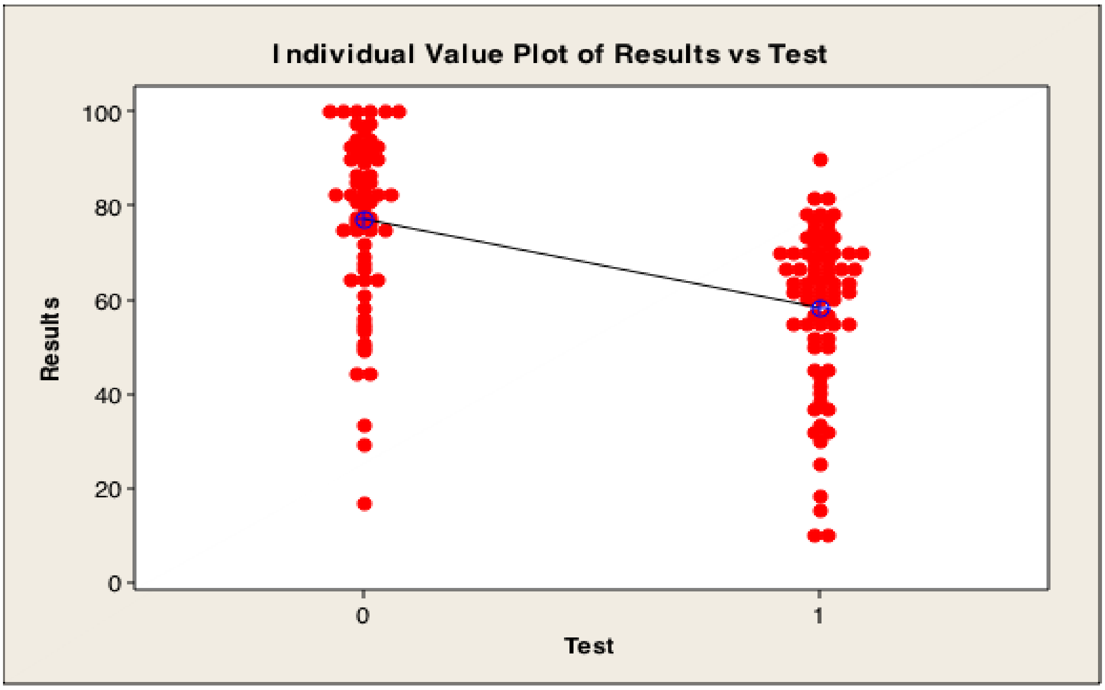
Appendix XII – Female Test Results

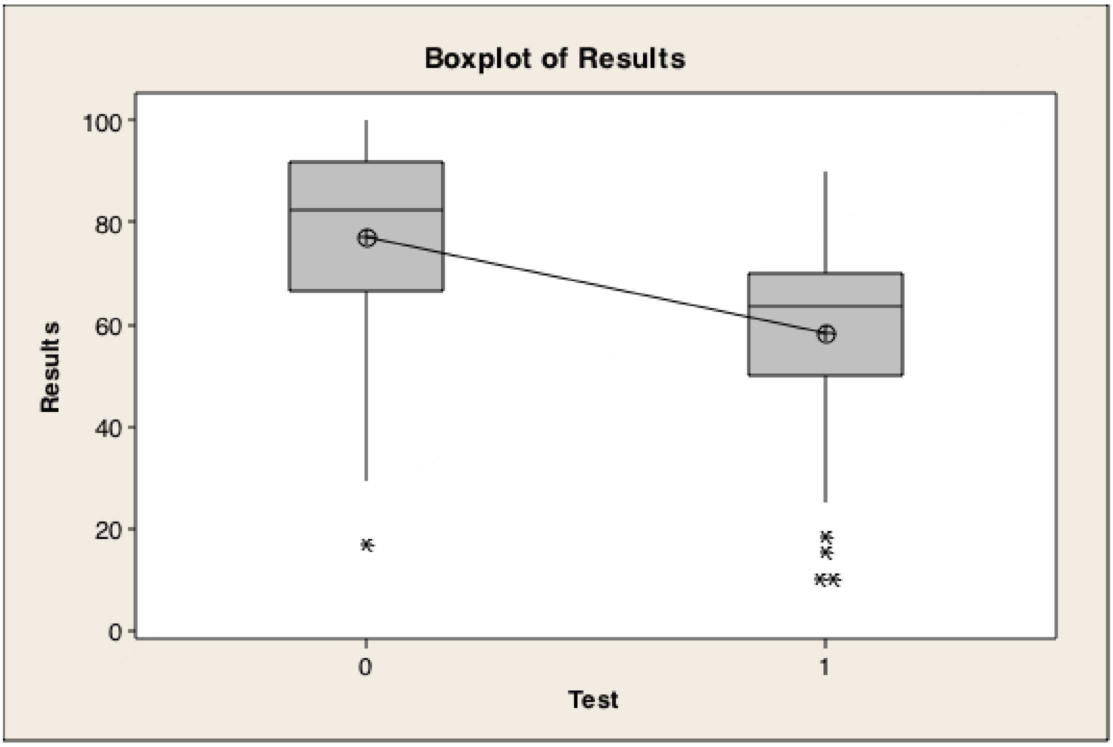


Appendix XII:i: Graphical Summary of descriptive statistics for Female study (t=0, t=1)



Appendix XII:ii: Graphical summary of test for equal variance for Female study (t=0, t=1)



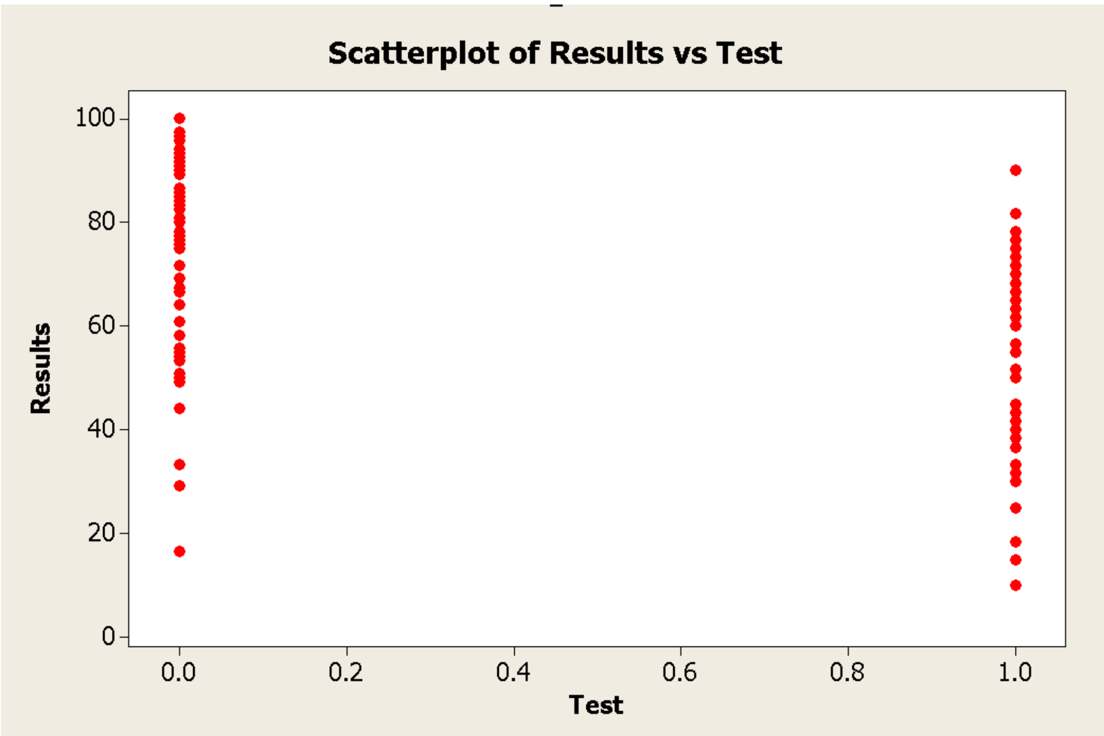


Appendix XII:iii: Two sample t-test Individual and box-plot graphics of Female study (t=0, t=1)

```
Source    DF     SS     MS      F      P
Test       1  13212  13212  40.39  0.000
Error    142  46452    327
Total    143  59665

S = 18.09   R-Sq = 22.14%   R-Sq(adj) = 21.60%
                         Individual 95% CIs For Mean Based on
                         Pooled StDev
Level    N   Mean  StDev  ---+---------+---------+---------+------
0       71  77.24  18.57                            (-----*-----)
1       73  58.08  17.61  (-----*-----)
                         ---+---------+---------+---------+------
                         56.0      63.0      70.0      77.0
Pooled StDev = 18.09

Tukey 95% Simultaneous Confidence Intervals
All Pairwise Comparisons among Levels of Test

Individual confidence level = 95.00%

Test = 0 subtracted from:
Test    Lower  Center   Upper        -+---------+---------+---------+-----
---
1      -25.12  -19.16  -13.20       (------*-------)
                                    -+---------+---------+---------+-----
---
                              -24.0     -16.0      -8.0       0.0
```

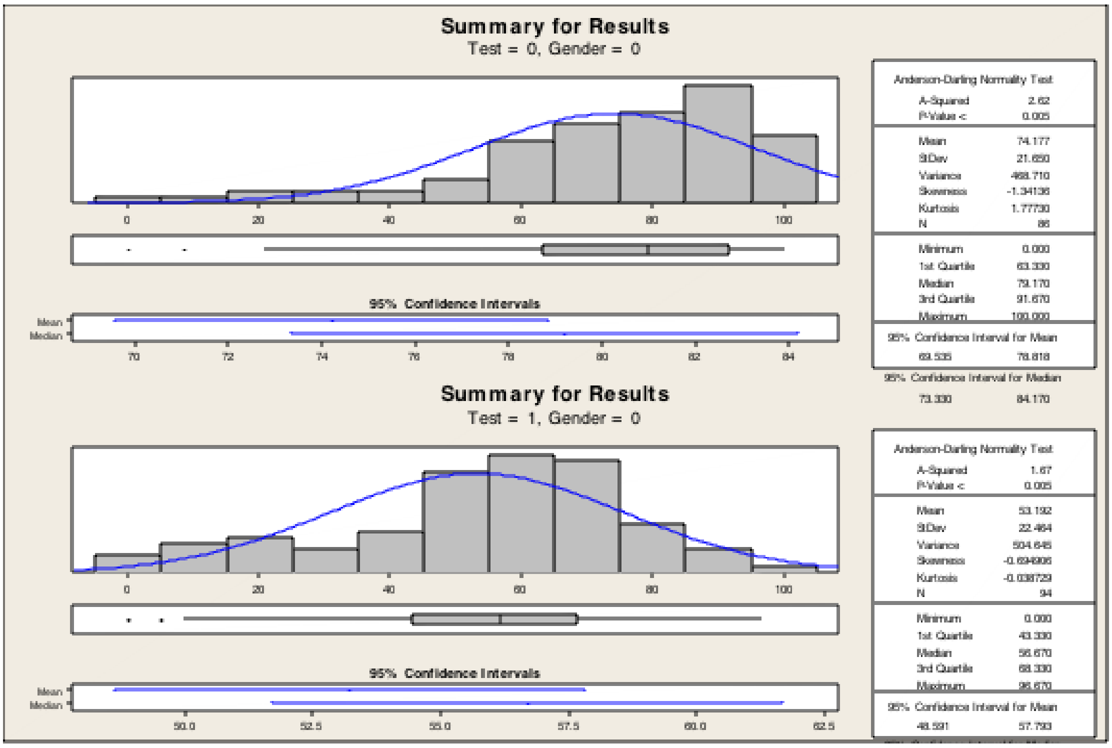
Appendix XII:iv: One way ANOVA statistics of Female study (t=0, t=1)

```
Correlations: Results, Test

Pearson correlation of Results and Test = -0.471
P-Value = 0.000
```

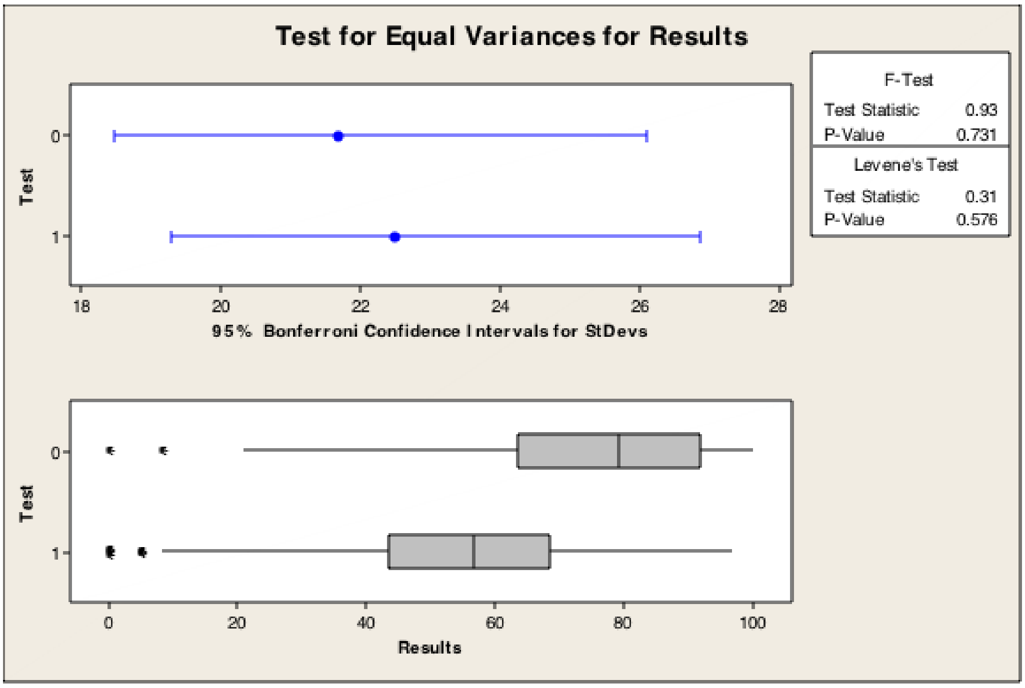


Appendix XII:v: Correlation statistics and matrix plot of Female study (t=0, t=1)

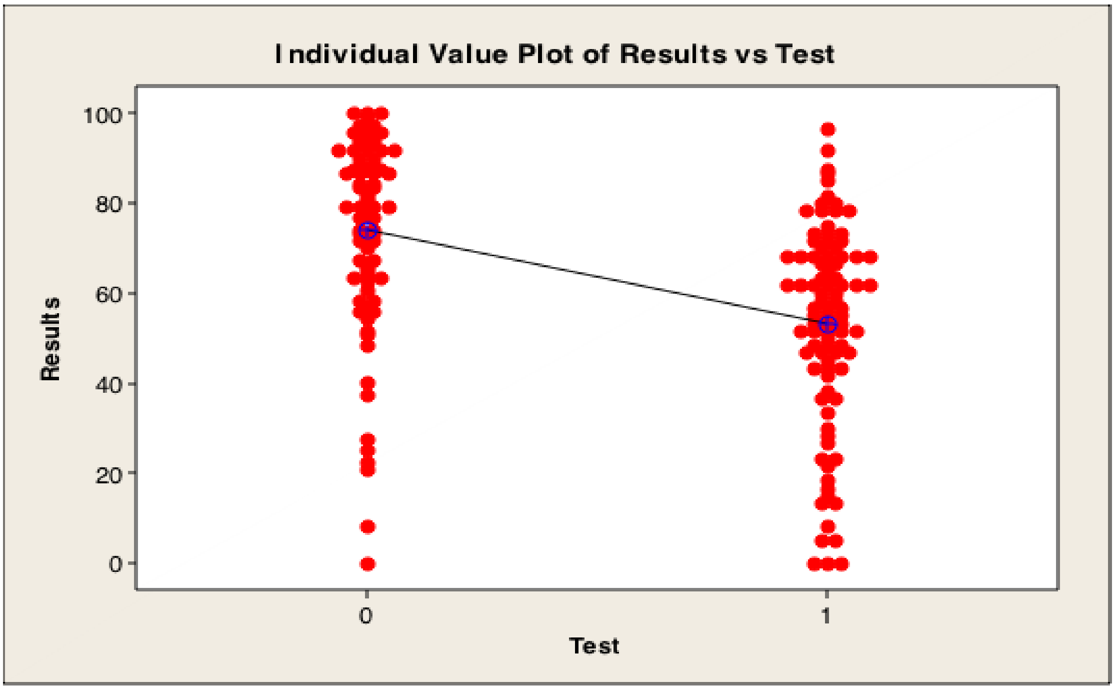
Appendix XIII – Male Test Results

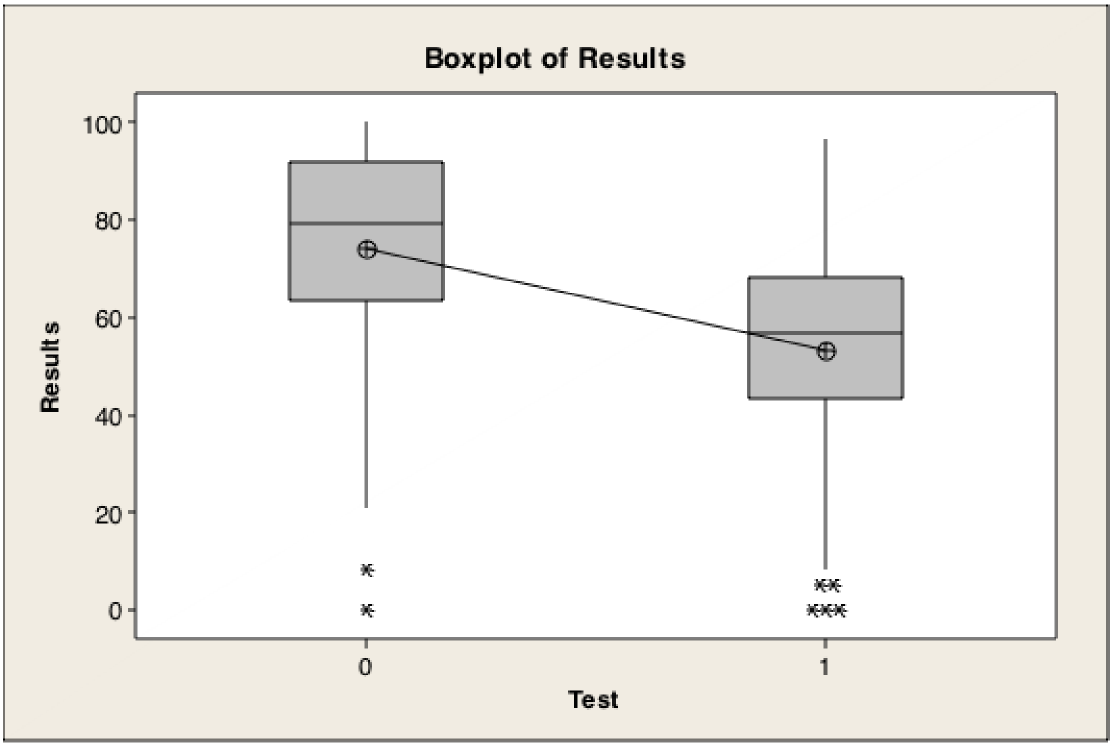


Appendix XIII:i: Graphical Summary of descriptive statistics for Male study (t=0, t=1)



Appendix XIII:ii: Graphical summary of test for equal variance for Male study (t=0, t=1)



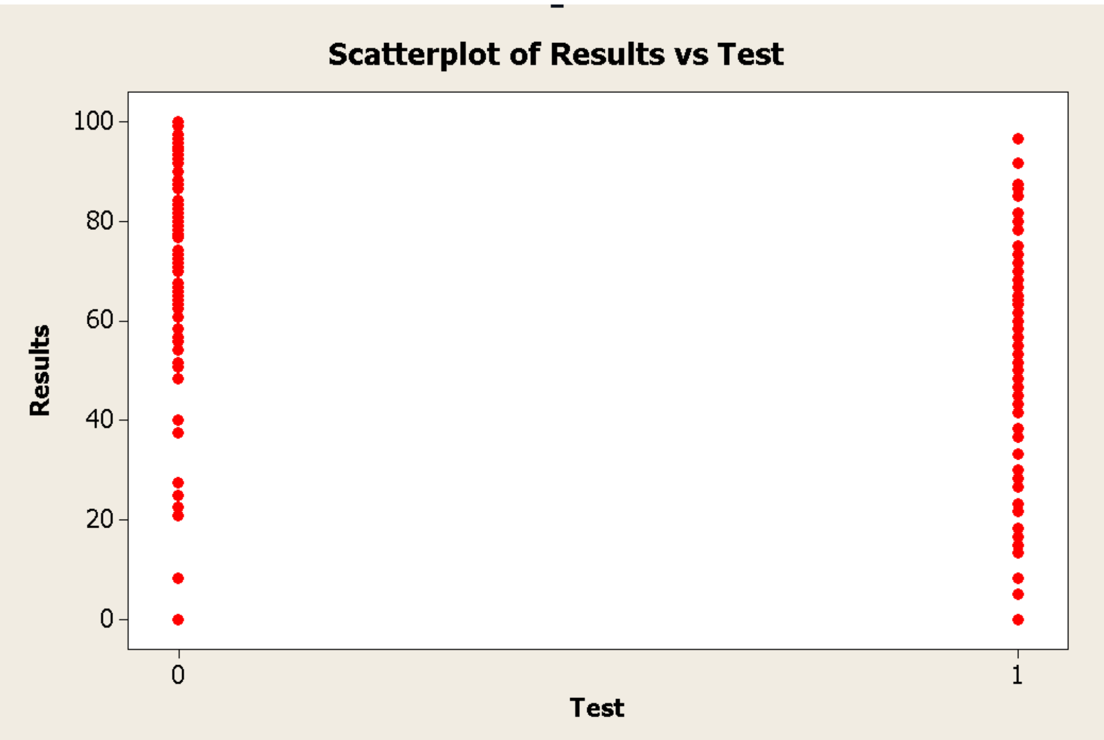


Appendix XIII:iii: Two sample t-test Individual and box-plot graphics of Male study (t=0, t=1)

```
Source    DF      SS     MS      F      P
Test       1   19777  19777  40.57  0.000
Error    178   86772    487
Total    179  106549
S = 22.08   R-Sq = 18.56%   R-Sq(adj) = 18.10%
                         Individual 95% CIs For Mean Based on
                         Pooled StDev
Level   N   Mean  StDev  ---------+---------+---------+---------+
0      86  74.18  21.65                           (-----*-----)
1      94  53.19  22.46  (----*-----)
                         ---------+---------+---------+---------+
                               56.0      64.0      72.0      80.0
Pooled StDev = 22.08

Tukey 95% Simultaneous Confidence Intervals
All Pairwise Comparisons among Levels of Test

Individual confidence level = 95.00%


Test = 0 subtracted from:

Test   Lower  Center   Upper  -------+---------+---------+---------+--
1     -27.49  -20.98  -14.48  (-----*------)
                              -------+---------+---------+---------+--
                                   -20       -10         0        10
```

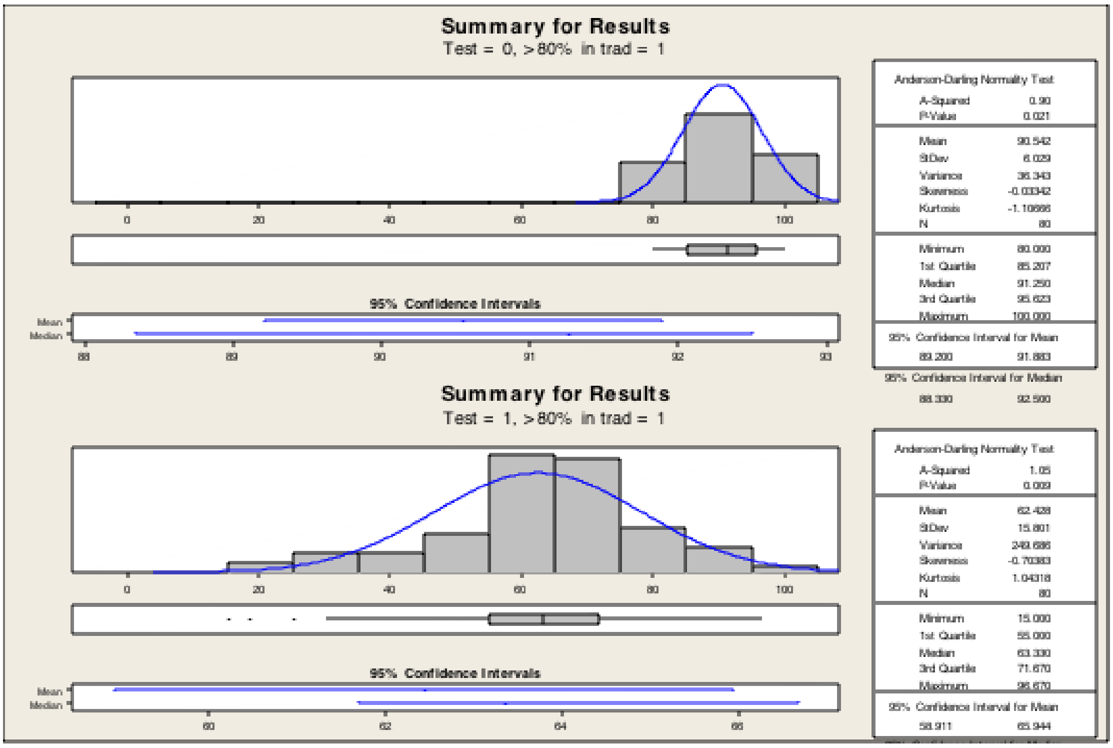
Appendix XIII:iv: One way ANOVA statistics of Male study (t=0, t=1)

```
Pearson correlation of Results and Test = -0.431
P-Value = 0.000
```

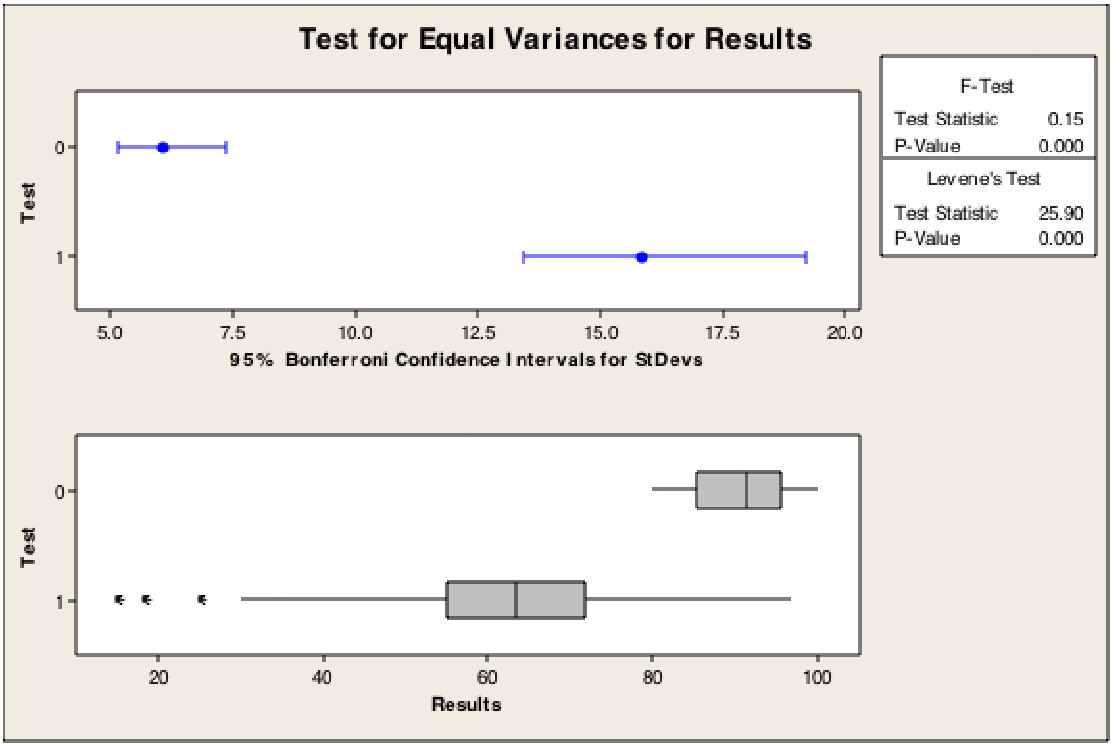


Appendix XIII:v: Correlation statistics and matrix plot of Male study (t=0, t=1)

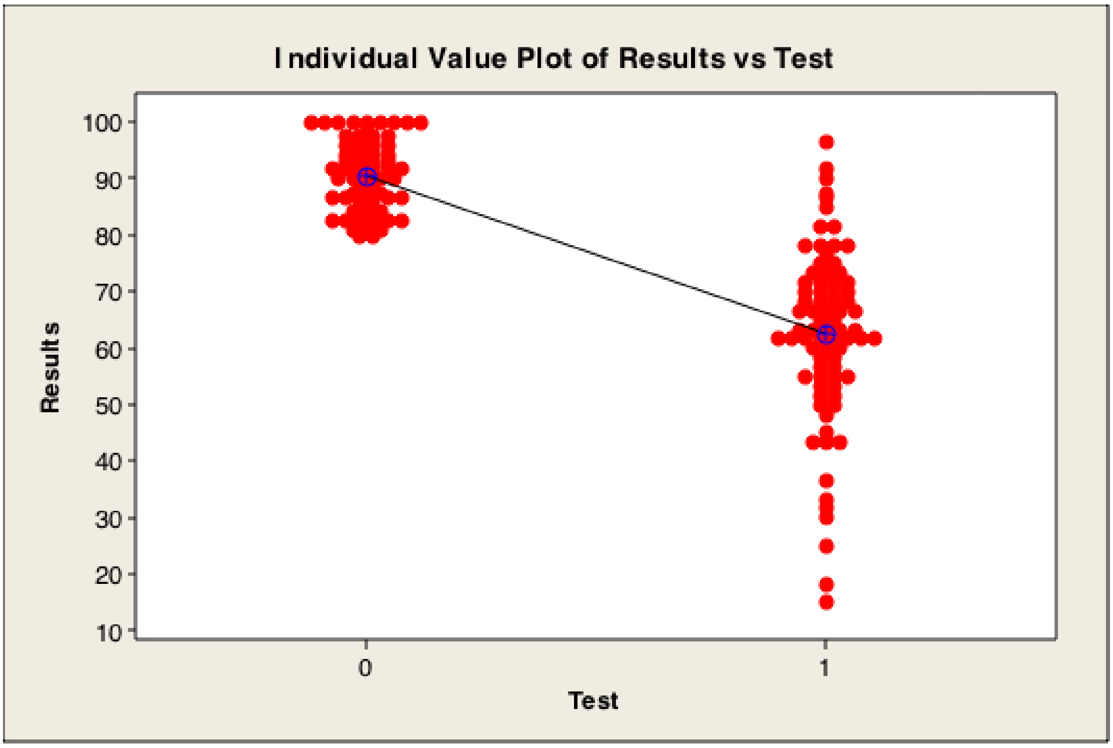
Appendix XIV – Traditional Test as an Indicator(≥80%)

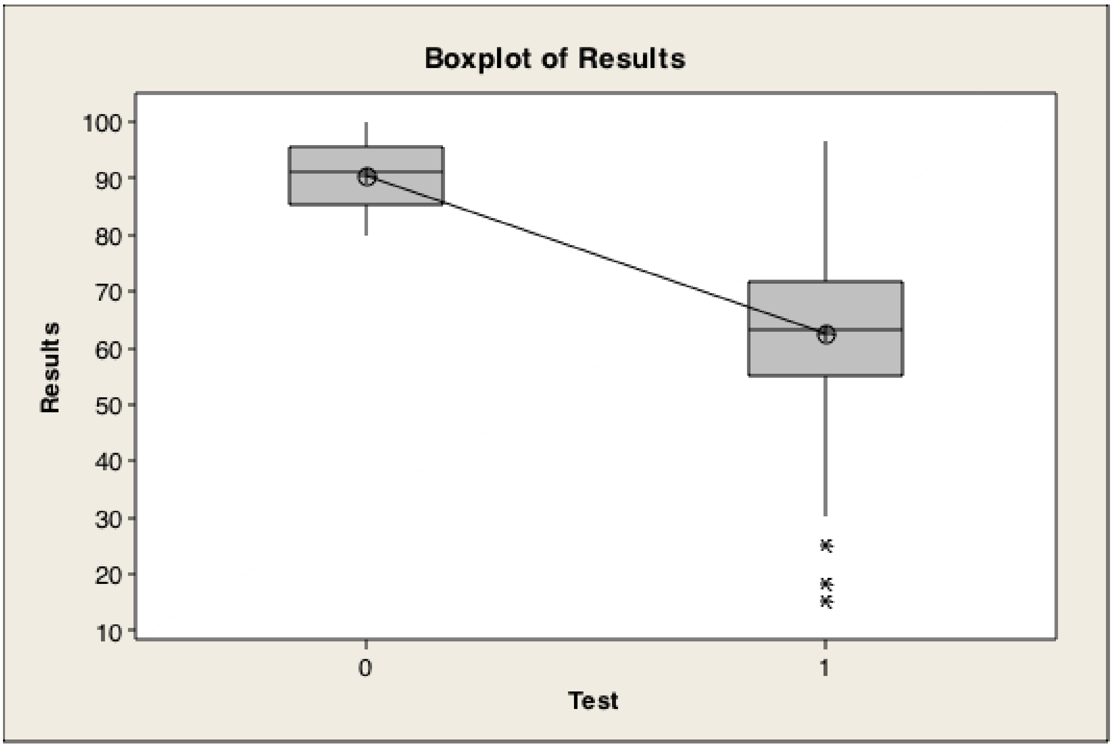


Appendix XIV:i: Graphical Summary of descriptive statistics of traditional ≥80% study (t=0, t=1)



Appendix XIV:ii: Graphical summary of test for equal variance of traditional ≥ 80% study (t=0, t=1)



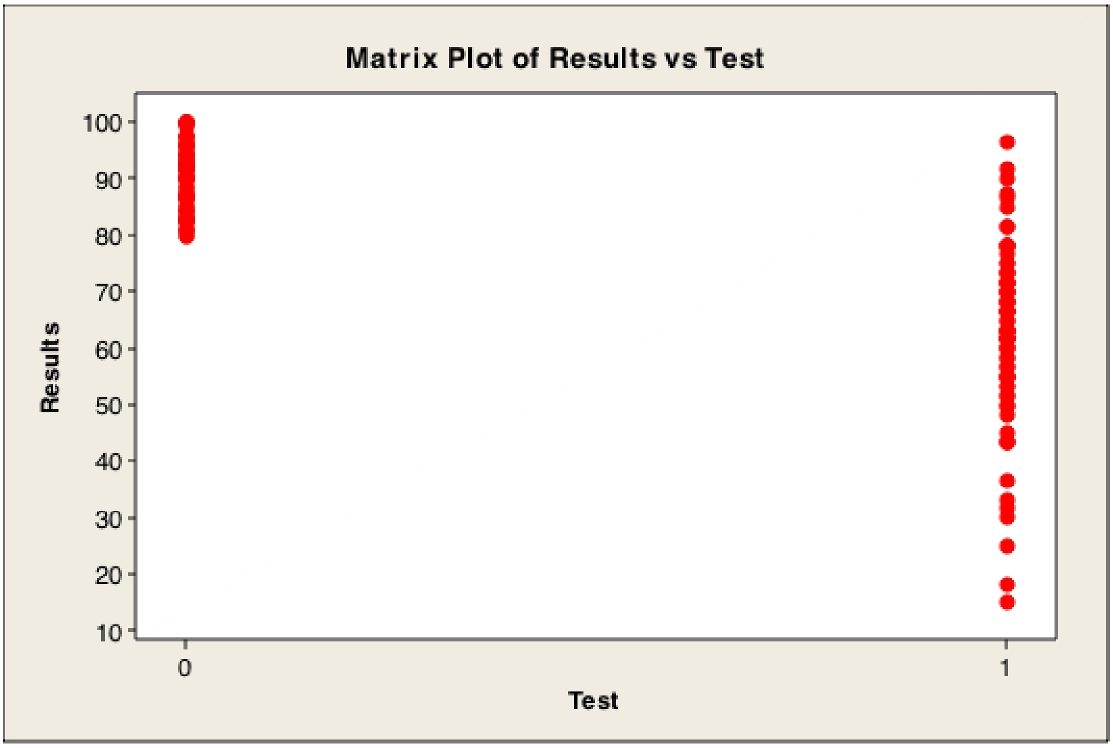


Appendix XIV:iii : Two sample t-test Individual and box-plot graphics of trad. ≥ 80% study (t=0, t=1)

```
Source    DF      SS      MS       F      P
Test       1   31616   31616  221.07  0.000
Error    158   22596     143
Total    159   54213
S = 11.96   R-Sq = 58.32%   R-Sq(adj) = 58.06%

                         Individual 95% CIs For Mean Based on
                         Pooled StDev
Level    N   Mean  StDev  ---------+---------+---------+---------
0       80  90.54   6.03                             (--*-)
1       80  62.43  15.80  (-*--)
                          ---------+---------+---------+---------
                         60        70        80        90

Pooled StDev = 11.96

Tukey 95% Simultaneous Confidence Intervals
All Pairwise Comparisons among Levels of Test

Individual confidence level = 95.00%

Test = 0 subtracted from:

Test   Lower  Center   Upper  --+---------+---------+---------+-------
1     -31.85  -28.11  -24.38  (---*---)
                              --+---------+---------+---------+-------
                              -30       -20       -10         0
```

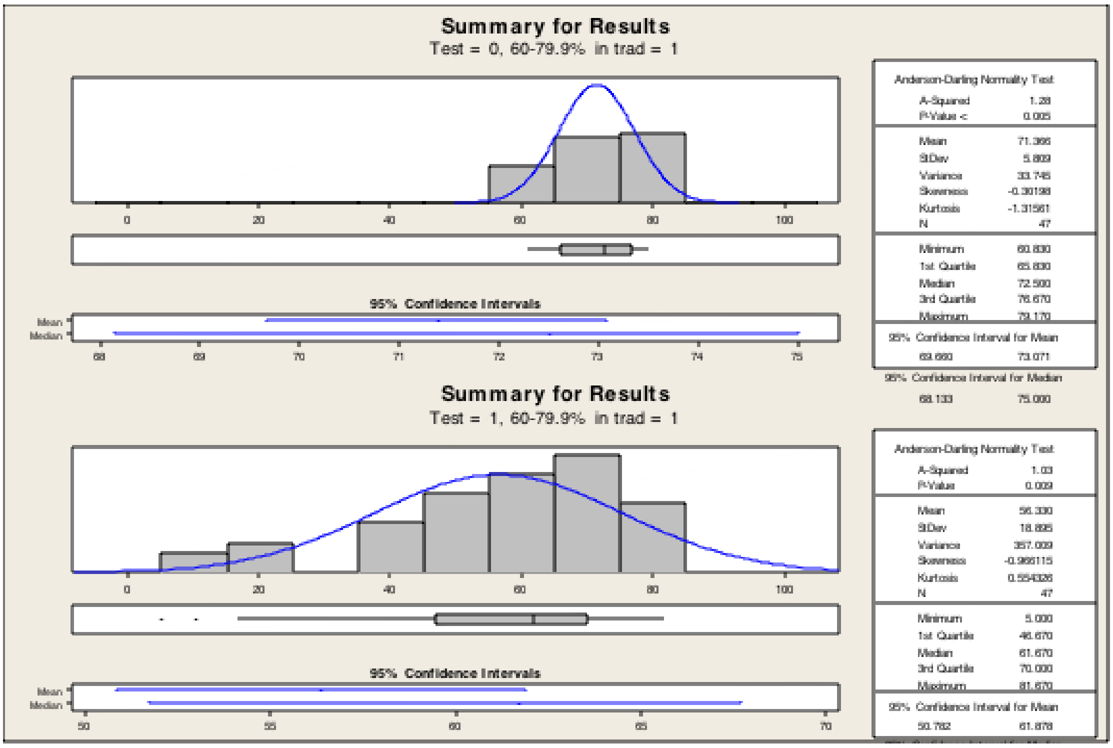
Appendix XIV:iv: One way ANOVA statistics of traditional ≥ 80% study (t=0, t=1)

```
Pearson correlation of Results and Test = -0.764
P-Value = 0.000
```

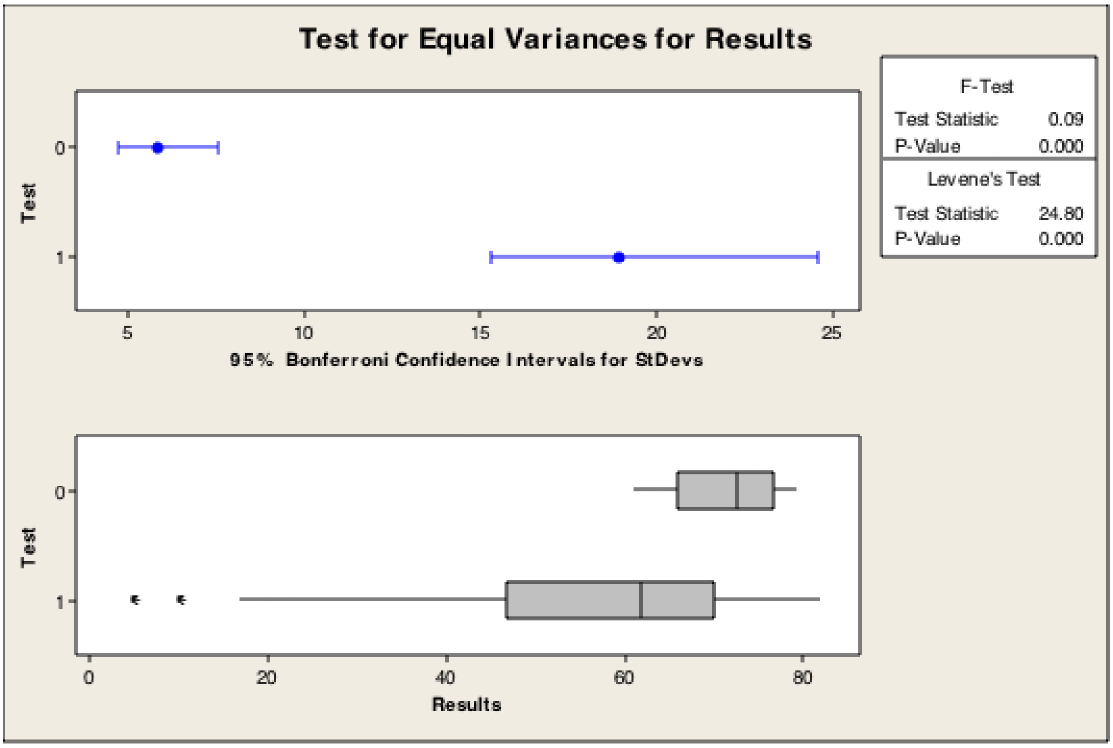


Appendix XIV:v: Correlation statistics and matrix plot of traditional ≥ 80% study (t=0, t=1)

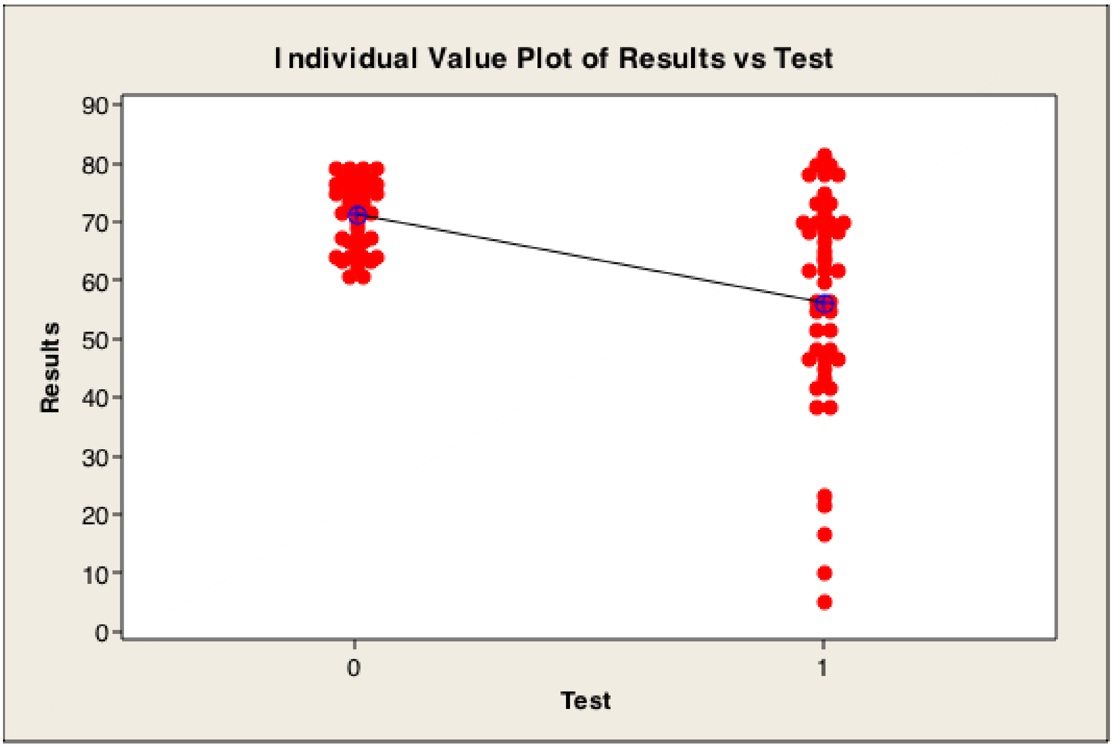
Appendix XV – Traditional Test as an Indicator (60%≤x<80%)

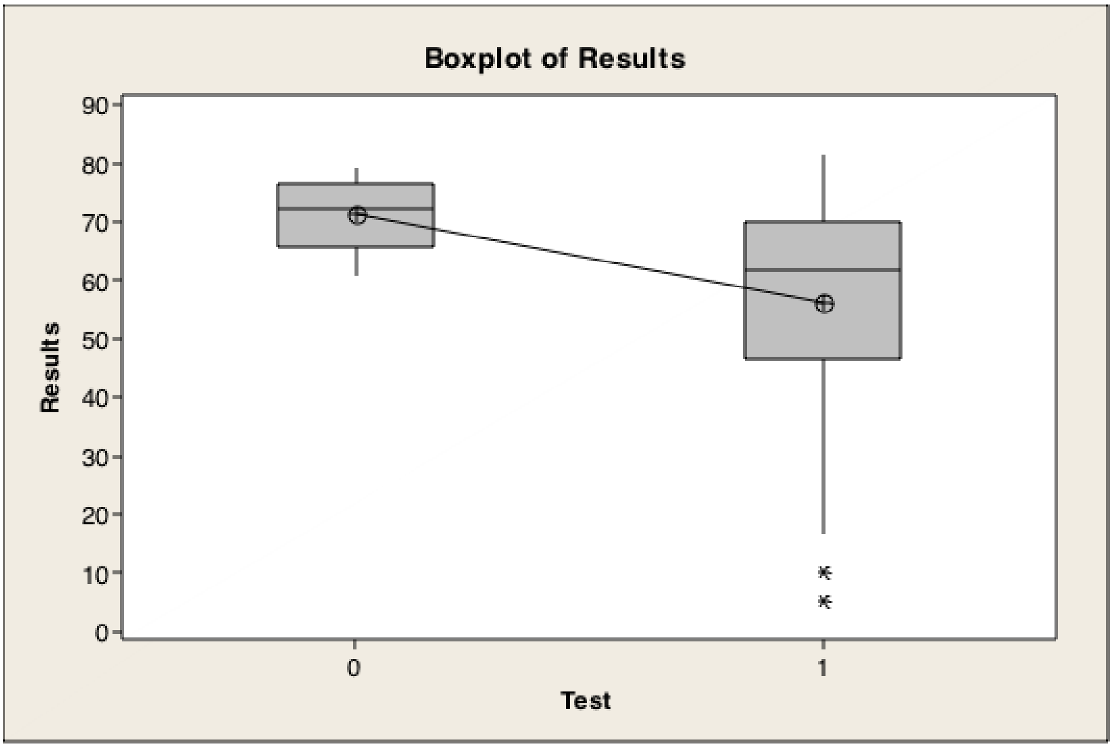


Appendix XV:i: Graphical Summary of descriptive statistics of traditional 60%≤x<80% study (t=0, t=1)



Appendix XV:ii: Graphical summary of test for equal variance of traditional 60%≤x<80% study (t=0, t=1)



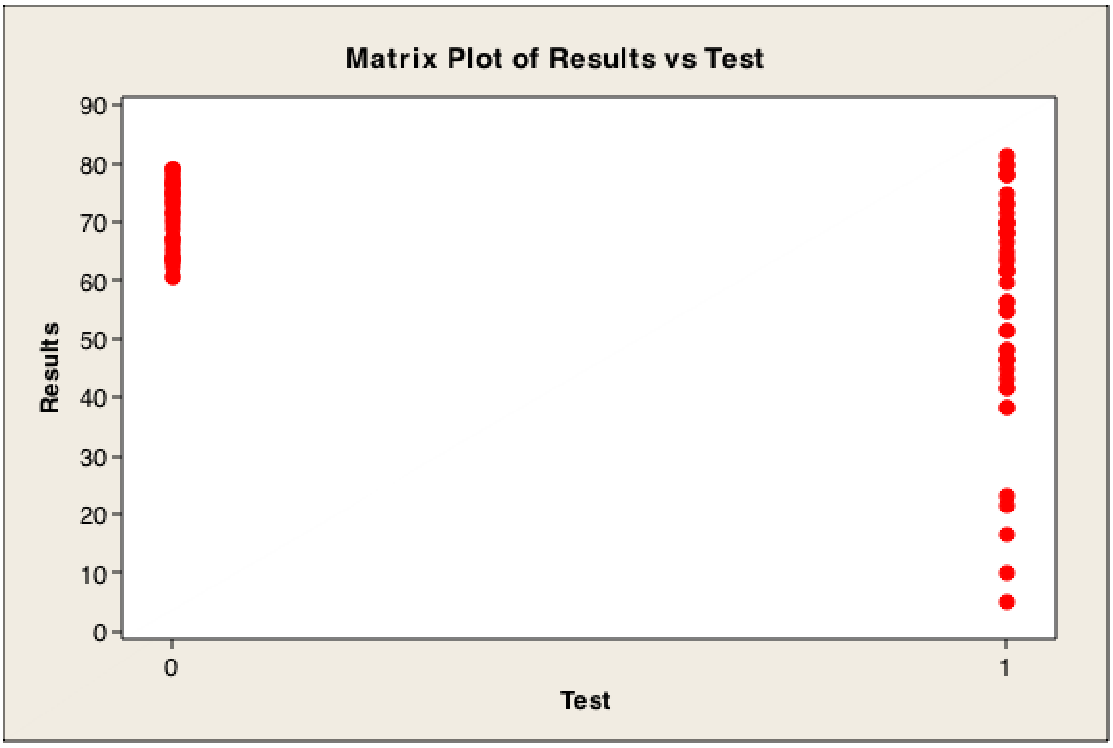


Appendix XV:iii: Two sample t-test Individual and box-plot graphics of traditional 60%≤x<80% study (t=0, t=1)

```
One-way ANOVA: Results versus Test
Source  DF     SS    MS      F      P
Test     1   5313  5313  27.19  0.000
Error   92  17975   195
Total   93  23287

S = 13.98   R-Sq = 22.81%   R-Sq(adj) = 21.97%

                         Individual 95% CIs For Mean Based on
                         Pooled StDev
Level   N   Mean  StDev  ---+---------+---------+---------+------
0      47  71.37   5.81                          (------*------)
1      47  56.33  18.89  (------*------)
                         ---+---------+---------+---------+------
                         54.0      60.0      66.0      72.0

Pooled StDev = 13.98


Tukey 95% Simultaneous Confidence Intervals
All Pairwise Comparisons among Levels of Test

Individual confidence level = 95.00%


Test = 0 subtracted from:

Test    Lower  Center  Upper      +---------+---------+---------+---------
1      -20.76  -15.04  -9.31      (--------*-------)
                                  +---------+---------+---------+---------
                               -21.0     -14.0      -7.0       0.0
```

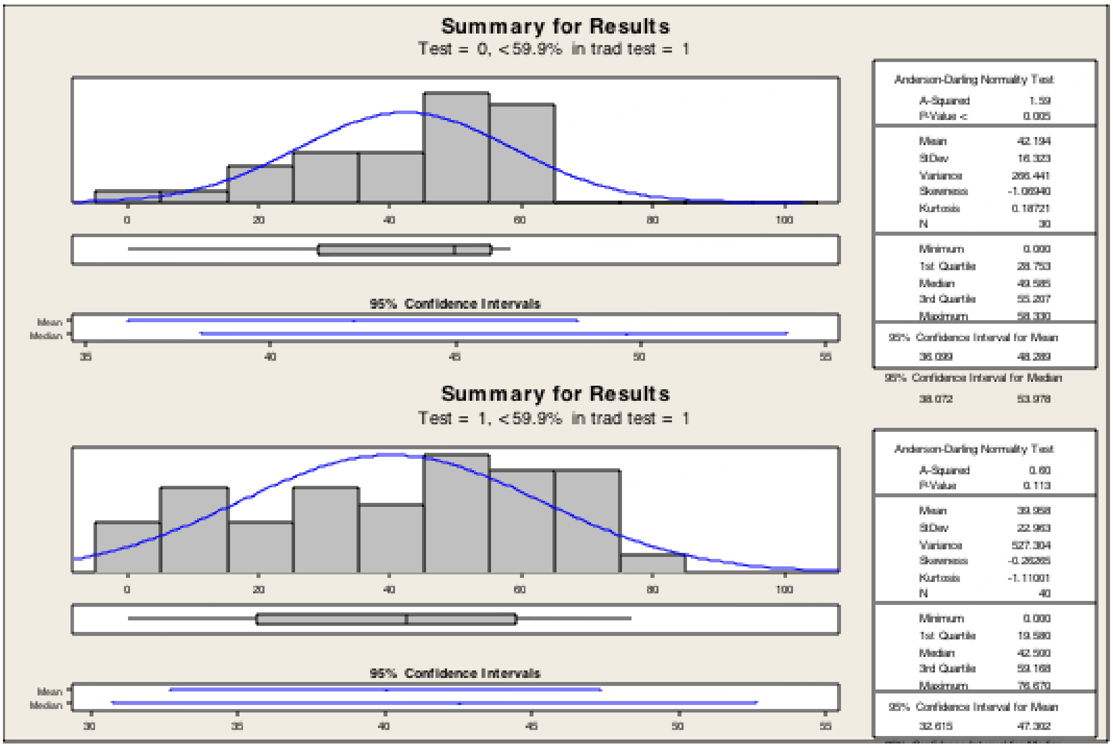
Appendix XV:iv: One way ANOVA statistics of traditional 60%≤x<80% study (t=0, t=1)

```
Pearson correlation of Results and Test = -0.478
P-Value = 0.000
```

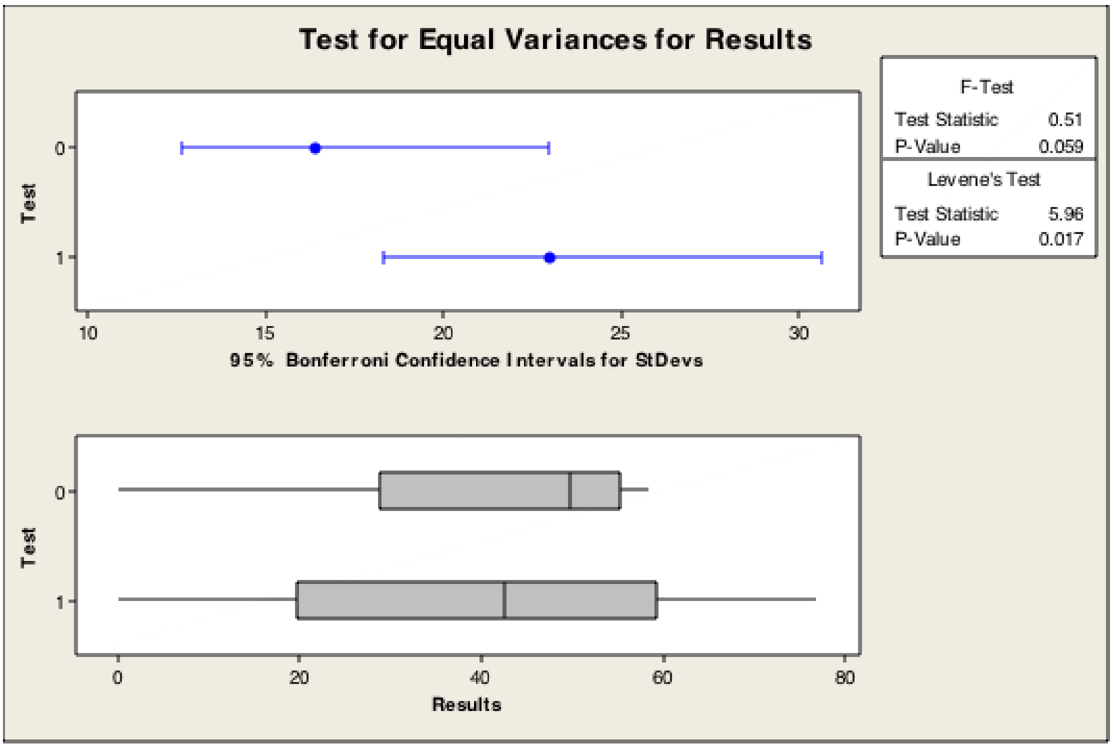


Appendix XV:v: Correlation statistics and matrix plot of traditional 60%≤x<80% study (t=0, t=1)

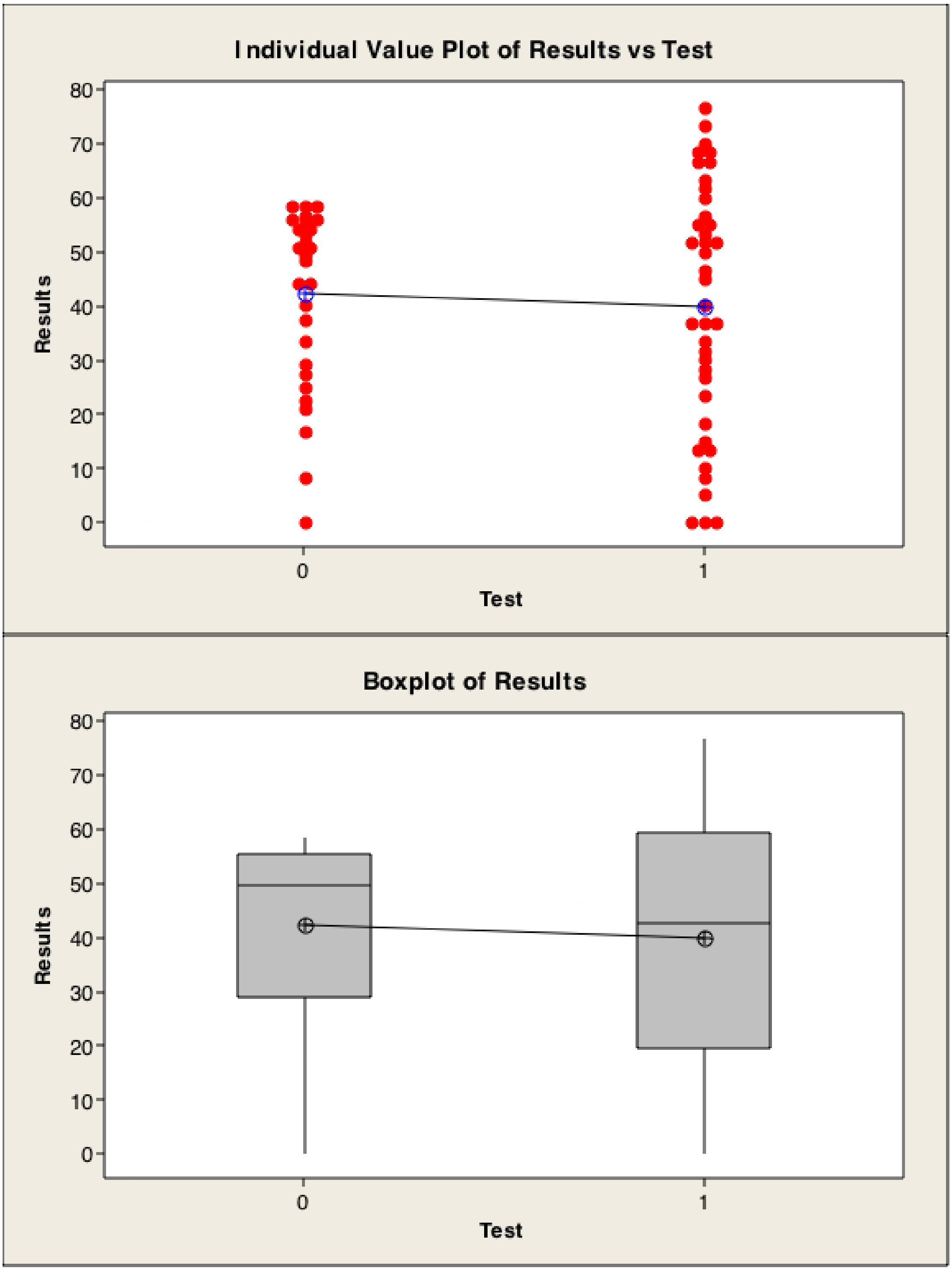
Appendix XVI – Traditional Test as an Indicator (<60%)



Appendix XVI:i: Graphical Summary of descriptive statistics of traditional 0%≤x<60% study (t=0, t=1)



Appendix XVI:ii: Graphical summary of test for equal variance of traditional 0%≤x<60% study (t=0, t=1)

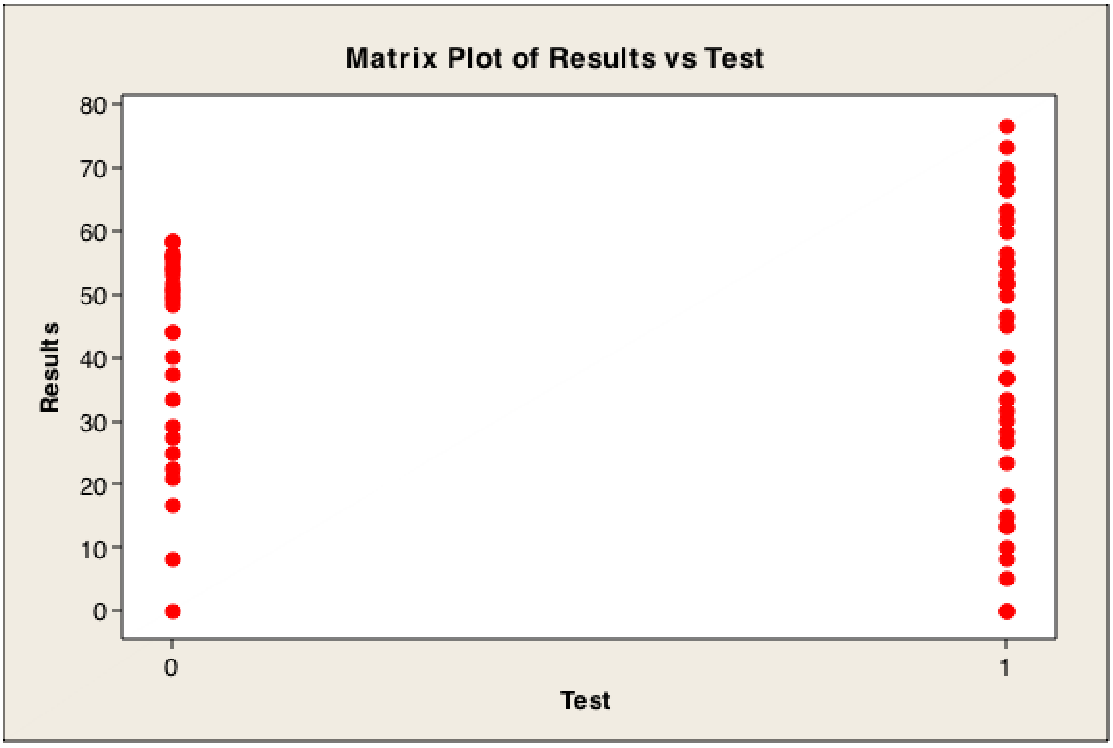


Appendix XVI:iii: Two sample t-test Individual and box-plot graphics of traditional 0%≤x<60% study (t=0, t=1)

```
Source  DF     SS   MS     F      P
Test     1     86   86  0.21  0.651
Error   68  28292  416
Total   69  28377

S = 20.40   R-Sq = 0.30%   R-Sq(adj) = 0.00%


                         Individual 95% CIs For Mean Based on
                         Pooled StDev
Level   N   Mean  StDev  ---+---------+---------+---------+------
0      30  42.19  16.32     (--------------*---------------)
1      40  39.96  22.96  (------------*------------)
                         ---+---------+---------+---------+------
                         35.0      40.0      45.0      50.0

Pooled StDev = 20.40


Tukey 95% Simultaneous Confidence Intervals
All Pairwise Comparisons among Levels of Test

Individual confidence level = 95.00%


Test = 0 subtracted from:

Test   Lower  Center  Upper     +---------+---------+---------+---------
1     -12.07   -2.24   7.60     (---------------*----------------)
                                +---------+---------+---------+---------
                            -12.0      -6.0       0.0       6.0
```

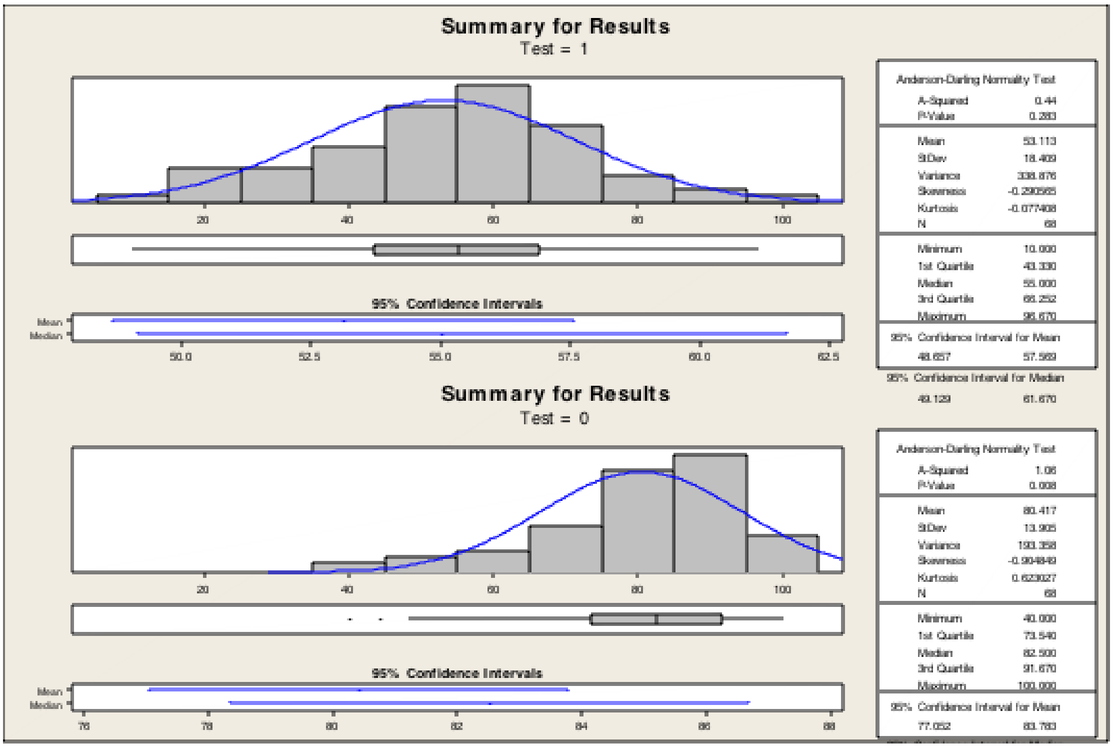
Appendix XVI:iv: One way ANOVA statistics of traditional 0%≤x<60% study (t=0, t=1)

```
Pearson correlation of Results and Test = -0.055
P-Value = 0.651
```

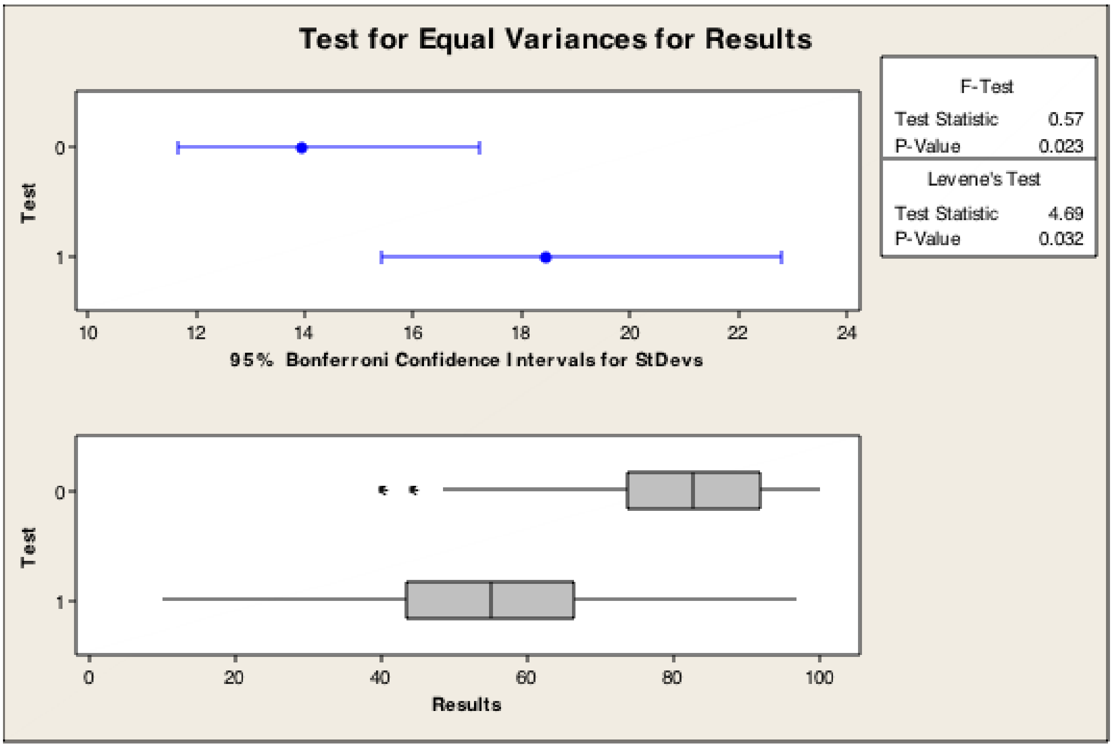


Appendix XVI:v: Correlation statistics and matrix plot of traditional 0%≤x<60% study (t=0, t=1)

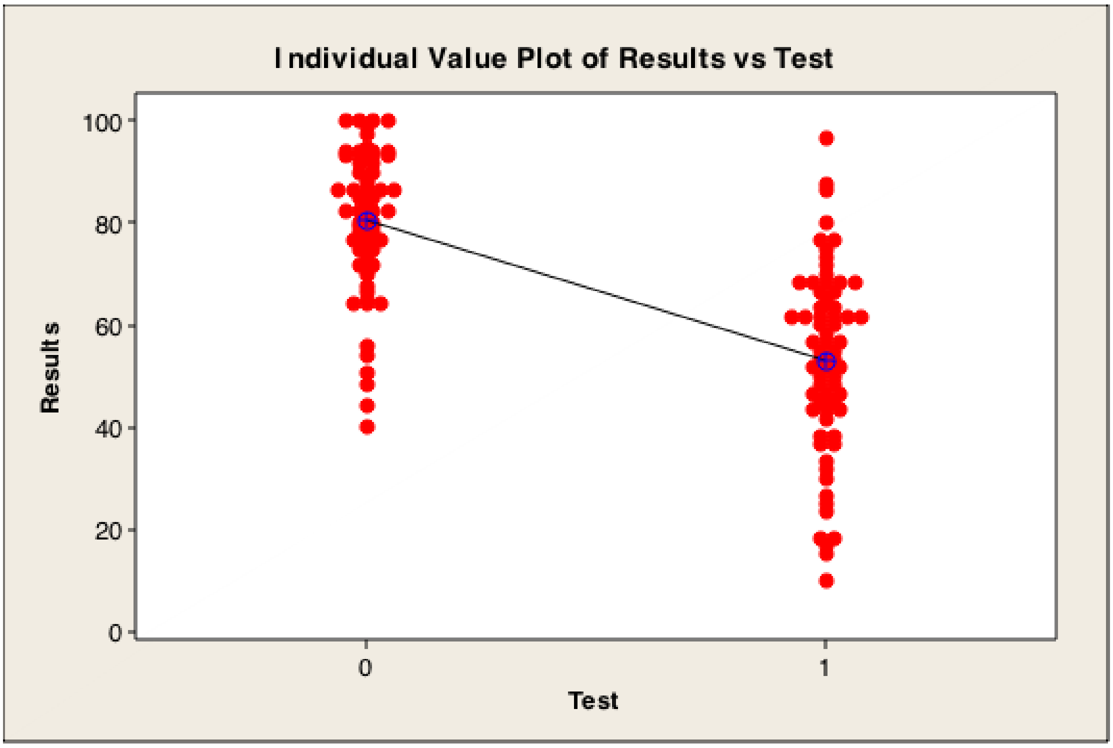
Appendix XVII: Higher Level Junior Certificate Course

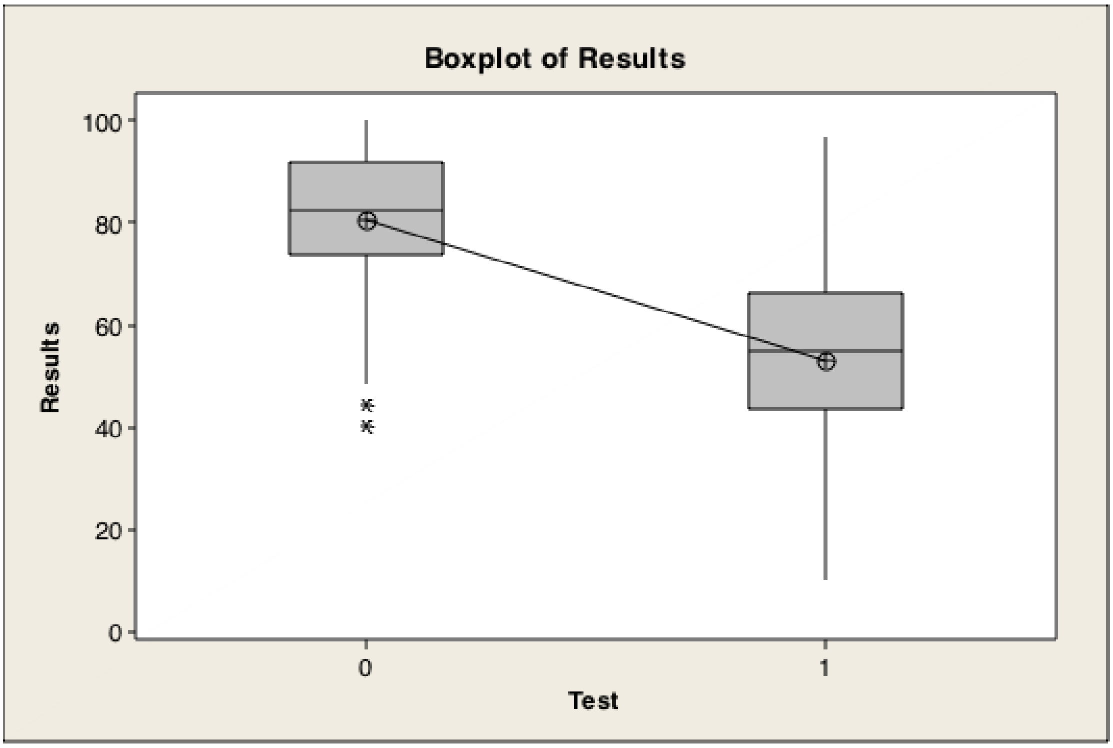


Appendix XVII:i: Graphical Summary of descriptive statistics of higher level junior certification study (t=0, t=1)



Appendix XVII:ii: Graphical summary of test for equal variance of higher level junior certification % study (t=0, t=1)



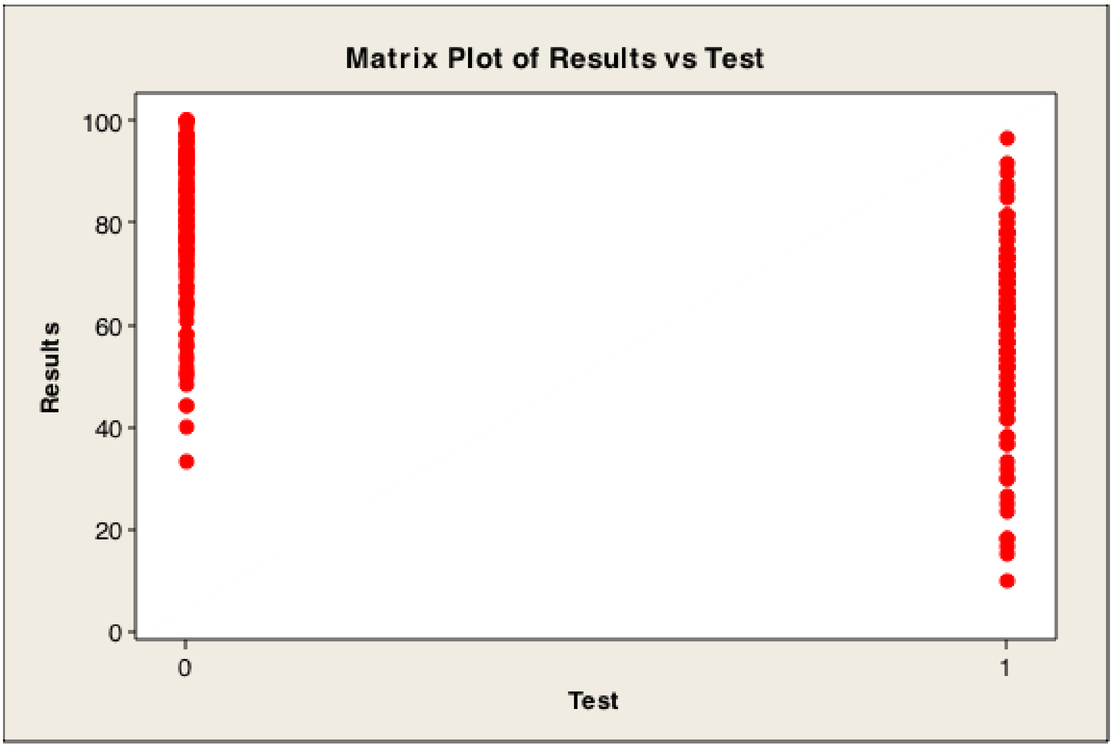


Appendix XVII:iii: Two sample t-test Individual and box-plot graphics of higher level junior certification study (t=0, t=1)

```
Source    DF     SS     MS      F      P
Test       1  25348  25348  95.25  0.000
Error    134  35660    266
Total    135  61007

S = 16.31   R-Sq = 41.55%   R-Sq(adj) = 41.11%


                         Individual 95% CIs For Mean Based on
                         Pooled StDev
Level   N   Mean  StDev  -+---------+---------+---------+--------
0      68  80.42  13.91                              (--*---)
1      68  53.11  18.41  (---*---)
                         -+---------+---------+---------+--------
                         50        60        70        80

Pooled StDev = 16.31

Tukey 95% Simultaneous Confidence Intervals
All Pairwise Comparisons among Levels of Test

Individual confidence level = 95.00%

Test = 0 subtracted from:

Test    Lower  Center   Upper  ---+---------+---------+---------+------
1      -32.84  -27.30  -21.77  (-----*----)
                               ---+---------+---------+---------+------
                                -30       -20       -10         0
```

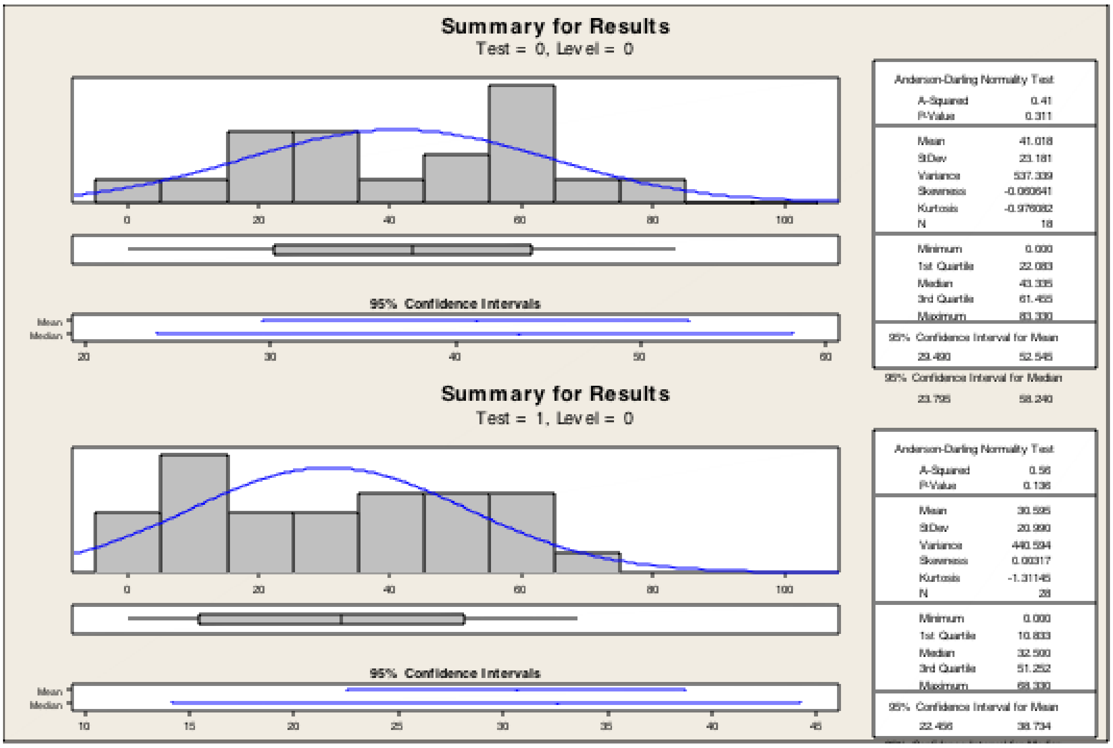
Appendix XVII:iv: One way ANOVA statistics of higher level junior certification study (t=0, t=1)

```
Pearson correlation of Results and Test = -0.531
P-Value = 0.000
```

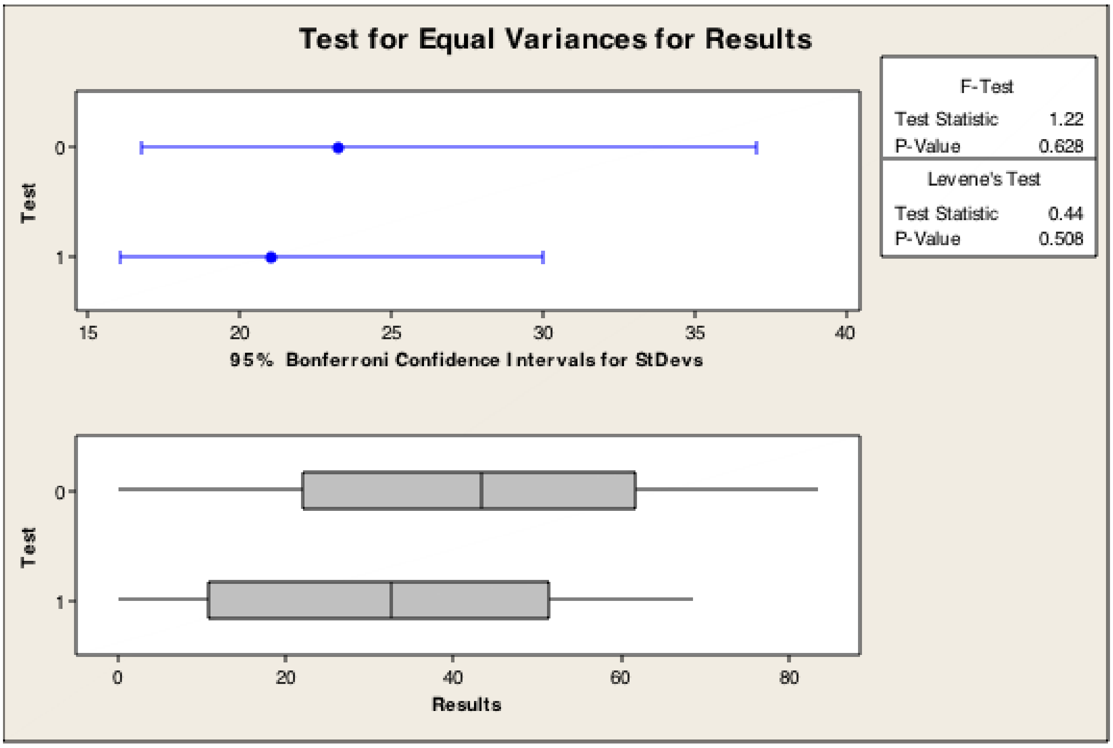


Appendix XVII:v: Correlation statistics and matrix plot of higher level junior certification study (t=0, t=1)

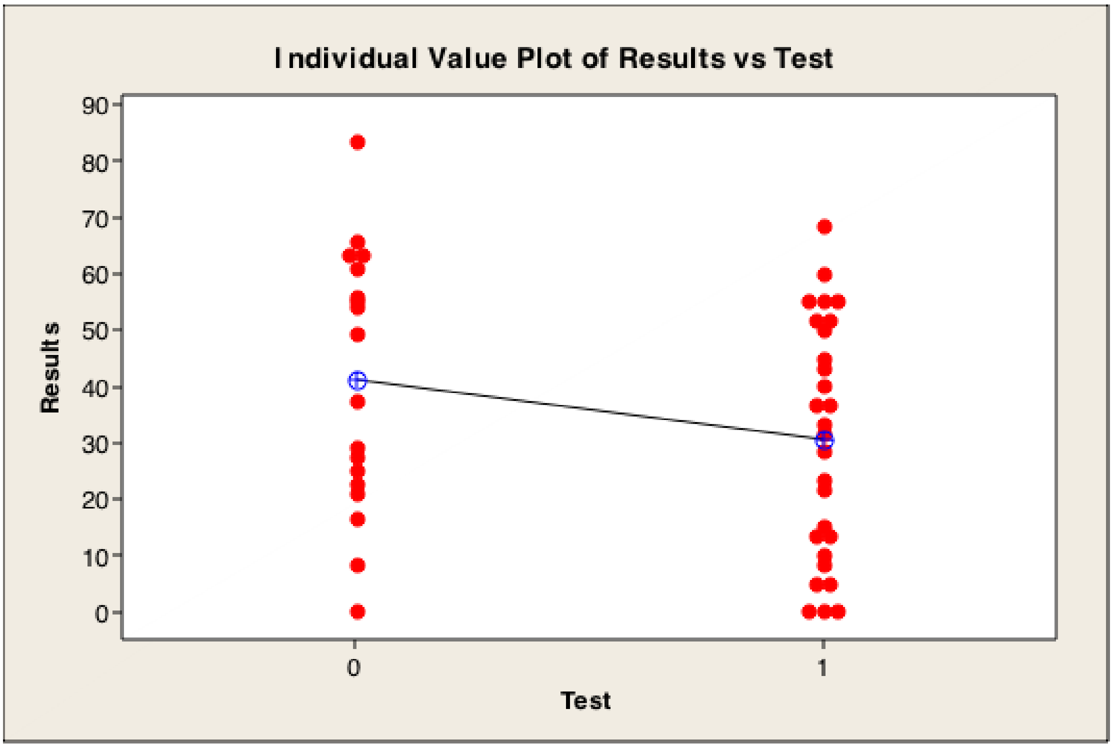
Appendix XVIII – Ordinary Level Junior Certificate Course

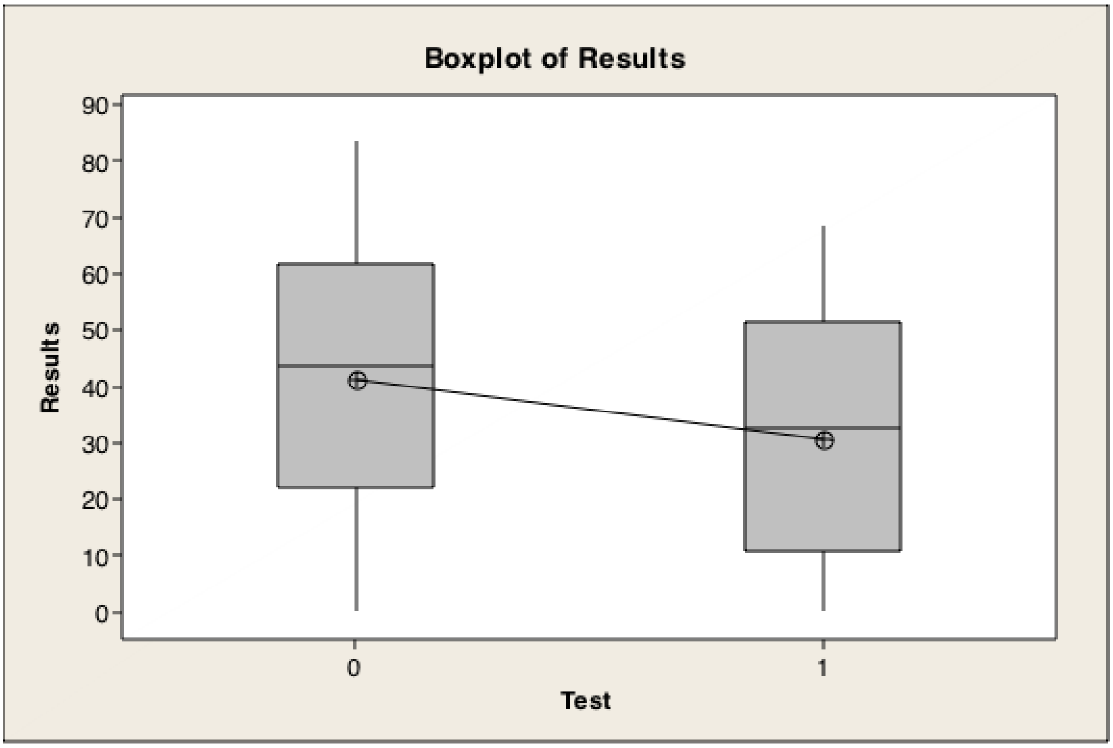


Appendix XVIII:i: Graphical Summary of descriptive statistics of ordinary level junior certification study (t=0, t=1)



Appendix XVIII:ii: Graphical summary of test for equal variance of ordinary level junior certification % study (t=0, t=1)



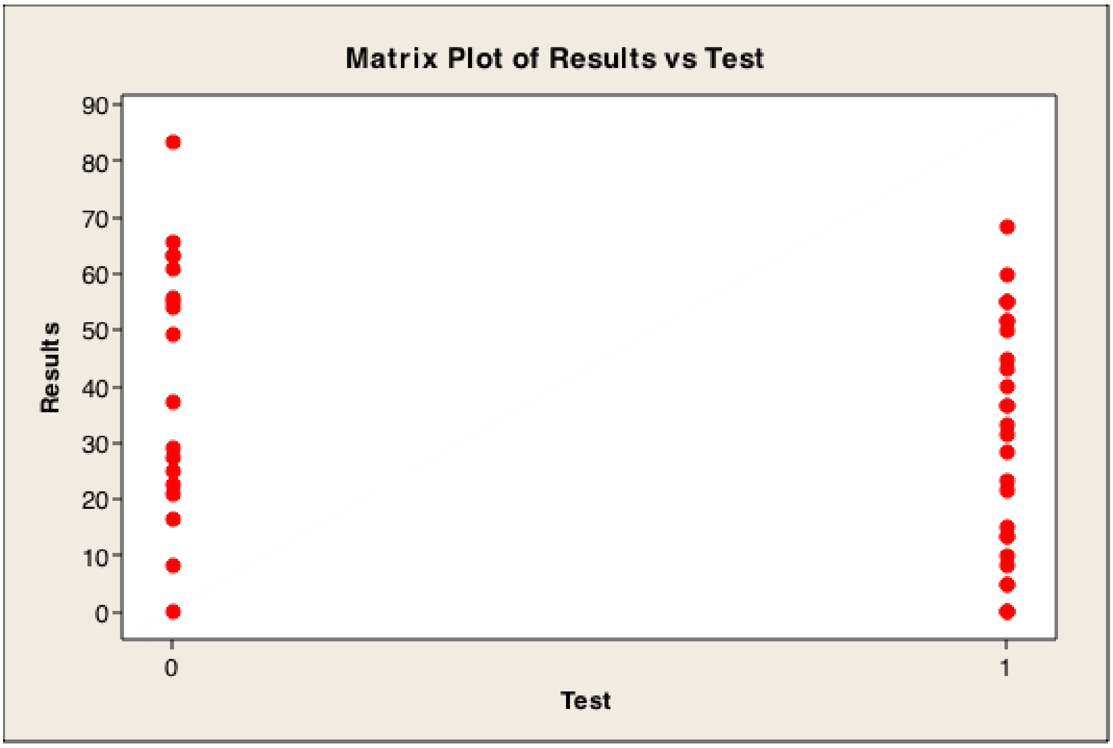


Appendix XVIII:iii: Two sample t-test Individual and box-plot graphics of ordinary level junior certification study (t=0, t=1)

```
Source  DF     SS    MS     F      P
Test     1   1190  1190  2.49  0.122
Error   44  21031   478
Total   45  22221

S = 21.86   R-Sq = 5.36%   R-Sq(adj) = 3.21%


                         Individual 95% CIs For Mean Based on
                         Pooled StDev
Level    N   Mean  StDev  --+---------+---------+---------+-------
0       18  41.02  23.18            (------------*------------)
1       28  30.60  20.99  (---------*----------)
                          --+---------+---------+---------+-------
                         24.0      32.0      40.0      48.0

Pooled StDev = 21.86


Tukey 95% Simultaneous Confidence Intervals
All Pairwise Comparisons among Levels of Test

Individual confidence level = 95.00%


Test = 0 subtracted from:

Test    Lower  Center  Upper  ----+---------+---------+---------+-----
1      -23.73  -10.42   2.89  (-------------*------------)
                              ----+---------+---------+---------+-----
                                -20       -10         0        10
```

Appendix XVIII:iv: One way ANOVA statistics of ordinary level junior certification study (t=0, t=1)

```
Pearson correlation of Results and Test = -0.231
P-Value = 0.122
```



Appendix XIX - Teacher Information Sheet

Teacher information sheet:

- The study to be carried out by the researcher, Peggy Lynch, hopes to examine the link between school learned mathematics and the ability to use these skills in real-life mathematical situations.
- The researcher, Peggy Lynch, is a PhD candidate and a student in the Education Department in NUI Maynooth.
- The NUI Maynooth Ethics Committee has approved this research project.
- The study consists of four main parts: 1. The class involved are observed in one of their regular class situations for the purpose of observing teaching and learning styles. The teachers ability is not under question and is not being examined; 2. The stduents solve real-life mathematics problems, in a pen and paper test; 3. The students involved answer some basic algebraic questions, in a pen and paper test; and 4. The teacher participates in a semi- structured interview (20 minutes in length) in order to provide information on their thoughts and opinions regarding their mathematics class involved in the research, the observed mathematics lesson and general thoughts on mathematics teaching and learning.
- It is hoped that the performance of the same students in the two different mathematical tests outlined above (2. and 3.) will give some insight as to whether students can use school-learned mathematical skills better or worse in unfamiliar, realistic mathematical situations.
- The results will form part of the researcher's PhD thesis which sets out to consider the performance of Irish mathematics students in an international context.
- The names of schools, students and teachers involved in the study will not be used in the publication of the study and/or results.
- The study solely seeks to consider the ability of students to transfer mathematics out of the classroom to realistic situations; outside of a basic report setting out the type of school, curriculum and teaching style used all other details of the students involved shall remain private.
- The research does not in any way constitute an evaluation of the child's mathematical abilities and will in no way affect the child's school results.

Appendix XX - Parent/Guardian information sheet:

Parent/Guardian information sheet:

- The study to be carried out by the researcher, Peggy Lynch, hopes to examine the link between school learned mathematics and the ability to use these skills in real-life mathematical situations.
- The researcher, Peggy Lynch, is a PhD candidate and a student in the Education Department in NUI Maynooth.
- The NUI Maynooth Ethics Committee has approved this research project.
- The study consists of three main parts: 1. The class involved are observed in one of their regular class situations for the purpose of observing teaching and learning styles, 2. The stduents solve real-life mathematics problems, in a pen and paper test, 3. The students involved answer some basic algebraic questions, in a pen and paper test.
- It is hoped that the performance of the same students in the two different mathematical tests outlined above (2. and 3.) will give some insight as to whether students can use school-learned mathematical skills better or worse in unfamiliar, realistic mathematical situations.
- The results will form part of the researcher's PhD thesis which sets out to consider the performance of Irish mathematics students in an international context.
- The names of schools, students and teachers involved in the study will not be used in the publication of the study and/or results.
- The study solely seeks to consider the ability of students to transfer mathematics out of the classroom to realistic situations; outside of a basic report setting out the type of school, curriculum and teaching style used all other details of the students involved shall remain private.
- The research does not in any way constitute an evaluation of the child's mathematical abilities and will in no way affect the child's school results.

Appendix XXI - Student information sheet:

Student information sheet:

- The study to be carried out by the researcher, Peggy Lynch, is a mathematics study. It is not a test of how clever students are: It will examine how well school prepares students to solve real-life mathematics.
- The study consists of three main parts: 1. The researcher will attend one of your mathematics lessons and watch what normally happens in your mathematics class; 2. You will be asked to solve some real-life mathematical problems in order to see if the mathematics you learn in school prepares you for questions of this kind; and 3. You will do a thirty minute mathematics test with some standard algebra questions. They will be questions similar to mathematics questions you have done before. The questions will not be very difficult. It does not matter if you can not answer all/any of them.
- The researcher hopes to see if the mathematics you do in school can also be used outside of school to solve every-day problems.
- There is no problem if you cannot answer any part of it.
- The researcher is using it as part of a project to get a degree called a PhD.
- Your name will not be used in the publication of the study.
- This will not affect your school results.
- You can change your mind about participating in the study at any stage before the results are written up.

Appendix XXII - Parent/Guardian letter of consent

Parent/Guardian letter of consent for student participation in the mathematisation project.

Parental/Guardian Consent:

I am happy for my child to participate in the mathematisation study as outlined in the information sheet.

Signed: ________________ Date: ______________

Appendix XXIII – Student letter of consent

Letter of consent for student participation in the mathematisation project.

Student Consent:

I agree to participate in the mathematisation project and have read the information sheet provided. I understand what is involved in this project and am happy to be involved.

Signed: ________________ Date:____________

Appendix XXIV – Ethical application approval letter

NATIONAL UNIVERSITY OF IRELAND, MAYNOOTH
MAYNOOTH, CO. KILDARE, IRELAND

Dr Carol Barrett
Secretary to NUI Maynooth Ethics Committee

NUI MAYNOOTH
Ollscoil na hÉireann Má Nuad

13 April 2010

Peggy Lynch
Education Department
NUI Maynooth

RE: *Application for Ethical Approval for a project entitled:*
"The transfer of mathematics from the classroom to real-life situations"

Dear Peggy,

The Ethics Committee evaluated the above project for approval and we would like to inform you that ethical approval has been granted, with the condition that a comment is added to the parent information sheet to state that:

a clause be included that reassures the parent that the research does not in any way constitute an evaluation of the child's mathematical abilities and will in no way affect the child's school results.

Kind Regards,

Dr Carol Barrett
Secretary to NUI Maynooth Ethics Committee

CC: Dr. Rose Malone
Education Department