

## ROBUST SHAPE CONTROL IN A SENDZIMIR COLD-ROLLING STEEL MILL

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**Abstract:** The shape control problem for a Sendzimir 20-roll cold rolling steel mill is characterised by operation over a wide range of conditions arising from roll changes, changes in rolling schedules and changes in material gauge, width and hardness. Previous approaches to the problem suggest storing a large number of precompensator matrices to cater for the full range of operating conditions. This paper, on the other hand, attempts to synthesise a controller which is optimally robust to changes in the conditions associated with the rolling cluster, resulting in a reduced storage requirement for the controlling computer. The performance of the robust controller is evaluated via nonlinear simulation.

**Keywords:** Steel manufacture, shape control, robust control, gain scheduling

### 1. INTRODUCTION

Accurate control of the shape (internal stress distribution) of steel strip in cold rolling presents a significant challenge, due to the multi-pass, multi-schedule nature of the activity. The different passes and schedules (approx. 2500 in all) required to achieve a given final gauge for different grades and widths of rolled strip involve variations in mill setup, such as roll diameters and strip speed and changes in material characteristics, such as input and output gauges for each pass, strip width and material hardness. These cause significant (up to 300%) changes in the mill model parameters, which point clearly to a requirement for a number of controllers.

The Sendzimir mill (see Fig.1) is a reversing mill, and a separate schedule containing a number of passes is specified for each different material rolled. A schedule can contain from 4 to 15 passes through the rolling cluster. Each pass involves different entry and exit gauges, with minor changes in the material hardness from pass to pass.

To date, the approach has been to design controllers using traditional multivariable techniques for a set of nominal cases (e.g. every schedule) and then employ a test to check for controller stability for schedules and passes outside this

nominal set (Ringwood, *et al.*, 1990; Ringwood, 1995). However, the deficiencies of this approach are (a) little attempt is made to actively build in robustness to model variations, resulting in possible wide variations in performance (although stability may be retained), and (b) no systematic method for scheduling different controllers across different passes and schedules is obvious.

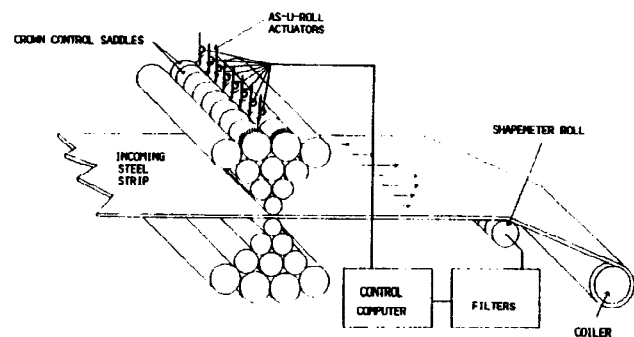


Fig.1. 20-roll cold rolling Sendzimir mill

This paper attempts to actively build in robustness to model parameter variations, due to pass and schedule changes, and clearly identify a scheduling strategy which can be implemented. In addition, other sources of model

imperfection are addressed. The approach relies on a problem formulation in  $H_\infty$ , where an attempt is made to guarantee robust stability over an unstructured model uncertainty due to pass changes, while maintaining reasonable performance (small sensitivity function) and high frequency measurement noise rejection.

## 2. SENDZIMIR MILL MODEL

The Z-mill has an ASEA 'Stressometer' for measuring the differential tension (or stress) profile across the strip. This device is mounted 2.91 m downstream of the roll gap and produces 8 (modelled) output measurements. Four pressure measurements per revolution of this device are provided, causing a four-period-per-revolution sinusoid to be superimposed on the output signal (40Hz at a speed of 10 m/sec.). Further noise on the output signal is introduced due to the 2kHz magnetising currents used with the pressure sensors.

Shape actuation is effected via the 'As-U-Rolls', which provide the equivalent of 8 independent (but equally spaced) point loads. This generates roll bending, causing differential elongation of the strip, thus influencing the shape profile.

The Z-mill model, therefore, has 8 outputs and 8 inputs. The rolling cluster is the most complex part of the system and accounts for all of the interaction between the 8 (modelled) paths in the system. A linearized gain matrix ( $G_a$ ) relates changes in the roll-gap shape profile to changes in the positions of the AUR's respectively (Gunawardene, 1982).

Diagonal dynamic blocks account for the actuators, strip dynamics (between roll-gap and shapemeter) and the shapemeter filters. The mill model is therefore of the form:

$$y = p(s) G_a f_a(u_a), \quad G_a \in \mathcal{R}^{8 \times 8} \quad (1)$$

where  $p(s)$  includes dynamics due to the strip and shapemeter, and the nonlinear function  $f_a(\cdot)$  represents the AUR actuators (see Fig.2). An actuator linearisation technique (Ringwood, 1994) is applied to the nonlinear actuators, resulting in a first order linear response for each actuator with a time constant of 2 secs.

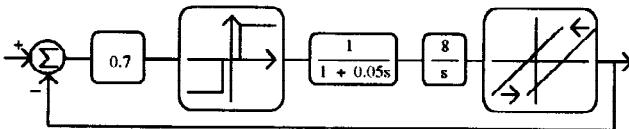


Fig.2. As-U-Roll actuator block diagram

The resulting overall mill dynamics are therefore given by:

$$g(s) = \frac{e^{-0.582s}}{(1+1.064s)(1+0.74s)(1+2s)} \quad (2)$$

for a medium strip speed ( $\approx 10$  m/s). A disturbance,  $d(s)$ , is included in the mill model to account for the shape of the incoming steel strip.

The scalar dynamical transfer function,  $p(s)$ , varies with strip speed, while the mill matrix,  $G_a$ , varies with mill setup (and therefore with pass and schedule no.). In this paper, an attempt will be made to design a controller which can cover all the (six) passes of a given schedule. The controller design is based on the nominal (average)  $G_a$  of:

$$G_a = \begin{bmatrix} 4.3907 & 4.9866 & -0.07199 & -2.1837 & -2.3299 & -2.0186 & -1.7916 & -1.7943 \\ 0.6112 & 2.5487 & 2.5138 & 0.2207 & -1.5248 & -1.9372 & -1.7115 & -1.7027 \\ -0.7673 & 0.4553 & 2.7242 & 1.7667 & -0.5024 & -1.7515 & -1.7883 & -1.7682 \\ -1.0494 & -1.0781 & 1.1593 & 2.6865 & 1.5551 & -0.6776 & -1.7538 & -1.7282 \\ -0.9135 & -1.6900 & -0.7009 & 1.4843 & 2.7079 & 1.2133 & -1.1479 & -1.1449 \\ -0.7882 & -1.7710 & -1.6810 & -0.3389 & 1.9747 & 2.7206 & 0.4609 & 0.4541 \\ -0.7566 & -1.7308 & -1.9495 & -1.5653 & 0.0505 & 2.3465 & 2.6623 & 2.6831 \\ -0.8345 & -1.8083 & -1.9580 & -2.2416 & -1.9845 & -0.1392 & 4.9882 & 4.9173 \end{bmatrix}$$

Mill matrices for passes 1 and 6 of the schedule are given in the Appendix.

## 3. ROBUST CONTROLLER DESIGN

### 3.1 Design Framework

Controller design in the  $H_\infty$  framework provides a guarantee of stability within a given set of model perturbations arising from pass or schedule changes (Maciejowski, 1989). Further objectives include the achievement of good dynamical performance across the set of perturbed plants and the attenuation of measurement noise and disturbances. Tradeoffs and conflicts arising from these different requirements are resolved using the weighting functions  $W_1(s)$  and  $W_2(s)$  in the  $H_\infty$  cost function:

$$J = \begin{bmatrix} W_1 S(s) \\ W_2 T(s) \end{bmatrix} \quad (3)$$

where:

$$S(s) = (1 + GK(s))^{-1} \quad (4)$$

is the system sensitivity function, which determines the disturbance rejection properties of the system, and

$$T(s) = GK(s)(1 + GK(s))^{-1} \quad (5)$$

is the complementary sensitivity function, which determines robust stability and measurement (shapemeter) noise attenuation. The components considered in a robust control

design are detailed in Fig.3. A further issue in weight selection is the requirement that the closed-loop bandwidth rolls off in frequency before the phase effects of the pure delay term in equation (2) become significant. This is achieved using  $W_2(s)$ .

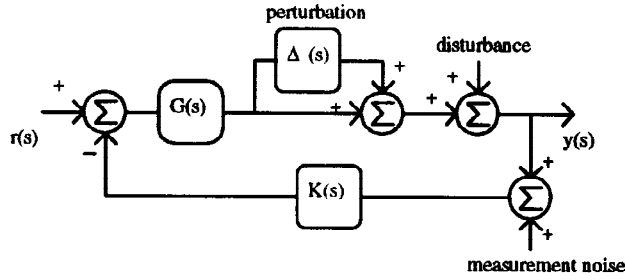


Fig.3. Plant configuration for robust control design

Robust stability is guaranteed by ensuring that the weight  $W_2(s)$  overbounds the plant (multiplicative) perturbation in the maximum singular value sense as:

$$\bar{\sigma}[W_2(j\omega)] \geq \bar{\sigma}[\Delta(j\omega)] \quad \forall \quad \omega \geq 0 \quad (6)$$

where

$$G(s) = G_{nom}(s)(I + \Delta(s)) \quad (7)$$

### 3.2 Reduction of Plant Dimension

Examination of the mill matrix,  $G_a$ , indicates singularity problems due to a significant spread in its singular values. In addition, an order of magnitude difference exists between the four largest and four smallest singular values. As an example, the singular value spectrum of the nominal plant mill matrix  $G_a$  is:

[12.3568 9.1120 4.9125 1.5625 0.3306 0.2101 0.0259 0.0051]

This poses a significant problem for a  $H_\infty$  design, since the sensitivity function,  $S(s)$ , will be always close to unity in the directions of the small singular values (i.e. the product  $GH(s)$  is approximately zero in those directions). This suggests a reparameterisation of the plant in terms of the four most significant singular values. Partition the plant as:

$$G(s) = g(s) \begin{bmatrix} U_1 & U_2 \end{bmatrix} \begin{bmatrix} \Sigma_1 & 0 \\ 0 & \Sigma_2 \end{bmatrix} \begin{bmatrix} V_1^T \\ V_2^T \end{bmatrix} \quad (8)$$

where:

$$U_1, U_2, V_1, V_2 \in \mathcal{R}^{8 \times 4}, \quad \Sigma_1, \Sigma_2 \in \mathcal{R}^{4 \times 4}$$

A parameterisation  $U_1^T$  is now applied to the mill output shape profile, while the control input is parameterised using  $V_1$ . The  $H_\infty$  design is now concentrated on the reduced dimension system, given by:

$$G_{red}(s) = g(s) U_1^T G_a V_1 \quad (9)$$

Such a parameterisation is consistent with previous approaches (Ringwood, 1995), motivated by rolling practice considerations.

### 3.3 Controller Design

The  $H_\infty$  controller design was now performed using the reduced plant in (9) against the objectives and considerations set out in Section 3.1. The weighting functions are chosen as:

$$W_1(s) = \frac{10(10^{-3}s+1)}{10^2s+1} I_4, \quad W_2(s) = \frac{0.2774(10^{-3}s+1)}{10^{-6}s+1} I_4 \quad (10)$$

and are shown together with  $\bar{\sigma}(\Delta(j\omega))$  in Fig.3.

$W_1(s)$  is chosen to:

- Penalise sensitivity,  $S(s)$ , at low frequency, giving good d.c. disturbance rejection (step changes in incoming strip shape profile due to welds), and
- Ensure that system performance (dynamic response) is maintained in spite of parameter variations at low frequency.

$W_2(s)$  is chosen to:

- Ensure robust stability by covering  $\Delta(s)$  i.e. that condition (6) is met, and
- Attenuate high frequency (shapemeter) measurement noise, by driving  $T(s)$  down at high frequency.

In addition, the relative positions of  $W_1(s)$  and  $W_2(s)$  determine the closed-loop bandwidth, controlling the dynamical response to setpoints and disturbances.

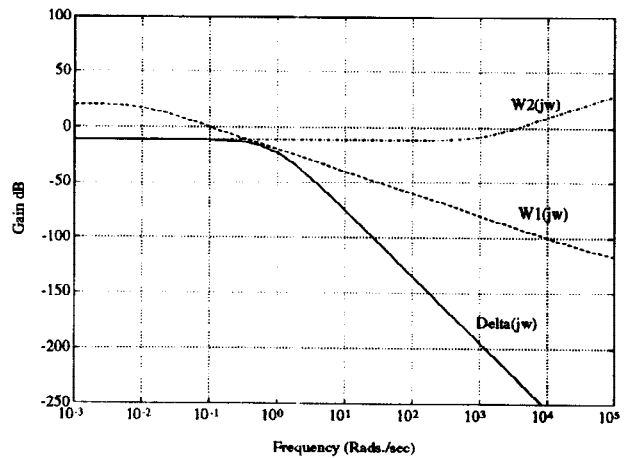


Fig.4. Weighting functions and perturbation.

The software used in the controller optimisation was based on the MATLAB® Robust Control Toolbox (Safonov and Chiang, 1988), with the derived controller detailed in the Appendix. A block diagram of the closed-loop system is shown in Fig.5. Note that the shape control problem is basically a regulator problem, since the desired shape (stress) profile in the output strip is uniform i.e. zero at all points.

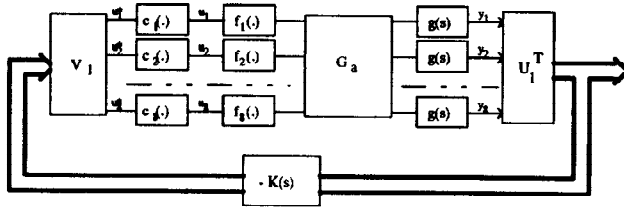


Fig.5. Closed-loop system

4. CONTROLLER PERFORMANCE ANALYSIS

Nominal performance and achievement of specifications were checked using frequency response plots, as given in Section 4.1. However, to assess the added effects of neglected time delay and residual nonlinearity in the actuators, nonlinear simulation tests were used to demonstrate realistic controller performance.

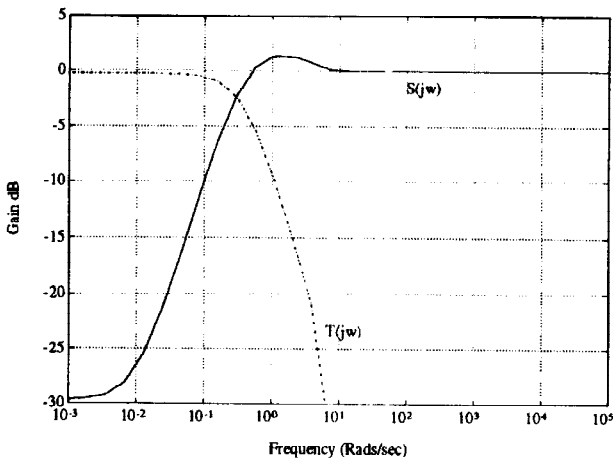


Fig.6. Sensitivity and complimentary sensitivity functions

4.1 Achievement of design objectives

Fig.6 shows the system sensitivity function,  $S(j\omega)$ , and the complementary sensitivity function,  $T(j\omega)$ . Note that  $S$  drops to  $-30\text{dB}$  at low frequency, ensuring good d.c. disturbance rejection and insensitivity to d.c. plant parameter variations. The profile of the complementary

sensitivity function, which also specifies the closed-loop transfer function, has a  $3\text{dB}$  bandwidth of approx.  $0.1\text{ rads/sec.}$ , giving a reasonable closed-loop bandwidth, while providing good attenuation of high frequency shapemeter noise. In addition, the closed-loop transfer function rolls off before the phase effects of the time delay in equation (2) become significant.

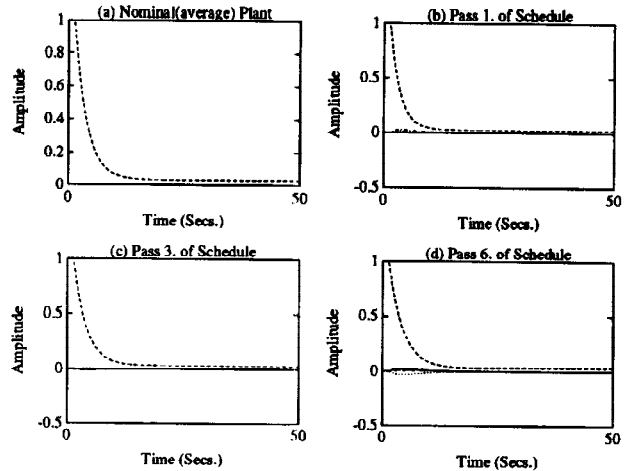


Fig.7. Disturbance rejection properties of system

To indicate the transient response and robustness of the system, a step disturbance in path 2 of the idealised (linear, delay-free) system was introduced and the output response for each path shown in Fig.7(a). Robustness to variations in  $G_a$  is demonstrated by observing the response for 3 different  $G_a$  matrices, corresponding to passes 1 (Fig.7(b)), 3 (Fig.7(c)) and 6 (Fig.7(d)) of a 6 pass schedule. It is seen that nominal performance and disturbance decoupling is preserved to a good degree of accuracy in all cases.

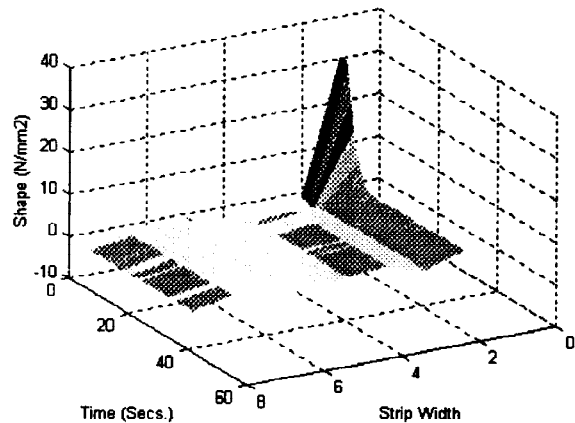


Fig.8. Shape control for nominal system

## 4.2 Simulation results

The controller developed above was now simulated with a simulation model of the mill, containing the transport delay (which was ignored in the design), nonlinear actuators together with their linearising precompensators and a realistic incoming strip shape disturbance. The shape profile variations are shown in Fig.8 for the nominal system. Parametric shape variations, which are shown as the output in Fig.5, are given in Fig.9. These plots confirm the results obtained in the previous section, while Fig 10, which shows the parametric shape profile variations for the nominal controller used in conjunction with  $G_a$  for Pass 6 verify the retention of robust stability and performance which is very close to nominal.

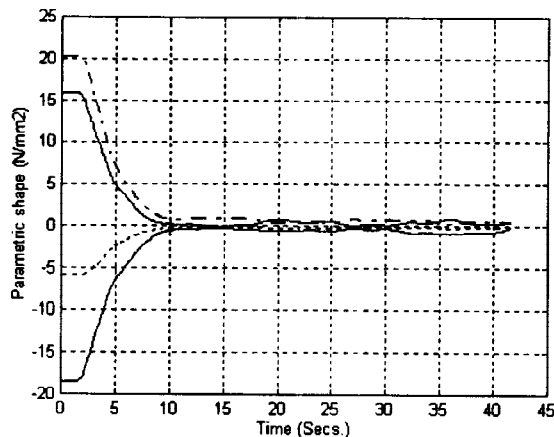


Fig.9. Parametric shape profile variations (nominal)

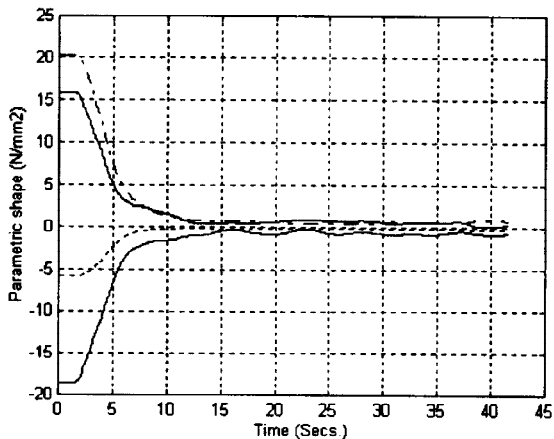


Fig.10. Parametric shape profile variations (Pass 1)

## 5. DISCUSSION OF APPROACH

Several comments regarding the approach taken, together with the outline results achieved, are pertinent. Firstly the  $H_\infty$  control design methodology provides an analytical

methodology for controller determination with a guarantee of robust stability across a given set of plant perturbations. Some user input is required in terms of weight function selection, in order to specify the desired controller properties, but this specification will be consistent across different plants. Such a solution procedure is in contrast to previous approaches, which have tended to involve a considerable amount of user input (e.g. for frequency response shaping) and retention of stability across different passes must be checked separately for each individual case. One of the principal benefits, therefore, in adopting the  $H_\infty$  methodology, is the provision of an automated design philosophy, which can provide a systematic means of developing a set of controllers to cover all passes and schedules.

The dimension reduction, which was employed in Section 3.2, accords with other design approaches, but is clearly motivated by the inability to formulate a design for the full  $8 \times 8$  system which has reasonable sensitivity properties.

Some further benefits of the  $H_\infty$  design, which are not explicitly stated in the specification, are also available. In particular, it is known that the mill matrices produced from the static model of Gunawardene (Gunawardene, 1982) contain modelling inaccuracies. The  $H_\infty$  controller naturally produces some immunity to such errors, by providing good stability margins and insensitivity to d.c. parameter variations. These benefits extend to unmodelled (soft) nonlinear dynamics (Safonov, 1980), which are known to be present in the actuators. Further nonlinear effects may manifest themselves in the real system, since the mill matrices produced by Gunawardene's model are linearised gain matrices.

A final comment concerns controller implementation. Observation of the controller state space description highlights the possible need to employ controller order reduction techniques. This will be the subject of further investigation.

## CONCLUSIONS

$H_\infty$  has been shown to be a sound framework in which the Sendzimir mill shape control problem may be tackled. The synthesis of a controller which actively builds in robustness to known parameter variations is particularly significant when the multi-pass, multi-schedule nature of the mill operation is considered. The design philosophy can also form an important building block upon which to base a systematic solution to the controller scheduling problem.

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## APPENDIX

$G_a$  for Pass 1 of schedule:

$$G_1 = \begin{bmatrix} 5.09092 & 6.34360 & 0.26968 & -2.48203 & -2.80510 & -2.38248 & -1.98539 & -2.02650 \\ 1.00279 & 3.34942 & 3.13050 & 0.25597 & -2.05200 & -2.61747 & -2.22675 & -2.24159 \\ -0.89666 & 0.48619 & 3.37291 & 2.50732 & -0.51331 & -2.36358 & -2.43742 & -2.42733 \\ -1.34573 & -1.48967 & 1.38566 & 3.30285 & 1.94524 & -0.89787 & -2.33347 & -2.34295 \\ -1.17256 & -2.25602 & -0.98889 & 1.75450 & 3.35399 & 1.52936 & -1.48323 & -1.56488 \\ -1.00443 & -2.38581 & -2.32983 & -0.62499 & 2.31819 & 3.36104 & 0.72382 & 0.63545 \\ -0.94654 & -2.29830 & -2.69351 & -2.21713 & -0.16058 & 2.83896 & 3.68777 & 3.78202 \\ -0.85555 & -1.87405 & -2.08204 & -2.37710 & -1.96711 & 0.59551 & 5.93841 & 6.06127 \end{bmatrix}$$

$G_a$  for Pass 6 of schedule:

$$G_6 = \begin{bmatrix} 4.12077 & 4.50166 & -0.19453 & -2.07818 & -2.13894 & -1.88670 & -1.72470 & -1.69745 \\ 0.48345 & 2.34648 & 2.35850 & 0.18784 & -1.37801 & -1.73299 & -1.54856 & -1.52501 \\ -0.72171 & 0.45367 & 2.35850 & 0.18784 & -1.37801 & -1.73299 & -1.54856 & -1.53501 \\ -0.94952 & -0.96559 & 1.07289 & 2.50512 & 1.41015 & -0.63816 & -1.57128 & -1.53163 \\ -0.82447 & -1.52496 & -0.62953 & 1.40829 & 2.51223 & 1.09019 & -1.04530 & -1.00397 \\ -0.71384 & -1.58239 & -1.49821 & -0.26633 & 1.87309 & 2.51739 & 0.38262 & 0.41848 \\ -0.69047 & -1.54612 & -1.73372 & -1.38238 & 0.10433 & 2.20707 & 2.36437 & 2.38232 \\ -0.79992 & -1.75596 & -1.86316 & -2.15344 & -1.87756 & 0.04788 & 4.66098 & 4.45294 \end{bmatrix}$$

$H_\infty$  controller state space description (Jordan form)

$$\text{eig}(A) = \begin{pmatrix} -1.6076 + 2.6923i \\ -1.6076 - 2.6923i \\ -3.5069 \\ -6.4409 \\ -5.8667 \\ -4.8711 \\ -3.0574 + 5.3484i \\ -3.0574 - 5.3484i \\ -2.2788 + 3.9461i \\ -2.2788 - 3.9461i \\ -2.7722 + 4.8383i \\ -2.7722 - 4.8383i \\ -0.0100 \\ -0.0100 \\ -0.0100 \\ -0.0100 \end{pmatrix}$$

$$D = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$B = \begin{pmatrix} 0 & 0.0004 & 0 & -5.1707 + 1.3296i \\ 0 & 0.0004 & 0 & -5.1707 - 1.3296i \\ 0 & 0.0005 & 0.0001 & -8.3054 \\ 5.6945 & 0 & 0 & 0 \\ 0 & 6.0045 & 0 & 0 \\ 0 & 0 & -6.7014 & 0 \\ -1.7091 - 3.0528i & 0 & 0 & 0 \\ -1.7091 + 3.0528i & 0 & 0 & 0 \\ 0 & 0 & 0.9755 - 4.0701i & 0 \\ 0 & 0 & 0.9755 + 4.0701i & 0 \\ 0 & -2.0329 + 3.1005i & 0 & 0 \\ 0 & -2.0329 - 3.1005i & 0 & 0 \\ -0.8599 & 1.7396 & -4.1888 & -3.9928 \\ 5.5923 & -6.5353 & 2.4728 & -2.3806 \\ -5.8928 + 2.3702i & 11.9527 + 0.4073i & 4.4220 - 2.0760i & 0.4375 + 0.7001i \\ -5.8928 - 2.3702i & 11.9527 - 0.4073i & 4.4220 + 2.0760i & 0.4375 - 0.7001i \end{pmatrix}$$

$$C^T = \begin{pmatrix} 0 & 0 & 0 & -0.3703 + 0.6589i \\ 0 & 0 & 0 & -0.3703 - 0.6589i \\ 0 & 0 & 0 & -0.5416 \\ 1.1543 & 0 & 0 & 0 \\ 0 & 1.0485 & 0 & 0 \\ 0 & 0 & -0.8501 & 0 \\ -1.3274 + 0.1567i & 0 & 0 & 0 \\ -1.3274 - 0.1567i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -0.7583 - 0.7282i & 0 \\ 0 & 0 & -0.7583 + 0.7282i & 0 \\ 0 & 0.6538 + 1.0507i & 0 & 0 \\ 0 & 0.6538 - 1.0507i & 0 & 0 \\ -0.0003 & 0.0008 & -0.0041 & -0.0306 \\ 0.0029 & 0.0009 & 0.0060 & -0.0213 \\ 0.0007 + 0.0036i & 0.0015 + 0.0029i & 0.0020 - 0.0028i & -0.0039 + 0.0098i \\ 0.0007 - 0.0036i & 0.0015 - 0.0029i & 0.0020 + 0.0028i & -0.0039 - 0.0098i \end{pmatrix}$$