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Conference Paper *in* Proceedings of SPIE - The International Society for Optical Engineering · May 2008

DOI: 10.1117/12.782786

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A comparison of wavelet analysis techniques in digital holograms

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ABSTRACT

This study explores the effectiveness of wavelet analysis techniques on digital holograms of real-world 3D objects. Stationary and discrete wavelet transform techniques have been applied for noise reduction and compared. Noise is a common problem in image analysis and successful reduction of noise without degradation of content is difficult to achieve. These wavelet transform denoising techniques are contrasted with traditional noise reduction techniques; mean filtering, median filtering, Fourier filtering. The different approaches are compared in terms of speckle reduction, edge preservation and resolution preservation.

1. INTRODUCTION

Digital holography[1] involves the recording of an interference pattern from an object beam and a known reference beam by an imaging sensor. Reconstruction is achieved on a computer by numerical propagation to the original object plane. The hologram encodes 3D information; intensity and directional information of the optical wavefront. Digital holograms can be obtained in a number of ways including in-line and off-axis architectures. In-line capture involves only one capture but requires post processing digitally and full resolution is maintained.[2] Off axis capture also entails only one capture and does not require the same level of post processing, however approximately a quarter of the resolution is maintained.[3] Phase shift interferometry (PSI) is an adaption of in-line holography where four captures are combined digitally and full resolution is maintained.[4]

The digital nature of the digital hologram reconstruction has led to many computational processing techniques[5–9] including methods to suppress speckle noise.[10–13] Speckle occurs when coherent light is diffused by an optically rough surface. Speckle is present for digital holography as coherent light is used in the recording process.[13] Speckle evolves over time and is approximately a signal independent multiplicative noise.[14] Like all noise in images, speckle degrades reconstructed images. Previous compression techniques[15] focused on the compression of digital holograms rather than reconstructed object intensity images. This paper considers the reduction of speckle noise in reconstructed images. Wavelets has been proposed as a method of speckle reduction in many areas of cohererent imaging e.g. OCT.[16]

Wavelet transforms (WTs)[17,18] are a mathematical approach for analysing time or space invariant frequency data. Data is expressed in terms of dilations and transforms. Wavelets are local in frequency, dilations, and in time or space, translations. WTs decompose a signal into sums of basic functions using a basis function called a mother wavelet. These basic functions are calculated by scaling the mother wavelet which is related to frequency and shifting the signal which is delaying the onset. These basic functions are high frequency details and low frequency approximations.

The stationary wavelet transform (SWT)[19] is mainly used to de-noise (1D or 2D) signals.[20] It is a tri-step method: (i) decompose the signal into approximations and details (ii) threshold the individual details and (iii) reconstruct by averaging the details.

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Photon Management III, edited by John T. Sheridan, Frank Wyrowski
Proc. of SPIE Vol. 6994, 699412, (2008) · 0277-786X/08/$18 · doi: 10.1117/12.782786

The SWT imposes the property of time invariance on the wavelet decomposition.[20] This time invariant characteristic makes SWT useful for denoising. Performance of SWT for denoising images has been shown to be superior to the discrete wavelet transform. We note that DWT is a multi-resolution non-redundant decomposition.[20] Time invariance means that a shift in time has no effect on the result. The equivalent of this for images is space invariance which can be achieved via a redundant method.[19] Wavelets can be redundant e.g. SWT or non-redundant e.g. orthogonal or bio-orthogonal wavelets.

For SWT, at each filter the resulting two sequences are the same length as the original. The filters at each level are modified by effectively padding them out with zeros. In DWT, down-sampling allows a tree summing structure and breaks into two halves at each level, down until the sequence is of unit length. Reconstruction of the original signal is achieved by summing the approximation at a particular level, Ai, with the detail coefficients at that level and all lower levels, $signal = Ai + \sum_{j=1}^{i} D_j$ for 1D signals. However for SWT up-sampling is used before filtering at each scale, so redundancy is introduced which helps identify unusual features like noise. For the 2D case i.e. images, the difference is that the detail at each level is given as and three sub-images: horizontal, vertical and diagonal.[20] So the low resolution components i.e. approximations are unaltered and only the high resolution components i.e. details are thresholded.

Compression is necessary for space intensive digital holograms. The discrete wavelet transform (DWT)[21] is effective for compression of digital holograms.[22] Fresnelets[23] are a wavelet based technique that has been applied to complex data like digital holography.[24,25] Using SWT for denoising introduces redundancy and the original image size is maintained. DWT is advantageous for compression as the sequence length is reduced.

This paper is broken down as follows. In Section 2 we describe the techniques applied, and their required parameters, to reduce speckle noise in reconstructions of digital holograms. The metrics that are used to quantify the performance of these methods are also outlined. In Section 3 we present some of our experimental results which best demonstrate the capabilities of the denoising techniques applied. In Section 4 we explain these findings and Section 5 concludes this paper.

2. ALGORITHMS AND METRICS

SWT denoising involves decomposition, level dependent thresholding and reconstruction by inverse transforming. For level dependent thresholding the threshold value, $\tau_{i,j}$, is calculated in each orientation i (1=horizontal, 2=vertical, 3=diagonal) at each level j ($j = 1 - n$ where n = number of levels). Wavelet coefficients are then thresholded using $\tau_{i,j}$ to remove noise energy. The appropriate $3 \times n$ matrix τ can be obtained automatically by one of three potential methods:

1. Balance Sparisty Normal (bal_sn): $\tau_{1i,j}$ is calculated so that the ratio of remaining signal energy to original signal energy (remaining signal energy + noise energy) is equal to the ratio of the number of zero coefficients after thresholding and the total number of coefficients.
2. Square Root of Balance Sparisty Normal (sqrtbal_sn): $\tau_{2i,j}$ is the square of the above $\tau_{1i,j}$
3. Fixed: $\tau_{3i,j}$ is $\sigma\sqrt{2 \times log(ij)}$ where i, j is the size of $X_{i,j}$ and σ is the standard deviation of estimated Gaussian noise.

Speckle noise is inherent in reconstructions of digital holograms recorded with coherent light. Speckle noise in coherent imaging may be apprximated as a multiplicative noise.[14] In this way the logarithm of the intensity image may be used to convert speckle to an additive term that is separable from the wanted image.

Speckle contrast is used to determine the relative level of speckle noise present in an image. It is the ratio of standard deviation to the mean in an homogenous area. The area chosen is assumed to be a satisfactory size, and it is difficult to identify such an area automatically. This approach also assumes that the speckle is invariant across the image. The value of speckle contrast ranges from $0 - 1$. A speckle contrast value of one indicates the presence of fully developed speckle, and a value of zero indicates an absence of speckle noise.

The resolution metric entails the use of two resolution strip patterns created in matlab. These resolution charts are embedded into a reconstructed digital hologram in a horizontal and a vertical orientation. Since coherent speckle is approximately additive in the natural logarithm, the patterns are added in the natural log of the reconstruction amplitude and then exponentiated. Despite the embedding process, the strips have fundamental frequencies at the same frequencies of the original sine waves before embedding. These strips have coherent speckle noise added to a good approximation.

To test for the presence of a particular frequency each particular substrip, of which there are ten, is isolated. Ten 1D vectors are created by integrating along the 10 pixel width. Fast Fourier transform (FFT) operations are performed on the ten vectors. A peak in the region of fundamental frequencies indicates the survival of resolution in the corresponding level. If this peak is strong enough i.e. has a power greater than some predefined threshold, then the signal contains information at that resolution. The appropriate threshold applied corresponds to our own ability to see the resolution chart with our eye and identify the presence of the strip. A resolution chart is shown in Fig. 1 (a), and a reconstruction with the resolution charts in horizontal and vertical orientations, and edge chart embedded is shown in Fig. 1 (c).

It should be noted that when searching for the presence of frequencies, the lowest through to the highest frequencies are detected in order. As soon as a frequency is found to be missing, the search terminates. The last resolution band that is maintained in the embedded chart, before the first band is lost, is accepted as the level of resolution for that orientation (horizontal or vertical).

To quantify edge preservation, correlation can be utilised.[26] We apply this technique by embedding a test pattern, see Fig. 1 (b), in the background of the image with the same embedding process as above. Edge correlation is the correlation of the section of the image where the test pattern is embedded (X) with the original test pattern (Y) and is computed as

$$EC_{X,Y} = \frac{\text{cov}(X,Y)}{\sigma_X \sigma_Y}$$

where cov is covariance and σ_X and σ_Y are standard deviations of the embedded test pattern image and the original test pattern. Squared correlation describes the proportion of variance in common between the two variables and is a percent of variance in common between the two variables.[27] We note that this correlation value is usually quite low even for a close match.

The ground truth data is designed to contain features representative of potential features in the image; i.e. it is a binary image containing curves, straight lines in various orientations etc.

An overall metric (OM) is defined as

$$\text{OM} = (1 - \text{SC}) \times \text{EC}^2 \times R$$

where SC = speckle contrast, EC = edge correlation, R = $\frac{R_h + R_v}{2}$, R_h = horizontal resolution and R_v = vertical resolution. Each of the above metrics are equally weighted and OM is a value in the interval $[0 - 1]$.

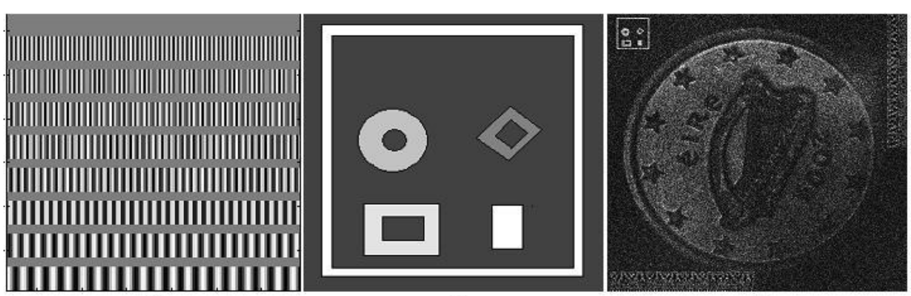

Figure 1. (a) Resolution chart (b) Edge Pattern (c) Intensity image of reconstructed coin hologram with embedded charts

There are two popular versions of thresholding (i) Hard thresholding[28] which is defined by

$$HT_\tau(X_{i,j}) = \begin{cases} 1 & \text{for } X_{i,j} \geq \tau \\ 0 & \text{for } X_{i,j} < \tau \end{cases} \tag{1}$$

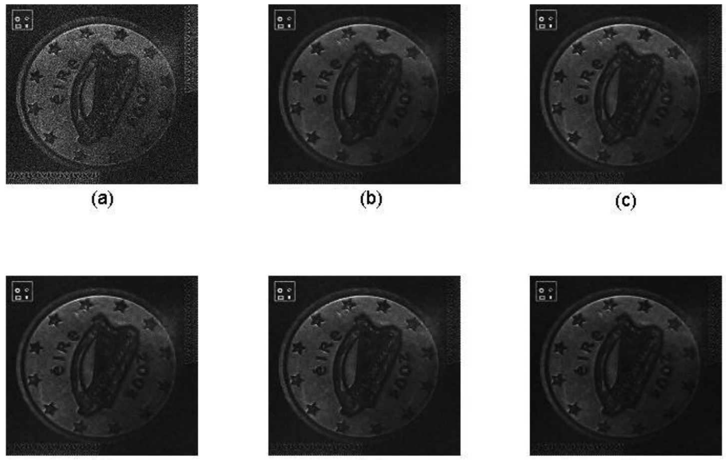

Figure 2. Application of SWT soft thresholding to coin hologram reconstruction using balance sparsity normal method of threshold estimation at decomposition level 5 using various mother wavelets (a) original (b) Haar (c) Db2 (d) Coif5 (e) Bior1.3 (f) Rbio1.3

and (ii) Soft thresholding[29] which is calculated by

$$ST_\tau(X_{i,j}) = \begin{cases} X_{i,j} - \tau & \text{for } X_{i,j} \geq \tau \\ 0 & \text{for } X_{i,j} < \tau \\ X_{i,j} + \tau & \text{for } X_{i,j} < -\tau \end{cases} \tag{2}$$

where τ is some threshold and X is an image indexed by i, j.

3. RESULTS

We recorded holograms using the recording approaches described previously for this experiment.

The best method for denoising was found to be soft thresholding. This was applied using level dependent thresholds computed using the bal_sn method. The Haar mother wavelet achieved the highest overall result and increasing levels (1-5) led to an improved speckle contrast. The best results from the three methods of threshold calculation are compared in the table below. Obviously the optimal result would be at speckle contrast = 0, resolution = 1 (full) and edge correlation = 1.

Soft threshold at level 5 with Haar				
Threshold	**SC**	**R**	**EC**	**OM**
Bal_sn	0.2374	1	0.0626	0.0478
Sqrtbal_sn	0.37373	1	0.0509	0.0316
Fixed	0.072264	0.2	0.1432	0.0266

Using wavelets to denoise speckled images is very effective. The best of the achieved soft thresholded results are shown in Fig. 2 for the chosen mother wavelet families at decomposition level 5.

Conventional wavelet based image denoising uses DWT which has been successfully applied to image compression.[22] The results in Fig. 3 show the overall metrics for each of the best performing mother wavelets for SWT and DWT. Both methods achieve full resolution, but DWT fails to reduce the speckle contrast to the same extent as SWT and edges are extremely degraded by DWT when compared to SWT. The major influence on the difference between DWT and SWT is the edge metric.

Traditional methods of denoising include filtering or smoothing the image. Some examples of this are mean, median and Fourier filtering. The best of these results are compared with the best results for wavelet denoising,

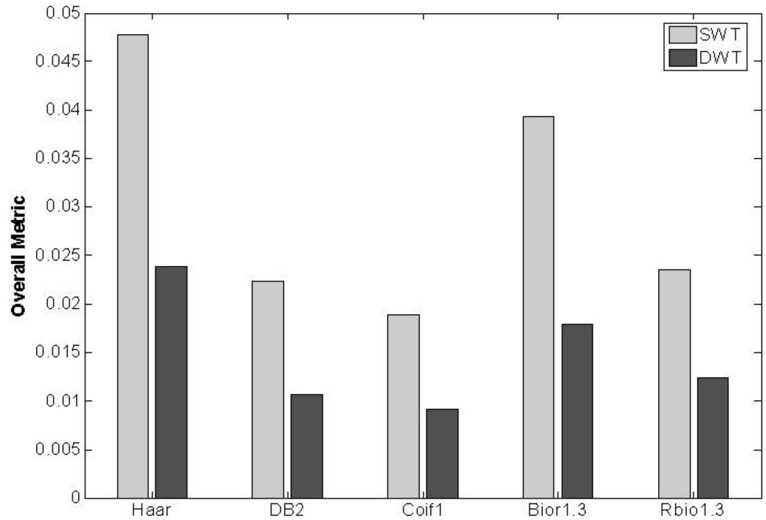

Figure 3. SWT compared to DWT using bal_sn soft thresholding at level 5

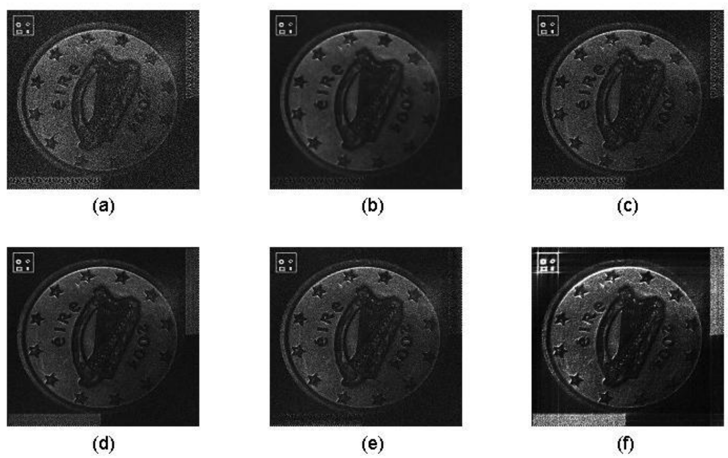

Figure 4. Application of various speckle reduction methods to reconstruction of coin hologram (a) original (b) soft threshold with balance sparsity normal at level 5 with Haar mother wavelet (c) hard threshold with balance sparsity normal at level 2 with Haar mother wavelet (d) mean filtering with a 3×3 neighbourhood (e) median filtering 3×3 neighbourhood (f) Fourier filtering with 512×512 neighbourhood

with both hard and soft thresholding in Fig. 4.

The test image used previously is an example of a hologram captured using the in-line method. We also tested the SWT method of denoising on a PSI and an off-axis hologram. The results shown in Fig. 5 demonstrate successful speckle reduction. The method performs well for PSI captured holograms, but the much lower resolution off-axis reconstructions suffers smoothing as a consequence.

4. DISCUSSION

The results are shown for the best mother wavelet from each family. These are computed at levels 1-5, for each threshold type using soft and hard thresholding. The wavelet families are: Haar (= db1 = bior1.1 = rbio1.1), Daubechies (db2-10), Symlets (sym2-8), Coiflet (coif1-5), Biorthoganal (bior1.3-6.8), Reverse biorthogonal (rbio1.3-6.8). The best choice of mother wavelets, from these 51 wavelets, are almost consistently (Haar = db1 = bior1.1 = rbio1.1 excluded): db2, sym2, coif1, bior1.3, rbio1.3. Of these db2 = sym2, and Haar = db1.

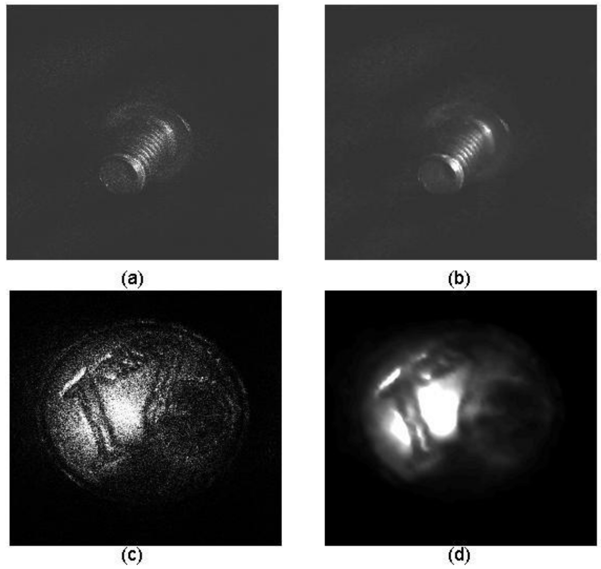

Figure 5. Application of SWT soft thresholding with the Haar mother wavelet at level 5 decomposition (a) original PSI (b)best denoised PSI (c) original off-axis (d) best denoised off-axis

These wavelet transforms are rendered the same as

$$Haar(\phi, \psi) = bior1.1(\phi\_dec, \psi\_dec) = bior1.1(\phi\_rec, \psi\_rec) = \\ rbio1.1(\phi\_dec, \psi\_dec) = rbio1.1(\phi\_rec, \psi\_rec)$$

where ϕ = scale and ψ = translation. Results for mean filtering, median filtering and Fourier filtering denoising methods are also observed and compared.

DWT and SWT achieve full resolution in the higher levels of soft thresholding. The difference between these methods is apparent in the difference in speckle contrast and edge metrics where the SWT approach consistently outperforms the DWT. Therefore we consider SWT to be a superior wavelet transform for speckle reduction, though DWT is better suited for compression.[22] We discuss the SWT below.

We found soft thresholding to outperform hard thresholding in all cases. The best soft thresholding overall metric is 0.0478 (threshold type = bal_sn with Haar wavelet at level 5) compared to 0.009 for the best overall metric obtained by hard thresholding (threshold type = bal_sn with Haar wavelet at level 2). For the soft thresholding results, as the levels increase so do the overall metrics for all of the mother wavelets. This is not the always the case with the hard thresholding results obtained.

The best result for hard thresholding is obtained at decomposition level 2. Thresholds are computed level dependently so there is an opportunity here for lower levels to outperform higher levels of decomposition, depending where and in what level of detail most of the speckle noise resides. Consider the case where a threshold x_{11} is applied to the horizontal detail of a level 1 decomposition. For level 2 decomposition, thresholds y_{11} and y_{12} are applied to horizontal details at level 1 and 2 respectively. $x_{11} > y_{11}$and $x_{11} > y_{12}$. If speckle resides in the intermittent band of frequency, in particular for higher levels, then it is kept rather than lost by 'keep or kill' hard thresholding.[30] Equally there is potential for false positive removal in this instance i.e. detail loss. In soft thresholding, those intermittent detail values are reduced or removed entirely.

For soft thresholding an increase in levels results in a decreased amount of speckle noise and an increase in the overall metric for all wavelets. As expected the edges are best preserved by the step edge wavelet - Haar. Best edge preservation is achieved by all thresholding methods at decomposition levels 2 or 3, and using higher levels only causes a minor degradation of the edges. The fixed form threshold behaves as had been originally anticipated, resolution is degraded as speckle is reduced. However this is not the case for the other thresholds.

Sqrtbal_sn holds full resolution for all levels. Bal_sn gets full reslution from level 2 and is the best thresholding method (achieves highest overall metric).

For hard thresholding the best over all metric is once again achieved by the bal_sn method, this time at decomposition level 2, and again with the Haar wavelet. For fixed thresholding resolution is lost, as speckle is reduced and as the number of levels increase. Full resolution is maintained using thresholds obtained by the other two methods, which decrease in speckle reduction effectiveness for an increase in decomposition levels. Edge preservation is not dramatically effected by the number of levels of decomposition.

The resolution is degraded for mean, median and Fourier filtering results as the speckle increases. This is due to the manner of convolution in which these techniques are applied. Fourier filtering appears to be superior to the other methods for noise reduction with the best overall metric = 0.0029 (neighbourhood size = 256). However resolution is reduced to 0.2 in this case. Resolution is seriously degraded in all of these methods, except for Fourier filtering with mask size = 1024, yet speckle reduction is poor in this case. Fourier filtering is a superior technique to mean and median filtering when considering edge preservation. However, speckle reduction, squared edge correlation and resolution are individually, and therefore combined, better for the bal_sn soft thresholding case at level 5 with a Haar mother wavelet.

5. CONCLUSION

Wavelet transforms, DWT and SWT, have been applied to reconstructions of holograms. Three different threshold estimation techniques were tested for both hard and soft thresholding approaches at five different levels of decomposition. These results were compared to those for traditional speckle reduction techniques; mean filtering, median filtering and Fourier filtering. The results were compared in terms of speckle contrast, edge preservation and resolution preservation. Wavelets have been shown to outperform the traditional techniques. In particular the SWT is shown to achieve the beat results. The best SWT results attained used a balance sparsity normal threshold applied by soft thresholding using the Haar mother wavelet at the highest decomposition level tested which was level 5.

6. ACKNOWLEDGEMENTS

We would like to thank Conor McElhinney for recording some of the holograms used in this paper. Gratitude is also extended to the Irish Research Council for Science, Engineering and Technology, and Science Foundation Ireland, under the National Development Plan for funding this research.

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