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Network Analysis of the Viking Age in Ireland as portrayed in *Cogadh Gaedhel re Gallaibh*

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Abstract

Cogadh Gaedhel re Gallaibh (“The War of the Gaedhil with the Gaill”) is a medieval Irish text, telling how an army under the leadership of Brian Boru challenged Viking invaders and their allies in Ireland, culminating with the Battle of Clontarf in 1014. Brian’s victory is widely remembered for breaking Viking power in Ireland, although much modern scholarship disputes traditional perceptions. Instead of an international conflict between Irish and Viking, interpretations based on revisionist scholarship consider it a domestic feud or civil war. Counter-revisionists challenge this view and a lively debate continues. Here we introduce quantitative measures to the discussions. We present statistical analyses of network data embedded in the text to position its sets of interactions on a spectrum from the domestic to the international. This delivers a picture that lies between antipodal traditional and revisionist extremes; hostilities recorded in the text are mostly between Irish and Viking - but internal conflict forms a significant proportion of the negative interactions too. Additionally, we quantitatively compare the network properties of *Cogadh Gaedhel re Gallaibh* to those of other epic-type narratives and find that, in many ways, they resemble those of the Iliad.

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1 Introduction

The year 2014 marked the 1000th anniversary of the Battle of Clontarf, an iconic event in the history of Ireland. The battle itself is traditionally remembered as marking the decline of Viking power after some two centuries in the country. The anniversary inspired academics to revisit the period through new journal papers, books, booklets, monographs, online commentaries and media engagements (e.g., Refs. [1–12]). As with earlier investigations, these approaches treat the subject matter using traditional tools of the humanities (e.g., Refs. [13–42]). For at least 70 years a debate has been taking place in that community, between what may be called “traditionalist” and “revisionist” interpretations, an element of which concerns the relationships between the conflicting parties [5–7, 11, 19, 22]. (Actually one could trace the discussion back 250 years [5, 43, 44].) Here we present an alternative, complexity science-based investigation, using one of the most famous accounts of the Vikings in Ireland: *Cogadh Gaedhel re Gallaibh*¹ (“The war of the Gaedhil with the Gaill” or “War of the Irish with the Foreigners”), which dates from the late 11th or early 12 centuries.

The text starts with the arrival of the Vikings² in Ireland (in 795) and gives a chronicle of their various raids. This is followed by a discussion of the Irish *Dál gCais* dynasty, their deeds, and those of their leader, Brian Boru, culminating in the Battle of Clontarf in 1014. Although its limitations are well documented, the text provides extensive information; it tells of multitudes of characters, alliances, conflicts, relationships and interactions of all sorts, from a perspective from the time it was written. Statistical tools to tackle the networks formed by such large casts of characters have recently been developed [45–47]. Here we apply them in a new investigation to shed quantitative light on the Viking age in Ireland as presented in *Cogadh Gaedhel re Gallaibh*.

Network science is a broad academic field, related to statistical physics, information visualization, mathematical sociology and other disciplines [48–51]. It enables statistical treatment of certain types of systems comprising large numbers of interdependent elements. In character networks, these elements are individual figures (personages), represented by nodes (or vertices), and the interactions or relationships between them are represented by edges (or links). Empirical approaches seek to capture statistics which characterise such systems [51]. Besides delivering new quantitative insights when applied to old problems, the networks approach inspires new questions and opens new avenues of

¹This is sometimes written *Cogad Gáedel re Gallaib* because that is the form in the earliest extant manuscript, the Book of Leinster (see Section 2) and other variants exist in the literature. We employ the spelling used by Todd [13] since his is the edition that we analyse. We sometimes refer to it simply as “the *Cogadh*” hereafter.

²There are a number of etymological theories for the word “Viking”. We use it to refer to the medieval Norse or Scandinavian raiders and invaders who attacked Ireland (and other countries) by sea, or those who subsequently settled in Ireland, between the late 8th and 11th centuries [40]. A stricter definition of the term “Viking” may involve the notion of “piratical” and in this sense, not all Vikings were Scandinavian and not all Scandinavians were Vikings [40]. But we use the term in the looser sense (in keeping with much of the literature, e.g., Refs. [24, 27, 30, 31, 33, 35, 37, 38]). These are the *Gaill* (singular *Gall*) referred to above. The Norse-Gaels or Hiberno-Norse (Gall-Gaidil or “foreigner Gaels”) emerged as a mixture of Gaelic and Norse culture through cross-cultural connections including intermarriage.

research. Also, by comparing statistics from various networks to each other, one can gain insight into similarities and differences between various systems.

The events associated with the Viking Age in Ireland and Battle of Clontarf are nowadays frequently considered as having entered the public imagination in an overly simplified manner. That popular picture is essentially of an “international” conflict — Irish versus Viking — in which victory for the former ended the latter’s ambitions in the country.³ The truth, we are told, is more nuanced and more complex [19, 22]. Instead of an international conflict, the issue at stake at Clontarf was an internal, domestic, Irish struggle for sovereignty: the determination of Leinster (in the east of Ireland) to remain independent of the dominant dynasties to its north in Ulster and south-west in Munster [19, 22]. Some such interpretations, wherein the Vikings are said to have played a secondary role, tend to downplay the significance of Clontarf [11] and have been partly ascribed to revisionist fashions [5, 33]. *Cogadh Gaedhel re Gallaibh* has been used to bolster arguments on both sides of the debate. Our aim is to determine what its character networks have to say on the matter.

It is important to emphasise from the outset that our analysis is of the content of *Cogadh Gaedhel re Gallaibh* and its portrayal of the Viking Age in Ireland. We do not have direct access to the actual social networks of the period and we recognise that the account in the *Cogadh* has been influenced by events and circumstances after 1014 and up to the composition of the text. We discuss the authenticity and deficiencies of the *Cogadh* as a source in Section 3. Nevertheless, the text is important in its own right and, at minimum, tells us how the author sought to represent reality.

The style of the text of the *Cogadh* is “inflated and bombastic” [13]. It is considered by modern scholars “as a piece of dynastic political propaganda on behalf of the principal lineage of the Dál Cais, the Uí Briain”⁴ [24]. (See Appendix A and Figure A.1 for a brief account of the political structure of Ireland in 1014.) This is achieved through extensive and elaborate passages extolling the virtues of Brian and his army while condemning the Vikings as brutal and piratical. However such qualitative, rhetorical features are largely irrelevant for quantitative character-network analysis. Instead, our approach draws only from the most basic information — the presence or absence of interactions between characters which may be positive or negative in a sense to be described in Subsection 5.1 below. The depictions of the characters themselves — as virtuous or piratical, for example — do not enter our analysis. Still, even such basic information is imperfect; e.g., the versions of the *Cogadh* which have come down to us contain imagined interactions in the form of interpolations (discussed in Section 3 and Subsection 5.2.4 below). If, despite this, the text contains networks which are reasonably or approximately reliable in the aggregate, they deliver useful information on the society of the time it presents.

Figure 1.1 contains a network visualisation of the full set of interactions recorded in the *Cogadh*. In it, and throughout this paper, nodes represent individual characters

³We are aware that terms related to the word “national” may be viewed as anachronistic here [7]; we use them in the sense of a large group of people with common characteristics such as language, traditions, customs and ethnicity [30] rather than in a governmental sense [52].

⁴“Uí” means “grandchildren” or descendants so that the Uí Briain are the descendants of Brian and the Uí Néill are descendants of Niall, etc. “Ua” is the singular form.

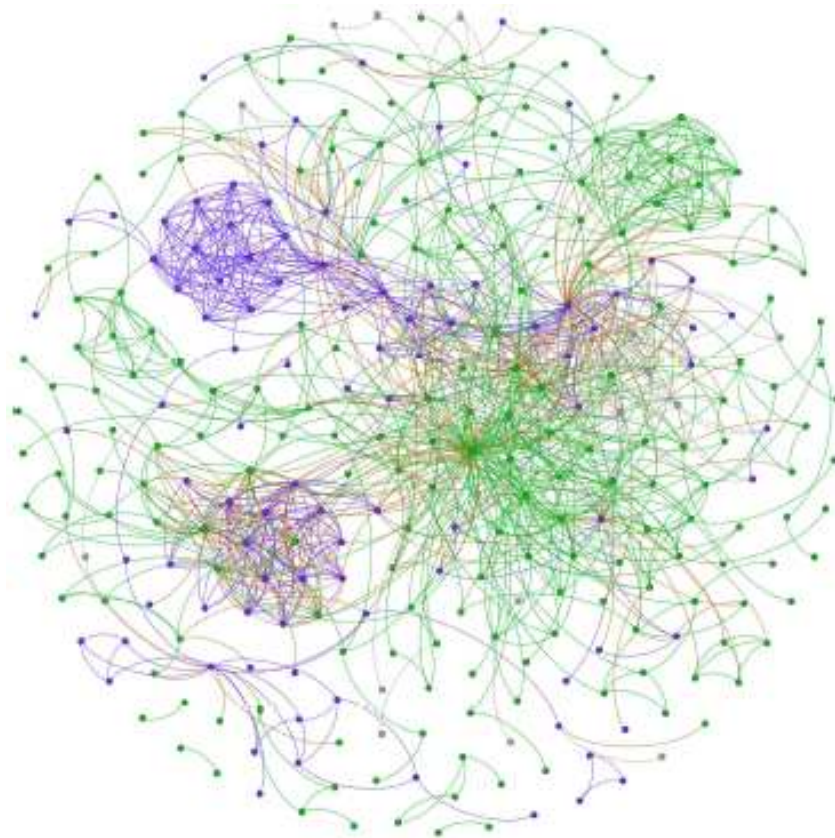


Figure 1.1: The entire *Cogadh* network of interacting characters. Characters identified as Irish are represented by green nodes and those identified as Vikings are in blue. Other characters are in grey. Edges between pairs of Irish nodes are also coloured green while those between Viking pairs are blue. Edges linking Irish to Viking nodes are brown and the remaining edges are grey.

which appear in the text and edges or curved line segments represent relationships or interactions between these characters. Green nodes represent Irish characters and green edges represent interactions between them. The counterpart set of Viking nodes and their interlinks are in blue. Brown edges represent interactions between Irish and Viking nodes. Any remaining nodes and edges are in grey. (Precisely how we assign the Irish/Viking/other identity tags and the interactions is given in Subsection 5.1.) The purpose of this paper is to analyse the network represented in Figure 1.1 and sub-networks associated with it (Figures 5.2 and 5.3, below) in order to gain quantitative insight into the complexity and conflicts of the Viking Age in Ireland as described in *Cogadh Gaedhel re Gallaibh*.

We are especially interested in the question whether (a) the networks underlying the *Cogadh* are consistent with the traditional depiction of a contest which is clear-cut international or (b) if they support the revisionist notion of a power-struggle which is

mostly domestic or (c) if they deliver something between both pictures. In other words, we aim to determine whether the conflictual edges in Figure 1.1 are predominantly blue and green or brown. We are also interested in the reciprocal question whether positive interactions are mostly between nodes of the same or different types. A simple tally of edges will not do as this would not account for different numbers of green and blue nodes, and a proper quantitative approach instead necessitates the networks-science concepts of *assortativity* and *disassortativity*. The former is the tendency for edges to connect nodes which have similar attributes. The opposite tendency is disassortativity; in a disassortative network, links tend to be between nodes of different types. The type of attribute we are interested in here is *narrative identity*⁵ — categorised as Irish, Viking or other, and taken from the text itself. We wish to gauge whether nodes linked by different types of edges represent Irish or Viking characters as presented in the narrative. We use the generic term *categorical assortativity* to denote associated measures which we denote generically by ρ . One such quantity will be used as the primary determinant to distinguish between the alternatives (a), (b) and (c) listed. A network with a positive value of ρ is said to be *categorically assortative*. A negative value of the metric signals disassortativity. A value close to zero indicated the absence of any such correlations (neither assortative nor disassortative).

The relationship attributes we wish to investigate in this context are positive (essentially social) and negative (physically hostile) interactions between characters of the text. The second of these is particularly interesting for the above-mentioned debate as it is there that the conflict resides. Taking the “international conflict” scenario (a) to an extreme, an overly simplified (traditional) picture might be expected to contain networks where hostility is “clear-cut” (to use the term employed by Ryan in Ref. [19]) between nodes of different identities. The categorical assortativity metric ρ for such a conflictual network would be expected to be strongly negative. A revisionist picture of domestic or intranational conflict (b), on the other hand, where hostility is primarily between sides of the same identity tags, would have a positive value of ρ .⁶ A moderate value of categorical assortativity would then be indicative of a more complex, mixed picture (c) of conflict in Ireland during the Viking Age.

We will report that the categorical assortativity for the conflictual network is (moderately) negative. We interpret this as signalling that the depiction of hostility in the narrative is primarily international. However, the power of our analysis is that we can quantify this statement. We will see that the spectrum of potential values of the categorical assortativity for networks of the conflictual type contained in *Cogadh Gaedhel*

⁵The term is motivated by a discussion in Ref. [30] of “the strong sense of identity, achievement, and cultural cohesion that had been created by the Irish learned classes.” Ó Corráin states “The island was united culturally and linguistically” and “Self-consciously, the literati saw the Irish as a people or *natio*, to be compared with the Goths, the Franks, or the peoples of classical antiquity. As far as the genealogists were concerned, the Vikings were outsiders, and were called *Gaill* ‘Foreigners’ to the end. Irish reaction to the Vikings is to be understood in terms of these cultural traits.” For further discussions of Hiberno-Scandinavian relations, see Ref. [31].

⁶We loosely use the term “intranational conflict” to describe Irish-on-Irish and Viking-on-Viking hostility although there are distinct sets of Vikings which frequently fight with each other. We deal with that issue in Subsection 5.3.

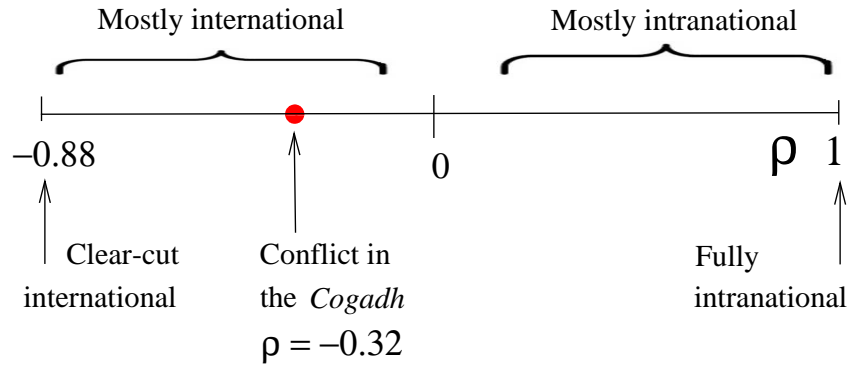


Figure 1.2: Graphical representation of the main conclusion of this paper. The spectrum of values of categorical assortativity for networks of the conflictual-*Cogadh* type ranges from $\rho = -0.88$ to $\rho = 1$. Negative values of ρ correspond to various degrees of the traditional picture of international hostilities with $\rho = -0.88$ representing a clear-cut Irish-versus-Viking conflict. Positive values correlate with the revisionist picture of mostly intranational conflict. The analysis presented in this paper shows that the *Cogadh* hostile network delivers a value -0.32 which, although not clear-cut, lies on the traditional side of the spectrum.

re Gallaibh ranges from $\rho = -0.88$ to $\rho = 1$. This spectrum is represented graphically in Figure 1.2. The “clear-cut” traditional picture corresponds to the left-hand limit. The revisionist picture is associated with the right-hand side. Within that spectrum, we find a value for the actual *Cogadh* network of $\rho = -0.32$. We interpret the moderateness of this negative value as indicative of significant intranational conflict too. These statistics suggest that the *Cogadh* account is not as clear cut as either the traditional or revisionist pictures in the debate referred to above, but instead lies between their two extremes, albeit on the traditional side. Thus the networks of *Cogadh Gaedhel re Gallaibh* give a complex picture of the Viking Age in Ireland comprising predominantly international conflict but with strong degrees of intranational hostilities too.

The principal aims of what follows, then, are (i) to present visualisations for the positive (social) and negative (conflictual) character networks, (ii) to use the notion of categorical assortativity tailored to estimate where a network of interactions is positioned on the spectrum from the international to the intranational and (iii) to apply that tool to the networks recorded in *Cogadh Gaedhel re Gallaibh*. We also (iv) quantify other statistical properties of the *Cogadh* networks to enable comparison with other epic narratives.

2 Context and text: The war of the Gaedhil with the Gaill

Cogadh Gaedhel re Gallaibh comes down to us in three manuscripts. The oldest is in the

twelfth-century *Book of Leinster* (so named because of the associations of its content with that province of Ireland), which contains part of the tale. The second (also incomplete) is the *Dublin Manuscript* (so called because the copy is preserved in Trinity College Dublin), dated to the 14th century. The third, and only complete text is the *Brussels Manuscript*. This was saved by the famous Franciscan friar Mícheál Ó Cléirigh who in the 17th century was sent from Louvain in Belgium to Ireland to collect and preserve Ireland’s ancient heritage. It is a copy of an earlier work, *Leabhar Chonn Chonnacht Ui Dhálaigh* (Book of Cuconnacht O’Daly who died in 1139 [13]) which is now lost. The Brussels and Dublin manuscripts are close but not identical. Ní Mhaonaigh gives a detailed textual history of *Cogadh Gaedhel re Gallaibh* in Refs. [26,27].

As a proxy for the originals, we use the nineteenth-century translation into English by James Henthorn Todd [13]. Todd’s edition, which is 150 years old in 2017, is accompanied by an extensive introduction and by detailed explanatory footnotes. It was “edited from the three manuscripts” [13], prepared for the series *Rerum Britannicarum Medii Aevi Scriptores* (“Chronicles and Memorials of Great Britain and Ireland during the Middle Ages”) and is considered well balanced and thorough. It serves as a source for some scholars wishing to access the narrative today [35].

Todd considered *Cogadh Gaedhel re Gallaibh* as divisible into two parts. The first recounts the arrival and deeds of the Vikings in Ireland in a rough chronological fashion. The second part concerns Brian Boru and his Munster dynasty whose powerbase was on the banks of the river Shannon. The lives and politics of his family are outlined along with numerous encounters with the Vikings, all leading to the events at Clontarf.

Brian Boru or Brian Bóruma⁷ mac Cennétig was son of Cennétig mac Lorcaín, king of the Dalcassians (Dál gCais) in the northern part of the province of Munster (a map of Ireland during the Viking Age is provided in Appendix A). On the death of his father, Brian’s older brother Mathgamain assumed control. After capturing Cashel, the traditional capital of Munster, he claimed authority over the entire province. On his death, Mathgamain was replaced by Brian who, like his brother, proved to be an excellent military commander. After various battles at the provincial level, Brian and the Dál gCais consolidated rule of Munster, defeating their Irish and Norse challengers. Brian then turned his attention to the easterly province of Leinster and the westerly province of Connacht.

Máel Sechnaill mac Domnaill was king of Meath (Mide) and most powerful king in Ireland. He belonged to the Clann Cholmáin sept of the Southern Uí Néill. Brian’s interests in Leinster brought him into contest with Máel Sechnaill who led repeated forays there, as well as into Munster. However, in 997, Brian and Máel Sechnaill agreed a truce, whereby the former would rule over the (approximate) southern half of Ireland, while Máel Sechnaill kept the (approximate) northern half. By these means, Brian came to control Munster, the area immediately north of Dál gCais territory in southern Connacht, and Leinster as well as the Hiberno-Norse cities within, while Máel Sechnaill held the provinces of Meath and Ulster and part of Connacht.

In 998, Brian and Máel Sechnaill worked together against the Dublin Norse. The

⁷The epithet “Bóruma” in Old Irish (“Borumha” in Todd [13]) is an alternative to “Boru” and may signify “of the cattle tributes” [16].

Vikings had established a settlement in Dublin in 838 and during the following century they developed a kingdom comprising large areas surrounding the town and controlling parts of the Irish Sea. Viking Dublin was politically linked to the Isle of Man and the Hebrides, as well as to Viking settlements in Britain and Scandinavia. Indeed, in 980, an unsuccessful Dublin campaign against Máel Sechnaill mac Domnaill had support from the Isle of Man and the Hebrides.

Dublin was joined by Leinster under a new king, Máel Morda mac Murchada, in opposing Brian and Máel Sechnaill. Leinster traditionally rejected the rule of both Munster and Meath and the Hiberno-Norse city of Dublin was ruled by Máel Morda's cousin, Sigtrygg Silkbeard. The two sides met at Glenmama in late December 999. The Irish annals agree that the combined forces of Munster and Meath decisively defeated those of Leinster and Dublin.

Sigtrygg, however, was allowed to retain his rule over Dublin. Indeed, one of Brian's daughters married him while Brian himself married Sigtrygg's mother Gormflaith (Máel Morda's sister). Gormflaith was also the former wife of Máel Sechnaill.

The river Shannon presented a barrier to Meath receiving support from his ally Cathal mac Conchobar mac Taidg, king of Connacht, when Máel Sechnaill came under attack by Brian in the year 1000. By 1002, Máel Sechnaill had submitted to Brian at Athlone [22]. The next target for Brian was Ulster. It took ten years, a combination of forces and coordinated use of sea and land attacks, and support from the Church in Armagh for the Northern Uí Néill and regional kings of Ulster to submit to Brian. By 1011, Brian had achieved his aim of bringing all the regional rulers of Ireland under his control.

In 1012, Máel Mórda mac Murchada of Leinster rose in rebellion. Allied with Flaithbertach Ua Néill, regional king of Ailech in Ulster, he again attacked Meath. Máel Sechnaill sought Brian's help and the following year Brian and his son led a combined force from Munster and Connacht into Leinster, reaching Dublin in September. Out of supplies near the end of the year, they abandoned their siege of the walled city, with an intention to return.

Thus was the background to the famous Battle of Clontarf. In 1014, Máel Morda's cousin, Sigtrygg, journeyed to Orkney and the Isle of Man seeking Viking support. These Norsemen came under Sigurd Hlodvirsson (Earl of Orkney, known as Sigurd the Stout) and Brodir, reputedly of the Isle of Man. Brian's forces came from Munster and southern Connacht possibly supported, at least initially, by Máel Sechnaill mac Domnaill's Meathmen (the precise role of Meath in the Battle itself is a matter of some contention [5, 19, 33]). The Battle of Clontarf is believed to have taken place on Good Friday, 23 April 1014 [13] (see, however, Ref. [7, 53]). According to the *Cogadh*, after a day's fighting, the battle ended with the routing of the Viking and Leinster armies. The account tells us that their retreat was cut off by the high tide. Many of the nobles died. Brodir killed Brian, having found the old man in his tent. *Njáls Saga* informs us that Brodir in turn was killed by Úlf Hreða (possibly Cuduiligh in the *Cogadh* [54], meaning Wolf the Quarrelsome), a relative of Brian Boru. Sigurd the Stout of Orkney was also killed, as was the Leinster king Máel Morda mac Murchada. Sigtrygg Silkbeard survived and remained king of Dublin, and the king of Meath, Máel Sechnaill mac Domnaill,

resumed his claim to high kingship of Ireland,⁸ supported by Flaithbertach Ua Néill.

3 Authenticity and deficiencies of *Cogadh Gaedhel re Gallaibh*

It is nowadays widely accepted that one of the main aims of *Cogadh Gaedhel re Gallaibh* was to document the achievements of the Dál gCais and eulogise Brian Boru “... to create an illustrious past for his dynasty and to underline thereby later Uí Brian claims to political power” [26]. Although it is a valuable resource for studies of the Viking Age in Ireland, it is considered a biased one. The question of its reliability has been the topic of a very long-standing debate [5, 13, 19, 22, 27]. Besides some clear interpolation, much of its bias appears in the descriptive detail of the narrative. Ours, however, is a statistical analysis and, as such, is rather concerned with the totality of the interactions between characters rather than rhetorical levels of detail. As with any statistical analysis, what it delivers is a summary which captures aggregate characteristics, largely insensitive to individual elements. In this sense, one may hope that it delivers useful statistical information on the Viking Age in Ireland.

Estimates for the date of *Cogadh Gaedhel re Gallaibh* are various. Todd stated its author “was a contemporary and strong partizan of King Brian Borumha” [13]. Following consideration of the possibility of a later date, he concluded that the author was “either himself an eye-witness of the Battle of Clontarf, or else he compiled his narrative from the testimony of eye-witnesses” [13]. Flower also considered the chronicle “almost contemporary” [21]. Goedheer gives a date as late as 1160 [18] but Ryan argues that *Cogadh Gaedhel re Gallaibh* “might have been composed about 1130 or earlier” [20] In Ref. [22], Ó Corráin described the *Cogadh* as “concocted ... some two-and-a-half centuries after the date of the events it purports to narrate”. Elsewhere in Ref. [22] Ó Corráin refers to it as “written in the twelfth century”. He also describes the hypothesised text known as *Brian’s saga* (discussed below) as written about 1100 in response to *Cogadh Gaedhel re Gallaibh*, a suggestion that implies a date before 1100 for the creation of the latter [22]. More recent scholarship by Ní Mhaonaigh gives the likely composition date of *Cogadh Gaedhel re Gallaibh* as between the years 1103 and 1113 [26]. (She dates the common source for the Dublin/Brussels recension as the 1120s or 1130s [24, 26].) Casey also reviews dating estimates in Ref. [38]. Duffy believes it may be “based on contemporary annals and, no doubt, local memory” [5]. He suggests that *Cogadh Gaedhel re Gallaibh* gives “a vivid picture of what happened at Clontarf as related perhaps to the writer of the *Cogadh* by a veteran” and gives the possibility that it “was written by someone who may well have

⁸Ireland’s most powerful kings were described — either by themselves, or retrospectively — as king of Tara and less commonly, ardrí (translated as “high king”). These concepts were emphasised by the Uí Néill dynasty who claimed high kingship on the basis of their holding of Tara which long had a special status in Ireland’s polity. The kingship of Tara rotated between the northern and southern branches of the Uí Néill until Máel Sechnaill mac Domnaill’s claim to the title was interrupted by Brian Boru (as we shall see in Section 2). However, assertions of high kingship were just that — claims rather than unopposed fact. The law tracts gave only three grades of king but no “high king” or king of Ireland. For discussion of the nature of kingship and its various grades in Ireland, see, e.g., Ref. [5, 22, 81].

lived through these last year's of Brian's life". This bringing us back to Todd's original estimate [13].

The interpretation of *Cogadh Gaedhel re Gallaibh* as propagandistic is linked to the question of the date of its composition because "Heroic stature presupposes nurturing by time" [24]. Thus its propagandistic nature "implied that it could no longer be considered contemporary with any of the events it describes" [24]. The greater the distance between the events of Clontarf and the setting down of *Cogadh Gaedhel re Gallaibh*, the more room there is for a distorted view to take hold. This is the reason why a good estimate date for the composition of the *Cogadh* is important in the present context. Ryan writes: "In the course of the eleventh century, . . . the view seems to have gained universal acceptance that the Battle of Clontarf was par excellence the great decisive struggle of Irish history. Brian in the retrospect was everywhere acclaimed as a national hero" [19]. The claim is that time distorted reality; "The Norse were a substantial section of the opposing force, and in the mellow haze of popular imagination the battle tended to be transformed into a clear-cut issue, Irish versus Norse, with the former victorious. Even in the Northern countries the battle passed rapidly from history into saga" [19]. The above estimates for the interval between Clontarf and composition of *Cogadh Gaedhel re Gallaibh* range between contemporary and two hundred and fifty years. Our approach cannot deliver an independent estimate for the date of composition and the above estimates should be kept in mind. While the above considerations suggest that the *Cogadh* may distort in favour of an overly international picture of conflict, on the other hand it should also be kept in mind that, in places, it identifies Leinster as the principal enemies of Brian [5, 7]. We return to these issues in Section 6.

In his Introduction to *Cogadh Gaedhel re Gallaibh*, Todd acknowledges the defects of the work and expresses regret that it is "so full of the feelings of clanship, and of the consequent partisanship of the time, disfigured also by considerable interpolations, and by a bombastic style in the worst taste . . .". In chronicle literature, an interpolation of the type mentioned by Todd is a later addition not written by the original author. As scribes copied ancient material by hand, extraneous material frequently came to be inserted for a variety of reasons [55]. These may have been for bona fide intentions, perhaps as explanations; for subjective purposes; or they may simply have crept in through errors and inaccuracies arising from manual copying or, indeed, as attempts "to enhance the appeal of the narrative" [24]. One way to detect interpolation is through comparing different manuscripts.

For example, there occurs in the Dublin version a passage describing the actions of Fergal Ua Ruairc of Bréifne and associate chieftains. (For the location of Bréifne, see Figure A.1.) The Brussels manuscript, however, "omits everything connected with Fergal and his presence in the battle" [13]. As stated by Todd, "the whole story bears internal evidence of fabrication, for Fergal O'Ruairc was slain A.D. 966 . . . , and our author had already set him down amongst Brian's enemies". Ryan [19], Duffy [5] and others also identify Ua Ruairc as an interpolation and Ní Mhaonaigh gives a detailed account of Bréifne bias in *Cogadh Gaedhel re Gallaibh* [24]. She states "one of the main aims of the interpolator was to portray Fergal Ua Ruairc and his followers in as favourable a light as possible, sometimes regardless of the effect this had on his text". The point is that pro-Ua

Ruairc reviser of the narrative may have deemed it politically expedient to alter the record of relations between the Uí Ruairc and the Dál gCais by demonstrating assistance given by the former to Brian at Clontarf. Ní Mhaonaigh estimates the period when the Uí Ruairc were likely to have gained maximum advantage from such an association to have been the mid- to late 1140s, over a hundred years after Clontarf [24].

The possibility of interpolation applies not only to Ua Ruairc and allies. Ryan claims that “Many of the names mentioned are names only, for nothing is known of the persons who bear them. Some of the levies in important positions were certainly absent. In a word, no effort is made to distinguish between the genuine and the spurious, to criticise suspect sources, and to reconcile contradictions” [19]. Ó Corráin states that the author of *Cogadh Gaedhel re Gallaibh* “drew his material from the extant annals but he telescoped events, omitted references to other Viking leaders and concocted a super-Viking, Turgesius, whose wholesale raiding and, particularly, whose attack on Armagh was intended to demonstrate the inefficiency of the Uí Néill as defenders of the church and of the country in contrast of the achievements of the great Brian” [22]. (Turgesius is elsewhere referred to as “exaggerated” rather than “concocted” [40].)

Duffy, on the other hand argues that whatever about the detail of *Cogadh Gaedhel re Gallaibh* “and its slightly cavalier approach to chronology”, the gist of the account “seems sound” [5]. Duffy also discusses difficulties in using the annals to check the historicity of *Cogadh Gaedhel re Gallaibh*. By his reckoning, although some of the names of individuals drafted in from beyond Ireland are indeed suspicious, “up to half of them appear to be real and their presence at Clontarf is historically credible, if not corroborated by some other source” [5].

In Ref. [27], Ní Mhaonaigh shows that genuine annals underlie *Cogadh Gaedhel re Gallaibh* and that the compiler of *Cogadh Gaedhel re Gallaibh* “remained fairly true to his exemplar”. “Provided, therefore, that we keep the redactor’s political purpose firmly in view, we may tentatively add the annalistic material preserved in *Cogadh Gaedhel re Gallaibh* to our list of sources for information on the history of Ireland in the Viking Age” [27].

Todd himself also reports what he considers to be “curious incidental evidence” for reliability of at least some of the *Cogadh* account in that it “was compiled from contemporary materials” [13]. “It is stated in the account given of the Battle of Clontarf, that the full tide in Dublin Bay on the day of the battle (23rd April, 1014), coincided with sunrise” [13]. In a piece of “mathematical detective-work” [5] that precedes our own by 150 years, Todd’s colleague established that the full tide that morning occurred at 5:30am and indeed coincided with sunrise. For Todd, this “proves that our author, if not himself an eye-witness, must have derived his information from those who were” [13]. We have already seen the importance of the time of the evening tide; calculated to have been at 5:55 pm, consistent with the account in *Cogadh Gaedhel re Gallaibh*; it prevented the escape of the Viking forces and considerably aided Brian’s victory.

This is certainly amongst the most striking evidence in support of the account of *Cogadh Gaedhel re Gallaibh*. Duffy provides multiple other instances where the *Cogadh* may be reliable [5]. Certainly bombastic statements that are not backed up by the annals have to be treated warily. Notwithstanding this, he considers the narrative as having

“some credibility”, although “unreliable in its precise detail” [5]. There is “usually a germ of truth” in its narrative and he often finds reason to give *Cogadh Gaedhel re Gallaibh* the “benefit of the doubt”. For example, although there is a tendency to assume that the author of the *Cogadh* “dreamt up much” of the detail, such as of fortifications built by Brian, Duffy argues that since these structures would still have been standing at the time of writing, the author could not get away with much invention; “the Cogadh’s audience comprised individuals in a position to contradict it if it were inaccurate” [5]. (For criticism of Duffy’s counter-revisionist views, see e.g., Ref. [7].)

Along with the Irish annals, one may compare the contents of *Cogadh Gaedhel re Gallaibh* to those of texts from other countries. There are similarities, for example to some of the content of *Njáls Saga*, which takes place in Iceland between 960 and 1020. *Njáls Saga* is believed to have been composed between 1270 and 1290 [56]. Einar Olafur Sveinsson postulated that some of the content of *Njáls Saga* may have been drawn from a now lost *Brjáns saga* (Brian’s saga) [57]. Following Sophus Bugge [15], Ó Corráin [30] suggests that *Brjáns saga* was written in about 1100 as a response to *Cogadh Gaedhel re Gallaibh*, in order to affirm the loyalty of the Hiberno-Norse Dubliners to descendants of Brian. There are also accounts of the Battle of Clontarf in France, Germany and Wales. The consistency between *Cogadh Gaedhel re Gallaibh* and these accounts is discussed in Ref. [5].

To summarise, there is a vast amount of humanities scholarship concerning *Cogadh Gaedhel re Gallaibh*. Although some dispute its reliability, others consider its version of events mainly credible and largely consistent with other sources and evidence. As stated by Duffy, “even though it is exaggerated and biased”, *Cogadh Gaedhel re Gallaibh* can be useful “if we use it judiciously” and “make allowance for its propagandist tendency”. The composer surely did not think in terms of network science but, in recording a cast of hundreds connected with well over a thousand links between them, he nevertheless imprinted networks in the narrative. (We explain how we harvest these data in Subsection 5.1.) Thus we may expect that the bulks of the networks contained in *Cogadh Gaedhel re Gallaibh* might not be too far away from the reality of the networks of the Viking Age in Ireland. Objections listed above are largely irrelevant to our approach as static networks are immune to “bombastic” descriptions, “telescoping” of events and “cavalier” attitudes to chronology. We will see that the aggregate approach is even resistant to isolated cases of interpolation. It is with this perspective that we interrogate the narrative with a networks-science methodology. To recap, our primary aim is to determine whether the character networks in *Cogadh Gaedhel re Gallaibh* are implicative of an “international contest” or “local quarrel” [6].

4 International contest or local quarrel?

O’Connor [43] in the 18th century, with Ryan [19] and Ó Corráin [22], in the 20th, are considered early debunkers of the traditional myth of Clontarf [5,26]. O’Connor describes the conflict as a “civil war” in which “the whole province of Leinster revolted, and called the Normans from all quarters to its assistance” [43]. Ryan’s main claim in Ref. [19] is that

“In the series of events that led to Clontarf it was not . . . the Norse but the Leinstermen, who played the predominant part”. His thesis is that the conflict is not a “clear-cut” one between Irish and Viking. Firstly, Brian’s army was not a national one, but one of Munstermen supported by two small Connacht states. Secondly, the opposition “was not an army of Norse, but an army composed of Leinster and Norse troops, in which the former were certainly the predominant element and may have constituted two-thirds of the whole” [19]. The battle, then, was not a contest for the sovereignty of Ireland — it was not a clear-cut issue of Irish versus Norse. Instead, the issue at hand was “the determination of the Leinstermen to maintain their independence against the High-King” [19].

It was in the course of the eleventh century, Ryan argues, that the picture of a decisive struggle of Irish history gained “universal acceptance” in the popular imagination. This came about because of the parts played by forces from the Isle of Man and the Orkney Islands together with the partisan nature of *Cogadh Gaedhel re Gallaibh*. It was only in this retrospect that Brian was acclaimed as a national hero.

Ó Corráin’s view is similar [22]: “The battle of Clontarf was not a struggle between the Irish and the Norse for the sovereignty of Ireland [It] was part of the internal struggle for sovereignty and was essentially the revolt of the Leinstermen against the dominance of Brian, a revolt in which their Norse allies played an important but secondary role”.

Duffy points out that this revisionist interpretation is not supported by the other ancient annals. E.g., the Annals of Inisfallen gives a short but reliable account “reflective of contemporary reaction to what occurred” [5]. It is stated that “the Foreigners of Dublin gave battle to Brian” and Leinstermen are also slain. According to Duffy, “Whereas some modern historians see the Leinstermen as Brian’s primary enemy at Clontarf, the annalist was in no doubt that the enemy was the Norse of Dublin. In fact he has the same black-and-white picture of the opposing sides that we tend to think of as later legend . . .”. “The entry in the Annals of Ulster also echoes the Annals of Inisfallen in emphasising the primacy of the Norse as Brian’s adversaries”. Duffy states that the Annals of Ulster suggest “it was fundamentally a contest between the Irish and Norse (although the latter too had Irish allies)”.

Duffy provides multiple items of evidence in support of his view that “Brian’s principle opponents were the Hiberno-Norse allied to Leinster; and [the Battle of Clontarf] was notable in particular for the great numbers of overseas Norse forces present, and for the huge losses they incurred by fighting and drowning”. “Implicitly, for the *Cogadh*’s author, two centuries of Irish opposition to Viking invasion, spearheaded by Brian’s dynasty, reached a climax at Clontarf. That picture was imprinted too, with remarkable correspondences, on the minds of those thirteenth-century Icelandic writers. Those who did battle with Brian came from the Norse world seeking a kingdom for themselves in Ireland”.

Thus, the debate about Clontarf has spanned the centuries and frames our present investigation. Here we broaden the question to how conflictual and social relationships are presented in *Cogadh Gaedhel re Gallaibh*.

5 Network analysis

In this section, we present our complex-network analysis. We seek to be as non-technical as possible in order to accommodate readers from a non-scientific background and who come to our work from a humanities perspective. For this reason, we reserve technical details for the Appendices, where we also present some supplementary results. In Subsection 5.1 we give some prefatory information concerning how the data were harvested. In Subsection 5.2 we give some basic network statistics which indicate that *Cogadh Gaedhel re Gallaibh* has many properties in common with other epic narratives. We also discuss a famous instance of interpolation and how it has negligible effect on our network statistics. In Subsection 5.3 we present our main findings, namely the results of our investigation into the mixing of the *Cogadh* networks. It is here that we position conflict in the narrative on the spectrum from the traditional to the revisionist. We defer details concerning various assortativity and disassortativity measures to Appendix B. Discussion of degree distributions is also deferred to Appendix C, to keep the main text manageable. We anticipate a humanities readership to be interested in some details of the roles played by the most important characters in forming the networks which underlie the narrative and we present these in Appendix D along with an analysis of network robustness.

The entire set of interacting characters in *Cogadh Gaedhel re Gallaibh* and the relationships between them is represented in Figure 1.1. The figure represents a network of considerable complexity and includes features such as hubs, clustering, small-worldness, structural balance and various types of correlation which we explain in this section.

5.1 Gathering the data

As with previous studies [45–47, 58, 59], we consider *Cogadh Gaedhel re Gallaibh* as playing out on a complex network comprising N nodes and M edges. The edges link the nodes through relationships or interactions. We distinguish between three categories — Irish, Viking and other — identifying to which group each node belongs from the text itself. We obviously cannot directly access the reality behind the text to determine any gradation between the groups. For example, we cannot know how Sigtrygg Silkbeard, who had a Viking father and an Irish mother, might have self-identified in reality; we can only take our lead from the *Cogadh* itself and since the Hiberno-Norse of Dublin are presented there as Vikings, they are placed that category. Nodes classified as “other” are those that are not readily assigned to either camp.

We distinguish between two types of edge: positive and negative. Positive edges are established when any two characters are related, communicate directly with each another, or speak about one another, or are present together when it is clear that they know each other. So positive edges ordinarily represent familial or social relationships. Negative links, on the other hand, are formed when two characters meet in physical conflict or when animosity is explicitly declared by one character against another and it is clear they know each other (such as declaration of war). So negative edges typically represent actual or intended physical hostility. It is possible that two characters are linked by both positive

and negative edges as relationships between characters may change over time.

As *Cogadh Gaedhel re Gallaibh* is temporally relatively non-extended, covering a period of only two centuries, its connectivity properties can be reasonably accommodated through a static analysis. Indeed, capturing the temporal totality of the narrative (“Making the past just as visible as the present”, as Moretti puts it [60]) is a benefit of the networks approach and one which has been used elsewhere [45,46]. Nonetheless, it should be noted that the study of dynamical properties of networks constitutes an active, broad and developing area of research and such an approach would be of interest in the future [58].

In the following analysis, we focus primarily on the *topology* of the networks underlying *Cogadh Gaedhel re Gallaibh*. For this purpose, we consider undirected, unweighted networks. This means that (i) the features which connect the various nodes are not oriented and (ii) the statistics we report upon do not take into account varying levels of intensity of interactions between nodes. To account for (i), one would have to introduce a level of detail which is finer than just positivity or negativity. However, what one gains in refining details, one loses in statistical power. To account for (ii), one may place higher weight on more intense interactions. However, besides using the number of interactions between characters in the narrative, there is no established standard mode of weighting edges in character networks. Moreover, we are primarily interested in the presence or absence of conflict, not on the details of varying intensity of such hostility. Therefore we defer consideration of directed, weighted and temporal networks for future studies and restrict the current study to network topology and related matters.

Finally, we remark that our approach to constructing the networks follows the methodology of Refs. [45–47] in that nodes and links are identified by carefully and manually reading the texts with multiple passes through all of the material by multiple readers. In our experience, such an approach is required to minimise errors and omissions as well as to reduce levels of subjectivity. *Cogadh Gaedhel re Gallaibh* is a very dense text and meticulous care is required to interpret extremely subtle tracts containing large amounts of explicit and implicit information. It is currently beyond technological capabilities to extract such information automatically owing to the inherent complexity of such texts (see, e.g., Ref. [61]). Establishing the technology for such an approach is another active area of research.

5.2 Network statistics

5.2.1 The entire network and its positive and negative sub-networks

We identified $N = 315$ individual interacting characters in Todd’s translation of *Cogadh Gaedhel re Gallaibh*.⁹ These nodes are interconnected by $M = 1190$ edges and we

⁹Actually, we identified 326 individual characters in total. Of these, 11 are isolated in the sense that they do not interact in the narrative. We consider these as not forming part of the *Cogadh* network and they are omitted from our analysis. The characters were identified in the main part of Todd’s text. Todd’s paratexts (introduction, footnotes, appendices and index) were used to aid the identification of characters and links between them but individuals mentioned only in the paratexts do not form part of the *Cogadh* network. A small number of characters appear in the main text but are omitted in Todd’s index. We

Table 5.1: Full-cast networks comprise Irish, Viking and other nodes together with interactions between them. Unsigned networks comprise positive and negative edges as well as the nodes they connect. Thus, for example, the positive, full-cast network comprises all nodes but only positive links. The unsigned, Irish network comprises only Irish nodes but both positive and negative links between them. The *entire* network comprises all interacting nodes and all links.

		Edges	
		Positive	Negative
Nodes	Irish		
	Viking		
	Other		

refer to the corresponding assemblage as the *entire* network. We can also consider the positive and negative sub-networks, formed only of positive or negative edges, respectively. Examination of these allows us to gain more insight into the social and conflictual statistics contained in the narrative. Indeed, it is long known from sociology that societies exhibit *homophily*, the tendency of individuals to associate with others who are similar to themselves [62–64]. In the field of social network analysis, this is known as assortativity. In previous studies of epic literature [45–47, 59], we studied *degree assortativity*, the tendency (or otherwise) of nodes to attach to other nodes with similar numbers of links. We found some positive sub-networks exhibit degree assortativity, or are uncorrelated, while the opposite feature — degree disassortativity — is characteristic of negative sub-networks. This means that positive social networks give a “cleaner” picture (relative to full networks) of the non-conflictual societies underlying such narratives, making it valuable to study them in isolation [65]. A new feature of the current study is our additional focus on the negative sub-network to statistically measure levels of hostility.

We use the term *unsigned* to refer to networks containing both positive and negative edges. Networks comprising only positive (or only negative) edges are then themselves termed *positive* (or *negative*, respectively). We use the term *full-cast* to refer to networks containing the full cast of characters, Irish, Viking and others. Networks containing only Irish (or only Viking) characters are themselves referred to as *Irish* (or *Viking*, respectively). This terminology is summarised in Table 5.1. Statistics for the entire network and various sub-networks are collected in Table 5.2.

The average number of edges per node for the entire network is $\langle k \rangle = 2M/N \approx 7.6$. (The factor 2 here is to take into account that every edge connects two nodes.) The

also identified 34 groups of unnamed characters. If considered as nodes, they bring an additional 187 edges. However because these are neither individuals or named, we omit them from our presentation too. Besides, and for completeness, we also analysed the networks with these nodes included and they deliver only very small changes to the statistics presented here.

Table 5.2: Statistics for the entire network and its various sub-networks. The first and second columns indicate whether the sub-network is unsigned, positive or negative with full cast of characters (Irish, Viking and other) or only the Irish or Vikings are taken into account. Here, N represents the number of nodes; M is the number of edges; $\langle k \rangle$ is the mean degree and k_{\max} its maximum; ℓ is the mean path length and C is the mean clustering coefficient. Random counterparts are indicated by the subscripts “rand”. The proportion of triads that contain an odd number of positive links is represented by Δ ; the proportion of nodes belonging to the giant component is represented by G_c and the degree assortativity is denoted by r .

		N	M	$\langle k \rangle$	k_{\max}	ℓ	ℓ_{rand}	C	C_{rand}	Δ	G_c	r
Unsigned	Full cast	315	1190	7.6	105	3.6	3.1	0.58	0.02	0.93	0.95	-0.09(2)
	Irish	193	530	5.5	63	3.7	3.3	0.53	0.03	0.93	0.88	-0.08(3)
	Vikings	91	313	6.9	26	4.4	2.6	0.67	0.08	1.00	0.90	0.31(7)
		N^+	M^+	$\langle k \rangle^+$	k_{\max}^+	ℓ^+	ℓ_{rand}^+	C^+	C_{rand}^+		G_c^+	r^+
Positive	Full cast	287	957	6.7	53	4.1	3.2	0.59	0.02		0.87	0.00(4)
	Irish	186	475	5.1	47	3.9	3.4	0.53	0.03		0.85	-0.02(4)
	Vikings	88	301	6.8	26	3.0	2.5	0.68	0.08		0.73	0.34(7)
		N^-	M^-	$\langle k \rangle^-$	k_{\max}^-	ℓ^-	ℓ_{rand}^-	C^-	C_{rand}^-		G_c^-	r^-
Negative	Full cast	180	264	2.9	63	3.7	4.8	0.06	0.02		0.89	-0.25(3)
	Irish	62	72	2.3	25	2.6	4.7	0.06	0.04		0.69	-0.26(6)
	Vikings	18	16	1.8	4	1.5	4.5	0.00	0.10		0.33	-0.08(18)

actual number of edges associated with the i th node is denoted by k_i . This is a number which varies between 1 for the least connected characters (nodes with $k_i = 0$ have no links and are not attached to the network at all) and k_{\max} for the most connected (in a sense, the most important) character. For the entire network, the most connected character is Brian himself who, with $k_{\max} = 105$ edges, is linked to 33% of the other characters in the narrative. Besides Brian’s degree, we are also interested in the connectedness of other characters. We discuss the distribution of k_i -values (the *degree distribution*) in Appendix C and we rank the first few characters according to their individual degrees, and according to other measures of importance, in Appendix D.

Cogadh Gaedhel re Gallaibh has $N^+ = 287$ interacting characters in its positive sub-network, interconnected by $M^+ = 957$ edges, corresponding to a mean degree of $\langle k \rangle^+ \approx 6.7$.¹⁰ Here and henceforth, we use the superscripts “+” and “-” to identify

¹⁰Again we have omitted isolated nodes from the positive and negative sub-networks.

statistics associated with the positive and negative networks, respectively. (We omit such a superscript from statistics for the unsigned networks. These are distinguished from generic symbols by context.) The counterpart figures for the negative network are $N^- = 180$, $M^- = 264$ and $\langle k \rangle^- \approx 2.9$, respectively. (The total number of positive and negative links $M^+ + M^- = 957 + 264 = 1221$ exceeds the number $M = 1090$ which we previously identified for the entire network because some relationships involve both positive and negative aspects.) As for the entire network, Brian has the highest degrees in both positive and negative subgraphs, with the former measured at $k_{\max}^+ = 53$ and the latter at $k_{\max}^- = 63$.

The *mean path length* of the network ℓ is the average distance between pairs of nodes, measured by counting the minimal number of edges linking them. For the full network $\ell \approx 3.6$, shorter than the *six degrees of separation* that is claimed to characterise modern society [66]. The mean path lengths for the positive and negative sub-networks are $\ell^+ \approx 4.1$ and $\ell^- \approx 3.7$, respectively.

The *clustering coefficient* of a given node is the proportion of an individual’s acquaintances which are themselves mutually acquainted. Its average value over all vertices of the network is the *mean clustering coefficient* C and is a number between 0 and 1. We found that the full network and its positive subgraph have significant amounts of clustering with $C \approx 0.58$ and $C^+ \approx 0.59$, respectively. The negative sub-network, in contrast, is relatively unclustered with $C^- \approx 0.06$.

We can also compare the structure of a given network to that of a random one. To do this meaningfully, the random network is chosen to have the same size and average degree as the one to we wish to compare. We apply the term *small world* to our network if two conditions hold: (a) the mean path length ℓ of the network is similar to that of the random graph ℓ_{rand} and (b) if the clustering coefficient of the network C is much larger than that of the same random graph C_{rand} [67, 68]. We find that the path lengths of the comparative random networks are of the same order of magnitude for the unsigned and positive networks but the clustering is far smaller (see Table 5.2). The unsigned and positive *Cogadh* networks are therefore small worlds. The negative network does not exhibit this feature.

The adage that “the enemy of an enemy is a friend” is related to the notion of *structural balance* in network science [69–71]. The maxim suggests that *triads* (sets of three mutually connected nodes) with one positive and two negative edges are commonplace. More generally, triads with odd numbers of positive edges are considered structurally balanced. One way to quantify the extent to which it holds in a character network is through the statistic Δ , defined as the percentage of triads that contain an odd number of positive links. A large value of Δ means that hostility between two characters is suppressed if they have a common foe. Clearly Δ is only meaningful for the unsigned network; on the positive sub-network it is 1 by definition, while in the negative sub-network it is necessarily zero. We find that the entire network underlying *Cogadh Gaedhel re Gallaibh* (which has 3041 triads) is indeed structurally balanced with $\Delta \approx 93\%$.

A network may be fragmented into a number of disconnected components. In this case we are interested in the size of the largest — the so-called *giant component* — expressed as a proportion G_c of the overall network size. The giant component of the unsigned

Cogadh network contains 95% of its nodes. The equivalent proportions for the positive and negative networks are 87% and 89%, respectively (Table 5.2).

As mentioned above, assortativity (disassortativity) is the tendency for the nodes of a network nodes to attach to other nodes that are similar (different) in some way. Network theorists frequently measure *degree assortativity* — the extent to which nodes of similar degree tend to link up — and this is defined in Appendix C. As with other character networks, we find that the negative full-cast network is disassortative by degree $r = -0.25(3)$.¹¹ This means that high-degree characters are hubs and their negative links preferentially attach to low-degree ones. This appears to be a generic feature of heroic tales in particular, where the hero or heroes encounter multitudes of lesser characters and defeat them in battle. The positive full-cast network, on the other hand, is uncorrelated within errors [$r = -0.00(4)$, meaning it is neither assortative nor disassortative]. These features are typical of social networks and of character networks with positive interactions [45,65].

5.2.2 The Irish and Viking sub-networks

Beside the networks comprising the full cast of characters, we can also consider the networks containing only Irish or only Viking nodes and these are also listed in Table 5.2.¹² We observe the following average properties of the various networks. In the Irish, and Viking networks (as in the full-cast cases), the mean degrees are maximal for the unsigned networks and minimal for the negative sub-networks. For the Irish sub-network, the mean path length is largest for the positive sub-network (as it is for the full-cast of characters). The mean clustering coefficients of the unsigned and positive networks are similar in magnitude and much smaller for the negative sub-networks. The unsigned networks and their positive sub-networks may be considered small worlds but the negative sub-networks do not exhibit this feature. This is quite typical of epic literature [45–47,59]. The unsigned Viking network is more structurally balanced than its Irish counterpart. Structural balance for the Irish network, which has 830 triads, is 93% whereas the 881 Viking triads all contain odd numbers of positive links.

5.2.3 Comparisons to epic networks

We may attempt to compare the network structures in *Cogadh Gaedhel re Gallaibh* to those from previously studied epic narratives [45,46]. We choose to compare to the Irish tale *Táin Bó Cuailnge* (*The Cattle Raid of Cooley*), a recension of which is also contained in the *Book of Leinster* (as stated earlier, this contains the oldest extant partial recension of *Cogadh Gaedhel re Gallaibh*). In fact, it has been claimed that the bombastic style of

¹¹The error here is estimated using the method described in Refs. [63,64]. Error estimates for other network statistics are small (see discussion in the final paragraph of Subsection 5.2.4) and we refrain from reporting them here. We only display assortativity errors because they provide useful information when comparing systems which are, or nearly are, uncorrelated (r close to zero).

¹²As usual, isolated (degree-zero) nodes are removed. E.g., there are 202 Irish nodes in total (see Table 5.4), but 9 of these are disconnected from other Irish nodes, so they are omitted from the unsigned Irish network in Table 5.2. Besides the value of N , reinstating them does not alter the statistics listed within the precision of Table 5.2.

Table 5.3: For comparative purposes we list the statistics for the unsigned, positive and negative networks of *Táin Bó Cuailnge*, *Njáls Saga* and the *Iliad*.

		N	M	$\langle k \rangle$	k_{\max}	ℓ	ℓ_{rand}	C	C_{rand}	Δ	G_c	r
Unsigned	<i>Táin</i>	422	1266	6.0	168	2.8	3.5	0.73	0.01	0.92	0.99	-0.35(2)
	<i>Njáls Saga</i>	575	1612	5.6	83	5.1	3.9	0.42	0.01	0.90	1.00	0.01(2)
	<i>Iliad</i>	694	2684	7.7	106	3.5	3.4	0.44	0.01	0.98	0.99	-0.08(2)
		N^+	M^+	$\langle k \rangle^+$	k_{\max}^+	ℓ^+	ℓ_{rand}^+	C^+	C_{rand}^+		G_c^+	r^+
Positive	<i>Táin</i>	405	1118	5.5	164	3.0	3.7	0.74	0.01		0.93	-0.33(2)
	<i>Njáls Saga</i>	564	1388	4.9	77	5.4	4.1	0.39	0.01		0.96	0.07(2)
	<i>Iliad</i>	640	2329	7.3	85	3.8	3.5	0.44	0.01		0.86	0.09(2)
		N^-	M^-	$\langle k \rangle^-$	k_{\max}^-	ℓ^-	ℓ_{rand}^-	C^-	C_{rand}^-		G_c^-	r^-
Negative	<i>Táin</i>	134	148	2.2	82	2.9	6.0	0.03	0.02		0.91	-0.45(4)
	<i>Njáls Saga</i>	145	224	3.1	22	4.9	4.4	0.0	0.02		0.82	-0.30(4)
	<i>Iliad</i>	321	355	2.2	40	4.5	7.0	0.0	0.01		0.90	-0.44(4)

Cogadh Gaedhel re Gallaibh was also used in the Book of Leinster *Táin* [17–20, 26]. We further compare to the Icelandic *Njáls Saga*, which also contains a record of the Clontarf episode. Finally we compare to the *Iliad* as it has been claimed that the *Cogadh*'s author may have been influenced by the story of Troy [14, 18, 26, 39].

The relevant data were collected for Refs. [45, 46] and are listed in Table 5.3.¹³ The positive *Cogadh* network contains 91% of all *Cogadh* nodes. The corresponding proportions for *Táin Bó Cuailnge*, *Njáls Saga* and the *Iliad* are 96%, 98% and 92%, respectively. By contrast, the negative *Cogadh* network contains 57% of all *Cogadh* nodes. The corresponding proportions for *Táin Bó Cuailnge*, *Njáls Saga* and the *Iliad* are 32%, 46% and 25%, respectively. These figures suggest that hostility plays a greater role overall in *Cogadh Gaedhel re Gallaibh* than in the other narratives.

Turning to the unsigned networks, we see that although that of *Cogadh Gaedhel re Gallaibh* is closer in size to that of *Táin Bó Cuailnge*, the network statistics are very different in absolute terms. The *Cogadh* network is also dissimilar to that of *Njáls Saga*. The data actually suggest the overall network is rather more similar to that of the *Iliad* to either that of the Irish or Icelandic works. The mean and maximal degrees and the mean path length are very similar ($\langle k \rangle \approx 7.6$ for *Cogadh Gaedhel re Gallaibh* and 7.7 for the *Iliad*; $k_{\max} = 105$ and 106; $\ell \approx 3.6$ and 3.5 respectively) as are the degree assortativities

¹³The data omit zero-degree nodes, of which there are only six for the *Iliad* and fewer for the *Táin* and *Njáls Saga*.

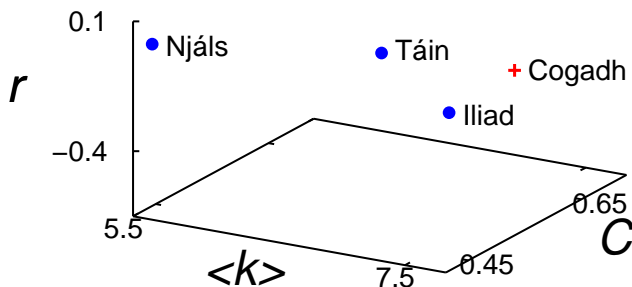


Figure 5.1: Representational depiction of some properties of the *Cogadh* network compared to those of *Táin Bó Cuailnge*, *Njáls Saga* and the *Iliad*. Only three dimensions can be represented in any one graph of this type, and the plot employs mean degree $\langle k \rangle$, mean clustering C and assortativity r . The statistics listed in Table 5.3 give other possibilities.

($r = -0.09$ for the *Cogadh* and $r = -0.08$ for the *Iliad*). The relative closeness of the entire *Cogadh* network to that of the *Iliad* is depicted in Figure 5.1 in a three-dimensional metric space spanned by the mean degree, clustering and assortativity. One could use the other parameters of Table 5.3 to extend to other combinations of dimensions.

5.2.4 The Uí Ruairc interpolation

Perhaps the most famous interpolation in the narrative is that associated with Ua Ruairc [19, 24]. Ní Mhaonaigh suggests “the period in which the Uí Ruairc stood to gain most from associating themselves with the Uí Briain seems to have been the mid- to late 1140s” [24], implying such a date potentially for the interpolation.

We have already seen that 93% of the 3041 triads in the unsigned network are structurally balanced as are 93% of the 830 triads in the Irish network. The triad formed by Ua Ruairc’s enmity to Máel Sechnaill, the latter’s alliance with Brian, and the interpolated support of Ua Ruairc for Brian is one of two positive edges and one negative one, which is structurally *imbalanced*. Ní Mhaonaigh provides a possible explanation for the redactor’s interpolation strategy in Ref. [24]. Since the vast majority of triads in *Cogadh Gaedhel re Gallaibh* are balanced, this makes the Ua Ruairc episode stand out as relatively unusual.

We removed Ua Ruairc and his three associates (Gilla-na-Naomh, Mac an Trin and Domhnall mac Raghallach, to use Todd’s spelling [13]) from the networks to test the effects on the statistics. Besides reducing the number of edges (e.g., M reduces from 1190

to 1146 in the entire network), the effects of this removal are minimal. For example, the degree assortativities are unchanged within error estimates for the unsigned, positive and negative networks.

Given the minor effect of the most famous and easily identified, Ua Ruairc, interpolation, we do not attempt to remove other interpolations from our analysis. Besides, any attempt to do so would be incomplete because we cannot be certain that all interpolations have been identified. Indeed, as we have repeatedly emphasised, ours is a network study of *Cogadh Gaedhel re Gallaibh* as represented by Todd in Ref. [13] and therefore we present it in its entirety. However, we attempt to simulate the effects of interpolation by randomly removing up to 15% of nodes or edges. The process is repeated 1000 times and the averages deliver no appreciable difference to the statistics given in Table 5.2, indicative of their robustness (see Appendix D for a network-robustness analysis). For example, removal of 15% of the vertices alters the assortativity from $r = -0.09$ to $r = -0.08$ (imperceptible change within errors). Removal of 15% of the edges leaves r unchanged within this level of precision. A more systematic and targeted quantitative study of the effects of interpolation would be interesting for future study.

5.2.5 Summary

In summary, *Cogadh Gaedhel re Gallaibh* displays quite typical features of epic narrative networks; the unsigned networks are structurally balanced, small worlds. The positive sub-networks are also small worlds and are either uncorrelated by degree (neither assortative nor disassortative to within errors) or are assortative (in the Viking case). The negative sub-networks lack the small-world property and are all disassortative by degree. The giant components contains greater proportions of the positive sub-networks than they do of their negative counterparts. *Cogadh Gaedhel re Gallaibh* has relatively more negativity than the three epic narratives to which it is compared here, and, of these, its network is most similar to that of the *Iliad*. The overall network properties appear to be robust to an isolated but famous incidence of interpolation. In the next section we explore the issue of intranational versus international conflict in more depth.

5.3 Mixing in *Cogadh Gaedhel re Gallaibh*: analysis of categorical assortativity

The traditional “memory” of the events leading up to the Battle of Clontarf is of an international conflict between two distinct sides: Irish vs Viking [19]. This is dismissed by revisionist historians who argue that the conflict is primarily Irish-on-Irish [19, 22, 43]. The traditional viewpoint of a clear-cut contest might be expected to lead to a network in which the bulk of negative (conflictual) edges correspond to Irish-Viking interactions representing the primacy of hostility being between the two groups. We might expect a network supporting the revisionist stance to be somewhat different: the majority of negative edges would mainly link Irish nodal pairs. We also have to monitor Viking-on-Viking conflict as there were different Viking factions in Ireland during this period [13, 41].

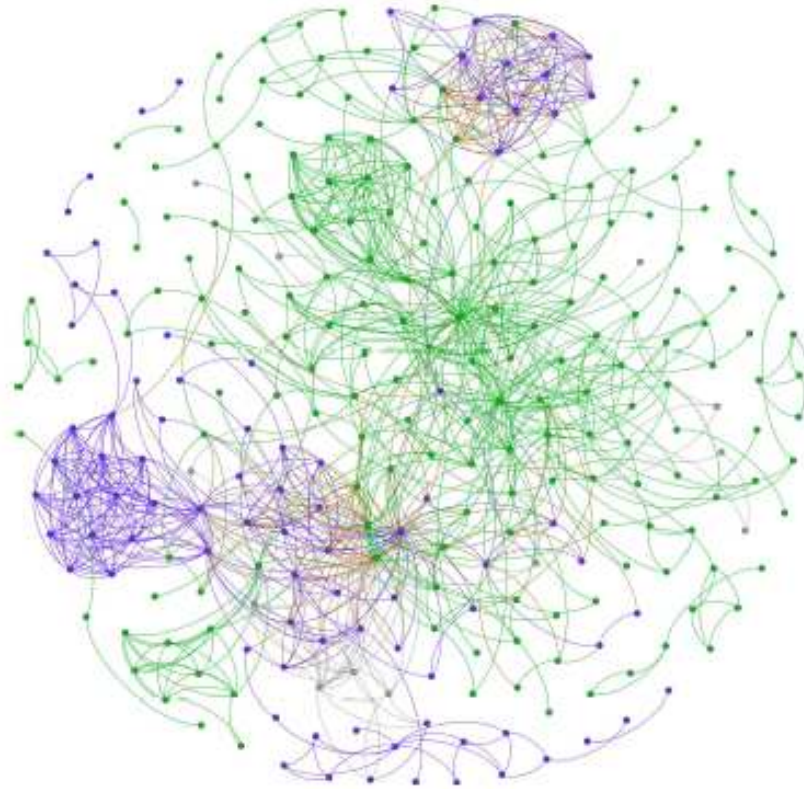


Figure 5.2: Representation of the complexity of the positive network underlying *Cogadh Gaedhel re Gallaibh*. The same colour coding is used as in Fig.1.1. Approximately 50% of the positive interactions connect pairs of Irish nodes (green); 31% are between Viking pairs (blue) and only 12% of positive interactions are between Irish and Viking (brown). The categorical assortativity for this network is $\rho^+ = 0.65$.

We present counterpart visualisations of the positive and negative networks in Fig.5.2 and Fig.5.3, respectively, using the same colour coding as for Fig.1.1 (green edges for Irish-on-Irish interactions, blue for Viking-Viking, brown for Irish-Viking and grey for interactions involving unassigned nodes). In Table 5.4, we record the proportions of Irish, Viking and other nodes in the unsigned networks and in its positive and negative sub-networks.¹⁴ At 61% – 65 %, the proportions of Irish nodes in each of the three graphs are approximately constant. The proportion of Viking nodes is also relatively stable between 31% and 34%.

In the same table, we list the proportions of interactions which link Irish to Irish nodes; Viking to Viking; and Irish-Viking pairs. Fifty percent of edges in the positive

¹⁴Some of the entries in the second and third rows of Table 5.4 differ from entries in the third column of Table 5.2 because isolated nodes are not removed from sub-networks in Table 5.4. This is because Table 5.4 concerns identity profiles of unsigned, positive and negative networks, in distinction to the Irish and Viking sub-networks of Table 5.2. Numbers of edges match across both tables, however, because, by definition, these do not involve isolated nodes.

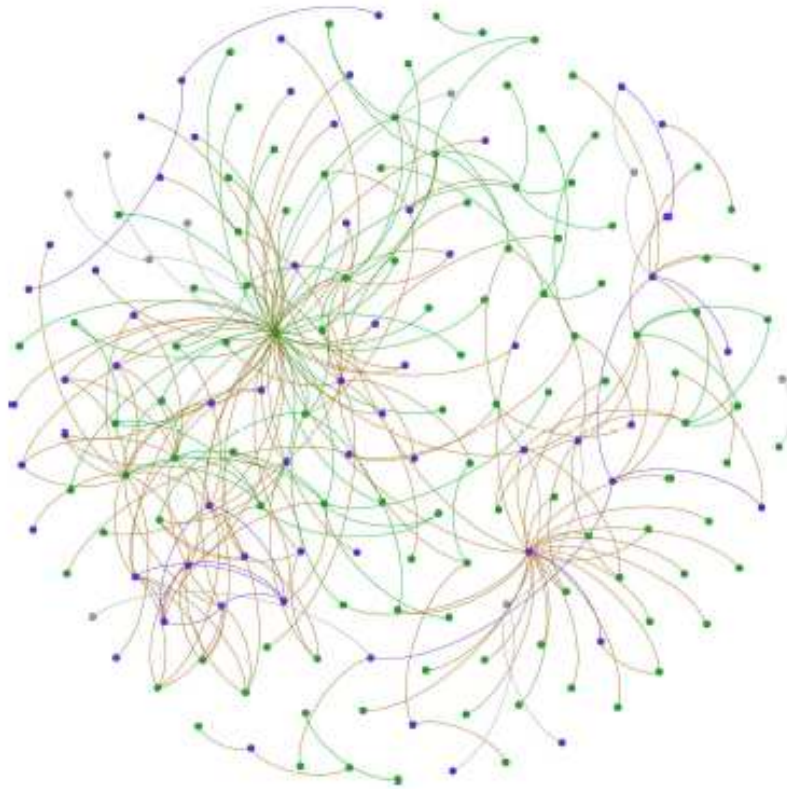


Figure 5.3: Representation of the complexity of the negative network underlying *Cogadh Gaedhel re Gallaibh*. The same colour coding is used as in Fig.1.1 and Fig.5.2. Twenty-seven percent of the negative interactions connect pairs of Irish nodes (green); 6% are between Viking pairs (blue). Sixty-two percent of the negative edges are between mixed Irish-Viking pairs (brown). The categorical assortativity for this network is $\rho^- = -0.32$.

network of Figure 5.2 link pairs of Irish nodes; 31% connect pairs of Viking nodes; and 12% of positive interactions connect mixed Irish-Viking pairs. Twenty-seven percent of links in the negative network of Figure 5.3 connect Irish to Irish nodes; 6% connect pairs of Viking nodes; and over 62% of negative interactions connect mixed Irish-Viking pairs. In other words, the positive (social) network of Figure 5.2 is dominated by interactions between characters of the same narrative identities (intranational interactions) and the negative (conflictual) network of Figure 5.3 is dominated by Irish-Viking (international) interactions. This suggest that the largest proportion of *Cogadh* conflict is international, but there are significant levels of intranational hostilities too (especially Irish versus Irish). Actually, from Table 5.4, we see that the number of international edges in the negative network is over twice the number of Irish-Irish negative edges, which, in turn is over four times the number of Viking-Viking negative edges.

However, to properly evaluate the levels of mixing, negative or positive, between Irish

and Viking, one has also to account for the fact that they do not have the same numbers of nodes in the networks (there are twice as many Irish nodes as Viking). To do this, we introduce the *categorical assortativity* of the various networks, represented generically by ρ . Its precise definition is given in Appendix B. It is a measure which ranges between ρ_{\min} and 1 where ρ_{\min} is a non-trivial, negative value, which itself lies between -1 and 0 if there are more than two categories under consideration [63, 64]. Thus, although the maximum value of ρ is one, its minimum value can be network-dependent. The reason for this is that, when there are more than two categories, disassortativity connects dissimilar nodes, just as randomness does. Assortativity, however, connects like nodes and is therefore quite different to randomness. We have to be mindful of this asymmetry when interpreting the categorical assortativity for the negative networks with three categories of node (Irish, Viking and unassigned). The only instance in which $\rho_{\min} = -1$ is when there are two categories.

The value $\rho = 1$ indicates 100% categorical assortativity. If this were the case for our positive network, for example, it would mean that the only positive interactions are *within* rather than between categories (friendly interactions would be intranational). The value $\rho = -\rho_{\min}$ implies that the network is fully categorically disassortative. If this were the case for our positive network it would mean that the only positive interactions are *between* rather than within categories (positive interactions would be international). A value $\rho = 0$ would indicate that the categorical assortativity is the same as would be

Table 5.4: Identity profiles of the cast and their interactions in *Cogadh Gaedhel re Gallaibh*. The second, third and fourth rows give the numbers (and percentages) of nodes which are identified as Irish, Viking and other (not identified as Irish or Viking) in the entire, unsigned network as well as in the positive and negative sub-networks. The fifth row gives the total number of nodes in each network (these values are N , N^+ and N^- for the full-cast networks, respectively). The sixth and seventh rows give the numbers (proportions) of edges which connect pairs of like nodes. The eighth row gives the numbers (proportions) of edges which connect Irish and Viking nodes. The last row gives the total numbers of edges in each case as (M , M^+ and M^- for the full-cast networks). The remaining edges involve other (not assigned as Irish or Viking) nodes.

	Entire network	Positive network	Negative network
Irish nodes	202 (64 %)	187 (65 %)	110 (61 %)
Viking nodes	97 (31 %)	88 (31 %)	61 (34 %)
Other nodes	16 (5 %)	12 (4 %)	9 (5 %)
Total # nodes	315 (100 %)	287 (100 %)	180 (100 %)
Irish-Irish edges	530 (45 %)	475 (50 %)	72 (27 %)
Viking-Viking edges	313 (26 %)	301 (31 %)	16 (6 %)
Irish-Viking edges	272 (23 %)	119 (12 %)	163 (62 %)
Total # edges	1090 (100 %)	957 (100 %)	264 (100 %)

expected for random mixing between nodes, oblivious of their Irish or Viking character. We find that $\rho^+ = 0.65(3)$ for the full-cast positive network. If we restrict our attention to Irish and Viking nodes only by removing other nodes, this rises to $\rho^+ = 0.72(3)$. These statistics are recorded in Table 5.5 and support the picture that most (but not all) positive interactions are intranational.

We now focus our attention on the negative networks as these connect with the debate in the humanities discussed in Section 4. A “clear-cut” version of the “international-conflict” picture would be characterised by the value $\rho^- \approx \rho_{\min}^-$ (where ρ_{\min}^- is the minimum possible value of ρ^- , and is -1 when unassigned nodes are excluded). Such a value would reflect a purely Irish-versus-Viking conflict. At the opposite end of the spectrum would be a world in which all conflict is intranational. In this case one would expect $\rho^- \approx 1$. The revisionist picture of a primarily (but not exclusively) intranational conflict may be expected to correspond to a positive value of ρ^- . Between the two extremes, we might imagine a more even distribution of negative edges, whereby conflict between nodes is “blind” to their identities. A completely colour-blind narrative would deliver $\rho^- \approx 0$ for the negative network.

We find that $\rho^- = -0.32(6)$ if all three kinds of node (Irish, Viking and other) are included in the negative network. This statistic is to be compared to the theoretical minimum $\rho_{\min}^- = -0.88(4)$. If unassigned nodes are omitted, one finds $\rho^- = -0.37(6)$ (with $\rho_{\min}^- = -1$). Thus our measured values for categorical assortativity on the negative (conflictual) networks are themselves negative. This means that picture of a primarily intranational conflict is not supported by data contained in *Cogadh Gaedhel re Gallaibh*. However, the conflict is not clear-cut international either; it is a narrative in which the highest proportion of conflict is presented as being between Irish and Viking but with significant amounts of green-on-green and blue-on-blue conflict too. On the spectrum from international to intranational conflict, representing various degrees of the traditional to the revisionist views, the negative *Cogadh* networks are firmly on the traditional side but at a moderate and not a limiting value (Figure 1.2). This is the main conclusion of our paper and is our contribution to the 250-years old debate mentioned in the Introduction.

The assortativity analysis thus far probes the extent to which conflict or harmony reigns within or between the two groups. However, one may argue that the revisionist concern is with the Irish side. The claim is that the conflict is primarily within the Irish community — not that it is both within the Irish cast and within the Viking set. Clearly there was a great degree of such conflict too; e.g., Ryan states “The Norse were traditionally unscrupulous in preying upon one another” [19]. Therefore, one may argue that Viking-on-Viking conflicts could contaminate our measurements. Our aim is to determine whether the Irish are mostly in conflict with other Irish or with Vikings; in this sense, the fact that the Vikings were also fighting amongst themselves is irrelevant.

To investigate further, we remove all Viking-on-Viking links from the negative sub-network. Recalculating the categorical assortativity delivers $\rho^- = -0.45(5)$ [$\rho^- = -0.53(4)$ if the unassigned nodes are removed] which indeed is larger in magnitude than the previous measure (the assortative Viking-on-Viking edges having been removed). But it is still not a clear-cut Irish-versus-Viking picture; i.e., it is not close to $\rho_{\min}^- = -0.88(4)$ (or -1 in the case where unassigned nodes are removed). Thus our conclusions are

Table 5.5: categorical assortativities. The first column identifies whether all nodes (Irish, Viking and other) are included in the determination of ρ or if the unassigned (other) nodes are excluded. In the former case, ρ_{\min} is determined by Eq.(B.5). In the latter case, it is -1 . The second column identifies whether all remaining links are included or whether Viking-on-Viking edges are omitted.

Nodes	Edges	Positive Network (ρ^+)	Negative Network (ρ^-)
All nodes included	Include all edges	0.65(3)	-0.32(6)
	Omit Viking-on-Viking edges	-0.62(3)	-0.45(5)
	ρ_{\min}		-0.88(4)
Other nodes omitted	Include all remaining edges	0.72(3)	-0.37(6)
	Omit Viking-on-Viking edges only		-0.53(4)
	ρ_{\min}	-1	-1

unchanged. These statistics are listed in Table 5.5.

In Appendix B, to overcome the awkwardness of network-dependent ρ_{\min} -values, we introduce a renormalised categorical assortativity measure that ranges from -1 in the case of fully disassortative networks through zero for uncorrelated networks to 1 for fully assortative networks. We also present in Table B.1 an alternative to Table 5.5, using these renormalized values.

In summary, we conclude that the character networks embedded in the *Cogadh Gaedhel re Gallaibh* do not support clear-cut traditionalist or revisionist depictions of the Viking Age in Ireland. Instead they support a moderate traditionalist picture of conflict which is mostly between Irish and Viking characters, but with significant amounts of hostilities between both sides as well.

6 Discussion

The popular tradition associated with the Viking Age in Ireland and the events of Clontarf in 1014 is that Brian’s principal opponents were Vikings. In 1938, Ryan [19] published what has been described as an “assault” [6] on that traditional interpretation. Instead of a “clear-cut” Irish versus Norse conflict, the revisionist claim is that it was a struggle primarily between two Irish forces — Munster and some allies on the one side, and Leinster and allies on the other, each having some Viking support. The fundamental issue was not, then, Irish versus Norse, but the desire of Leinster for independence [19].

With the millennial anniversary of the Battle of Clontarf, Seán Duffy attacked “the new orthodoxy” [6] and launched a counter-revisionist defence of the traditional picture [5]. The bombastic and partisan tone of *Cogadh Gaedhel re Gallaibh* notwithstanding, Duffy provides multitudes of evidence that it is a valuable source if used judiciously. His judicious use of that and other texts leads him to conclude that “The Battle of Clontarf was an international contest” [6]. This counter-revisionist view has itself come in for criticism [7] and the anniversary reinvigorated lively discussions and healthy debate amongst experts and the wider public. This and the 150th anniversary of Todd’s famous translation [13] form the context in which the above results are presented.

Cogadh Gaedhel re Gallaibh is a skillfully written propagandistic text, replete with bias, exaggerating virtues and vices of many of its characters [13,24,27]. It has been used to support arguments from both sides of the debate. Duffy describes it as a “long narrative of Irish conflict with the Vikings” [5]. Etchingham, on the other hand, in reviewing Ref. [5], stresses that “even *Cogadh* actually identifies the Leinstermen as principal rebels” [7]. From the side opposing Brian at Clontarf, the *Cogadh* gives the majority of the slain (3100 out of 5600) as Irish [13,19], tallies which could be viewed as supporting the picture of a mostly domestic conflict. At least these tallies show that *Cogadh Gaedhel re Gallaibh* does not pretend that Viking slain exceed the numbers of Leinstermen in order to “internationalise” the story. This suggests that, even if the *Cogadh* exaggerated qualities, it may not have exaggerated quantities (at least not by much). Indeed, Ryan believes that the account of the actual battle of Clontarf in the *Cogadh* is “incomparably the most reliable”.

In this paper we have gone beyond a simple tally of the slain and performed a character-network analysis of *Cogadh Gaedhel re Gallaibh*. Since this is wholly independent of the tone of the account (bombastic and partisan) and its shortcomings (“telescoping” of events and “cavalier” attitude to chronology), we considered this approach a judicious use of the text. To contribute to the debate as to the nature of the the Viking Age in Ireland as set down in the *Cogadh*, we applied a measure of categorical assortativity which is capable of taking proportions of Irish and Viking nodes into account.

A literal interpretation of “the popular tradition of Clontarf as wholly an Irish-Norse” conflict [19] would suggest a strongly negative value of categorical assortativity for the negative (conflictual) network. On the other hand, the revisionist picture of a “civil war” [43], an “internal struggle” [22], with Leinster as the “predominant element” [19] or “principal rebels” [7], suggests a positive value of categorical assortativity for the negative network. The primary outcome of our investigation is our measured value of the associated metric and we find a negative value, supportive of the traditional picture. But its magnitude is moderate, suggesting that, at least in network terms, *Cogadh Gaedhel re Gallaibh* does not describe a “clear-cut” Irish versus Norse conflict. We can gain complementary information from the positive network. The categorical assortativity for the positive network suggests that positive (social) interactions are mainly, but by no means entirely, between nodes of the same narrative identity. In other words, *Cogadh Gaedhel re Gallaibh* describes the Viking Age in Ireland as predominantly an Irish-Norse conflict but it is not wholly so.

As we have stressed throughout, any statistical analysis is only as good as the data it

draws upon and here all of our data comes directly from the *Cogadh* text. Any conclusions about the implications of our study for the reality of the Viking Age in Ireland have to be made in combination with knowledge from humanities literature on the topic. Humanities scholars agree that, to some degree, historical sources lie behind the *Cogadh*. But they differ as to their extent. If, having assessed the evidence, one believes *Cogadh Gaedhel re Gallaibh*, in the main, to be unreliable, invented or concocted then little can be drawn from our study about reality. Even in this case, however, the text (and hopefully this paper) still delivers information on how medieval writers sought to, or were able to, portray the composition of societies.

A more optimistic assessment of the evidence may offer hope that a reasonable proportion of characters and their interactions reflect the reality of the age (and we have seen that our network statistics are robust; even omitting Viking-Viking interactions does not alter the broad conclusions of our study). Indeed, since the *Cogadh* author scarcely anticipated a complexity-scientific analysis nearly 1000 years thenceforth, one might expect the networks to be less encumbered by the bias and partisanship that permeates more qualitative aspects of the text. In this sense, the networks approach delivers unique insights in that it extracts a perhaps unintended message from his time, namely new, quantitative knowledge of the Viking Age in Ireland.

A second question which we addressed herein is how the *Cogadh* narrative compares to others of the epic genre [45–47]. To this end, we measured a number of other statistics characterising the *Cogadh* network. We found it has properties typical of the genre and resembles the *Iliad* more than *Njáls Saga* or *Táin Bó Cuailnge*. The resemblance to the *Iliad* is intriguing as a link to an Irish account of the Trojan War (*Togail Troí* — “The Destruction of Troy”) has been suggested before by humanities scholars, using traditional methods [14,18,26,39]. It would be interesting to continue such comparative investigations at a more detailed level in future studies.

There are a number of other ways in which this work can be extended. Like Refs. [45, 46], the present analysis is based on static networks. These freeze the narrative progress and capture the plot “all at one glance in a visual display of its character network” [72]. Static networks are particularly advantageous for *Cogadh Gaedhel re Gallaibh* which, although believed to have been composed following some of the annals, paid limited regard to chronology [27]. Nonetheless, dynamical properties are also of interest and should be investigated in the future [58]. It would be interesting to see if temporal networks can help restore some of the chronology to *Cogadh Gaedhel re Gallaibh* [55]. Directed and weighted networks also offer obvious routes for wider study. Furthermore, motivated by the Ua Ruairc example, it would also be interesting to investigate if the structural imbalance in some network triads could be developed to give a way to spot other potential interpolations. A plethora of other quantitative approaches and suggestions are contained in the compendium [73]. Substantially more work is needed to extend this approach to compare with the Irish annals, genealogies, and other contemporary sources. We feel this is desirable in the long-term, and we hope that the current paper will help rally enthusiasm for such a project.

A criticism sometimes leveled at the character-network approach is that it brings little new; merely confirming knowledge already gained from traditional approaches to

humanities. The rebuttal to such criticism is that agreement is precisely what one would expect from a new approach which is valid and still evolving. The quantitative determination of categorical assortativity in this paper, and its precise placement of *Cogadh Gaedhel re Gallaibh* along the spectrum from the international to the intranational, is a new development in the evolution of this field. In that sense, our paper goes beyond limitations identified in some previous works in that it generates a new quantitative element to an unfinished debate in the humanities.

Acknowledgments

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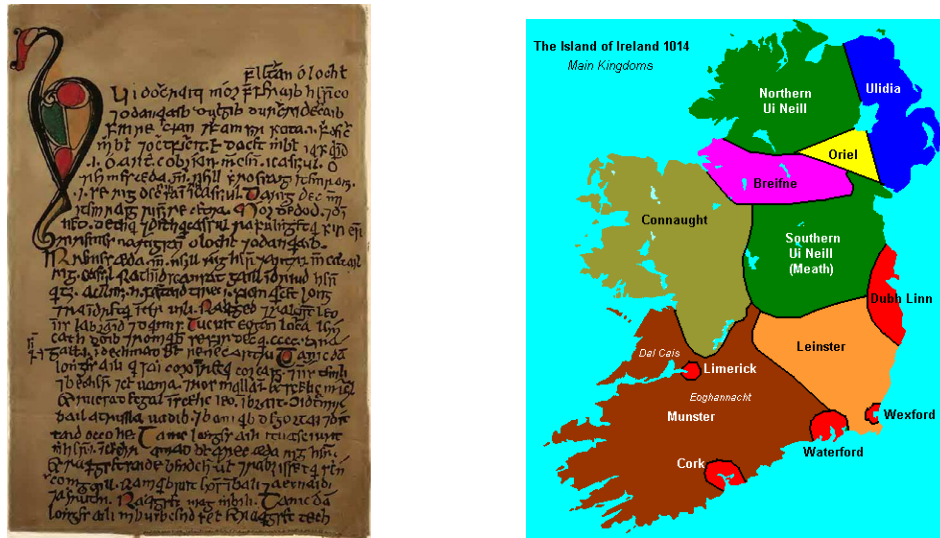


Figure A.1: Left: The opening page of the oldest extant copy of *Cogadh Gaedhel re Gallaibh* from the twelfth century Book of Leinster. Right: The main kingdoms of Ireland in 1014 (courtesy of Wesley Johnston, <http://www.wesleyjohnston.com>).

A Ireland during the Viking Age

The five provinces referred to in the main text are Connacht, Leinster, Meath, Munster and Ulster. These five provinces were called *cóiceda* or fifths. The medieval Ulster, Connacht and Munster roughly correspond to today’s provinces of the same names while the ancient provinces of Meath and Leinster, if combined, approximate the modern Leinster (Meath and Westmeath being counties within it). *Cúige* (literally “fifth part”) remains the modern Irish word for “province”. In the tenth century the main rivalry for claims to high kingship of Ireland was between the northern and southern branches of the Uí Néill. Their dominance was ended by Brian Boru.

In the ninth century, Cork, Dublin, Limerick, Waterford and Wexford all developed from Vikings base camps to more permanent settlements. See Figure A.1 which, alongside an image of the opening page of the oldest extant copy of *Cogadh Gaedhel re Gallaibh* from the Book of Leinster, includes a map outlining the political structure of Ireland in 1014.

B Scalar and categorical assortativity

In the main text we used two different forms of assortativity: the degree assortativity r and the measure ρ . The first of these is an example of *scalar assortativity* — it quantifies the tendency of nodes whose degrees have similar values to associate with each other. In determining r , it is important to account for nodes possibly having similar but not identical values; e.g., high degree nodes may tend to mix with other high degree nodes without them having to have precisely the same k -values. The second is *categorical* — it

measures tendencies for nodes belonging to the same category to link to each other. In the categorical case, two nodes either have the same attributes or they do not; there is no question of degrees of similarity here. Therefore we require two different formulae to quantify scalar and categorical assortativity.

Scalar assortativity is simply given by Pearson’s correlation coefficient, i.e., it is the covariance of two variables normalised by the product of their standard deviations. The normalization factor ensures that the assortativity takes values in the range $[-1, 1]$. Networks with a degree value $r > 0$ are termed *degree assortative*. If the measured value of r is negative, the network is deemed *degree disassortative*. Since the theoretical bounds on scalar assortativity are the same for all networks, comparisons of assortativity between them are straightforward and meaningful.

Many networks tend to evolve towards their maximum-entropy state unless otherwise constrained [74]. Such maximum-entropy states are usually disassortative because disassortative configurations are more abundant than assortative ones [75]. For this reason, non-social networks are usually degree-disassortative. Social networks, on the other hand, are usually uncorrelated or assortative. This can be explained by homophily; highly connected people tend to link together [63, 64]. The lack of disassortativity in the positive networks, as seen in Table 5.2, is a common feature of epic narratives. It is a signal of the presence of a non-trivial social or narrative force — driving them away from their maximum-entropy, anticorrelated (disassortative) states. In this sense, positive character networks, including those of *Cogadh Gaedhel re Gallaibh* are more like social networks than unlike them.

For categorical assortativity, consider the nodes i of a network having attributes c_i which could be colours (e.g., green, blue or grey) as in the main text. We require the difference between the fraction of edges that exist between nodes of the same attribute and the fraction of such edges we would expect if the nodes were connected at random regardless of the nodes’ attributes (i.e., if the linking process were “colour blind”). It is defined as follows [63, 64].

The total degree of the network is $\sum_{i=1}^N k_i = 2M$ (twice the number of edges because each edge is double-counted). Let c and c' denote categorical variables and let $e_{cc'}$ denote the density of directed edges in the network pointing from nodes of type c to nodes of type c' . We note that $e_{cc'} = e_{c'c}$ if the network is undirected. We define the density of degrees associated with nodes of type c as

$$a_c = \sum_{c'} e_{cc'} = \frac{1}{2M} \sum_i k_i \delta_{c_i c}, \quad (\text{B.1})$$

and have the sum rule

$$\sum_c a_c = \sum_{cc'} e_{cc'} = 1. \quad (\text{B.2})$$

The *modularity* is defined as

$$Q = \sum_c (e_{cc} - a_c^2). \quad (\text{B.3})$$

The categorical assortativity ρ is obtained by normalising the modularity so that its maximum value is 1 (as is the case for the scalar assortativity). If the network is fully

assortative, *all* edges connect nodes of the same type. Therefore the normalising factor for Q is given by Eq.(B.3) with $\sum_c e_{cc}$ set to 1. This motivates the definition

$$\rho = \frac{\sum_c (e_{cc} - a_c^2)}{1 - \sum_c a_c^2}. \quad (\text{B.4})$$

The minimum possible value of this quantity is obtained when all edges connect nodes of different types ($e_{cc} = 0$ for all c) and is

$$\rho_{\min} = \frac{-\sum_c a_c^2}{1 - \sum_c a_c^2}. \quad (\text{B.5})$$

Fully disassortative, undirected networks with only two categories have $\rho_{\min} = -1$. However, the minimum value for ρ is not generally -1 if more categories are involved. While the absence of assortativity means that $\sum_c e_{cc} = 0$ for any number of categories, the lack of directedness that assures the symmetry between the categories only happens when there are two of them. This property, together with Eq.(B.2) trivially gives $\rho = -1$. More generally, ρ_{\min} lies between -1 and 1 .

The reason why ρ_{\min} is not -1 is for a perfectly disassortative network is that such a network more closely resembles a random network than does a perfectly assortative one when there is more than two categories. I.e., random mixing mostly mixes unlike nodes and disassortativity does the same. But assortativity mixes like nodes. This is why the minimum value of ρ is closer to the value for a random network $\rho = 0$ than is the maximum value $\rho = 1$. In the main text, we have to be mindful of this when interpreting the categorical assortativity for the negative network. However, we could easily introduce a measure which is -1 for a fully disassortative network as follows.

The modularity in Eq.(B.4) is defined with respect to the expected density of edges between nodes of the *same* category if the network were assembled without regard to category. This was appropriate for the measurement of *assortativity*. To directly measure *disassortativity* instead, we focus on edges between node of *different* categories and introduce

$$\bar{\rho} = -\frac{\sum'_{c,c'} (e_{cc'} - a_c a_{c'})}{1 - \sum'_{c,c'} a_c a_{c'}}, \quad (\text{B.6})$$

where the prime on the summation means that it is taken over c and c' values such that $c \neq c'$ and the leading minus sign is to ensure that disassortative networks have negative $\bar{\rho}$ -values, in line with their negative ρ -values.

Eq.(B.2) gives

$$\sum'_{c,c'} e_{cc'} = 1 - \sum_c e_{cc},$$

enabling us to write

$$\bar{\rho} = \rho \left(\frac{1}{\sum_c a_c^2} - 1 \right). \quad (\text{B.7})$$

From Eq.(B.5), this may be written

$$\bar{\rho} = -\frac{\rho}{\rho_{\min}}. \quad (\text{B.8})$$

Table B.1: The set of renormalised categorical assortativity values $\hat{\rho}$ from Eq.(B.6) presented here is an alternative to Table 5.5. Fully disassortative, uncorrelated, and assortative networks have $\hat{\rho} = -1$, $\hat{\rho} = 0$ and $\hat{\rho} = 1$, respectively.

Nodes	Edges	Positive Network ($\hat{\rho}^+$)	Negative Network ($\hat{\rho}^-$)
All nodes included	Include all edges	0.65(3)	-0.32(6)
	Omit Viking-on-Viking edges		-0.43(5)
	ρ_{\min}	-1	-1
Other nodes omitted	Include all remaining edges	0.72(3)	-0.33(6)
	Omit Viking-on-Viking edges only		-0.44(5)
	ρ_{\min}	-1	-1

In other words, $\bar{\rho}$ is simply the assortativity normalised by its minimum possible value (which is negative). This has the advantage that its value is 1 for a fully disassortative network; however a fully assortative network may have a value of $\bar{\rho}$ which exceeds 1.

We therefore introduce a renormalised version of the categorical assortativity that is suitable for all circumstances:

$$\hat{\rho} = \begin{cases} \rho & \text{if } \rho > 0, \\ -\frac{\rho}{\rho_{\min}} & \text{if } \rho < 0. \end{cases} \quad (\text{B.9})$$

This measure has the desired features that it vanishes in the case of colour-blindness, and it is 1 and -1 for fully assortative and fully disassortative networks, respectively. In Table B.1, we list the values of $\hat{\rho}$ for the various networks. This may be considered as a renormalised version of Table 5.5 of the main text. The differences between the values entered in the two tables are very small.

C Degree distributions

The degree distributions give further insights into the nature of complex networks and facilitate comparisons between them. We present, here, a brief report on the degree distributions of the cast of *Cogadh Gaedhel re Gallaibh* and how they compare to other epic narratives.

We are interested in the probability $p(k)$ that a node has degree k . The convention is to display the complementary cumulative degree distribution $P(k)$ rather than $p(k)$ itself. This is because the latter can be rather noisy, especially in the tails of the distribution.

The preferred quantity $P(k)$ is the frequency of nodes whose degree is greater than or equal to the given value k .

The simplest possibility is to attempt to describe a degree distribution by a power law of the form $p(k) \sim k^{-\gamma}$ [76, 77]. Empirically, exponents for such distributions are mostly found with $2 \leq \gamma \leq 3$ [48]. This behaviour is more usual in the tail of the distribution above some minimum value $k_{\min} > 0$ [78]. Here, as elsewhere, we use $k_{\min} = 2$ to capture as much information as possible [47].

Power-law degree distributions sometimes exhibit a cut-off at some value $k = \kappa$. In this case the degree distribution may be described by

$$p(k) \sim k^{-\gamma} e^{-k/\kappa}. \quad (\text{C.1})$$

We will find that the positive network underlying *Cogadh Gaedhel re Gallaibh* is of this type.

Another frequently encountered form is the log-normal distribution:

$$p(k) \sim \frac{1}{k} \exp \left[-\frac{(\ln k - \mu)^2}{2\sigma^2} \right]. \quad (\text{C.2})$$

Here μ and σ^2 are related to the mean and variance of the distribution, respectively. We will show that the negative network of *Cogadh Gaedhel re Gallaibh* is best fitted by a log-normal distribution.

Another commonly encountered distribution takes the form

$$p(k) \sim \left(\frac{k}{\kappa} \right)^\alpha \exp \left[-\left(\frac{k}{\kappa} \right)^\beta \right]. \quad (\text{C.3})$$

where κ and β are constants which endow the distribution with a scale. This is called the Weibull distribution when $\alpha = \beta - 1$. We will show that the unsigned network is best fitted by a Weibull distribution.

Notwithstanding the above fits, truncated power laws and Weibull distributions cannot be ruled out as an alternative for any of the networks considered here.

We fit each data set to each of the above functional forms and use maximum likelihood estimators to determine the various parameters for each probability distribution. The

Table C.1: Relative minimum-information-loss probabilities for the various degree distribution functions of the unsigned, positive and negative networks. The values of the most likely distributions are highlighted in boldface.

	Power law	Truncated power law	Log normal	Exponential	Weibull
<i>Unsigned</i>	~ 0	0.26	0.22	~ 0	0.52
<i>Positive</i>	~ 0	0.62	~ 0	0.04	0.34
<i>Negative</i>	0.04	0.27	0.44	~ 0	0.25

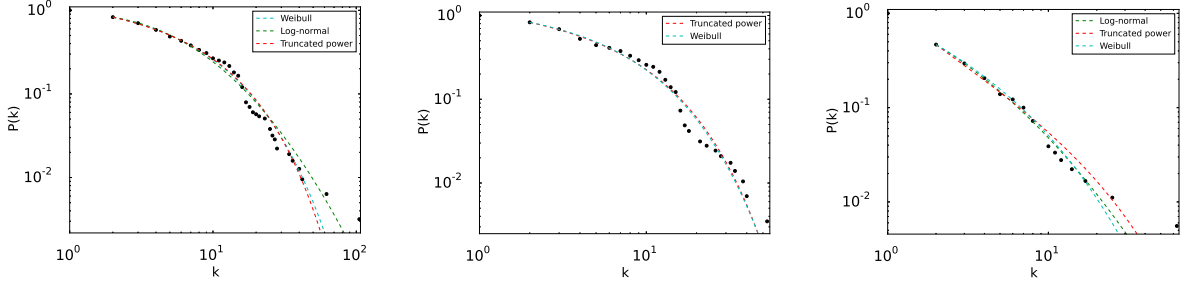


Figure C.1: The complementary cumulative degree distributions for the unsigned network of *Cogadh Gaedhel re Gallaibh* (left) and its positive and negative sub-networks (middle and right, respectively).

likelihood \mathcal{L} is the probability that independent and identically distributed data of N observations were drawn from a model $p(k)$ with parameter θ ,

$$\mathcal{L}(\theta|k) = \prod_{i=1}^N p_{\theta}(k_i).$$

The estimated parameters are those that maximise the probability of obtaining the observed distribution. The Akaike Information Criterion (AIC) is then employed to check and compare the probability distribution that best describes the data [79]. Given a set of models (fits) for a data set, the AIC estimates the relative quality. An adjusted version, denoted AIC_c , penalises small sample size and is given by

$$\text{AIC}_c = \frac{2qn}{n - q - 1} - 2 \ln \mathcal{L},$$

where q is the number of parameters and n is the sample size. To compare between various models, one uses the relative probability that a given model minimises the estimated

Table C.2: Maximum-likelihood estimates for the various parameters associated with the probability distributions fitted to the data for the unsigned, positive and negative, full-cast networks. Values corresponding to the most likely distributions are highlighted in boldface.

	Power law	Truncated power law		Log normal		Exponential	Weibull	
	γ	γ	κ	μ	σ	κ	β	κ
<i>Unsigned</i>	1.7(1)	0.5(2)	11.4(1)	1.6(1)	1.0(1)	7.4(1)	0.8(1)	5.3(1)
<i>Positive</i>	1.8(1)	0.5(2)	9.4(1)	1.5(1)	1.0(1)	6.3(1)	0.8(1)	5.0(1)
<i>Negative</i>	2.2(2)	1.8(1)	30.9(1)	0.0(1)	1.3(1)	3.6(1)	0.5(1)	0.5(2)

Table D.1: The most important characters of *Cogadh Gaedhel re Gallaibh* ranked according to their degree, betweenness centrality, closeness, and eigenvector centrality.

	Rank	Degree	Betweenness	Closeness	Eigenvector
Unsigned	1	Brian (105)	Brian (0.42)	Brian (0.44)	Brian (0.53)
	2	Sitriuc (62)	Sitriuc (0.21)	Sitriuc (0.41)	Maelmordha (0.28)
	3	Maelmordha (42)	Ottir (0.16)	Ottir (0.39)	Malachy II (0.22)
	4	Ottir (40)	Aedh Finnliath (0.13)	Gormflaith (0.38)	Sitriuc (0.21)
	5	Malachy II (36)	Ossill (0.11)	Maelmordha (0.38)	Gormflaith (0.21)
Positive	1	Brian (53)	Brian (0.28)	Sitriuc (0.34)	Brian (0.48)
	2	Sitriuc (40)	Sitriuc (0.17)	Brian (0.34)	Murchadh (0.30)
	3	Maelmordha (38)	Malachy II (0.11)	Gormflaith (0.34)	Maelmordha (0.26)
	4	Gormflaith (34)	Ottir (0.10)	Maelmordha (0.32)	Malachy II (0.26)
	5	Ottir (32)	Gormflaith (0.10)	Malachy II (0.32)	Conaing (0.23)
Negative	1	Brian (63)	Brian (0.63)	Brian (0.44)	Brian (0.66)
	2	Sitriuc (25)	Ottir (0.23)	Malachy II (0.35)	Maelmordha (0.23)
	3	Mathgamhain (17)	Sitriuc (0.23)	Sitriuc (0.34)	Brodur (0.22)
	4	Cathal (14)	Aedh Finnliath (0.16)	Ottir (0.33)	Malachy II (0.17)
	5	Olaf Cuaran (12)	Olaf Cuaran (0.12)	Ivar (0.32)	Ivar (0.17)

information loss given by $\exp[(AIC_{c,\min} - AIC_c)/2]$ where $AIC_{c,\min}$ is the minimum AIC_c value for a given set of data [47]. In Table C.1 we present the relative probabilities with the most likely values highlighted in boldface. In Table C.2 we present the corresponding maximum-likelihood estimates. Again statistics in boldface are those from the most likely distributions. The most likely distributions along with other candidates are plotted for the unsigned, positive and negative networks in Figure C.1.

D Network robustness and importance of individual characters

Having investigated the giant component in the main text, we may ask how reliant its integrity is on the most important characters. This is a question of robustness and one investigates it by determining the effects of systematic and random removal of nodes or edges. In the former approach, we remove the most important nodes one-by-one and monitor how the giant component reduces in size. We can then compare this to the results of the latter approach, in which removal of nodes is a random process.

There are a number of ways in which we can decide which are the most important or influential nodes. One way is to consider that those with highest degree are most important and to remove them first. Another possibility is to consider nodes with the highest *betweenness centralities* [80]. This counts the number of shortest paths (geodesics) which pass through each node [80]. To define it, we first write the number of geodesics (shortest paths) between nodes i and j as $\sigma(i, j)$. We denote the number of these which

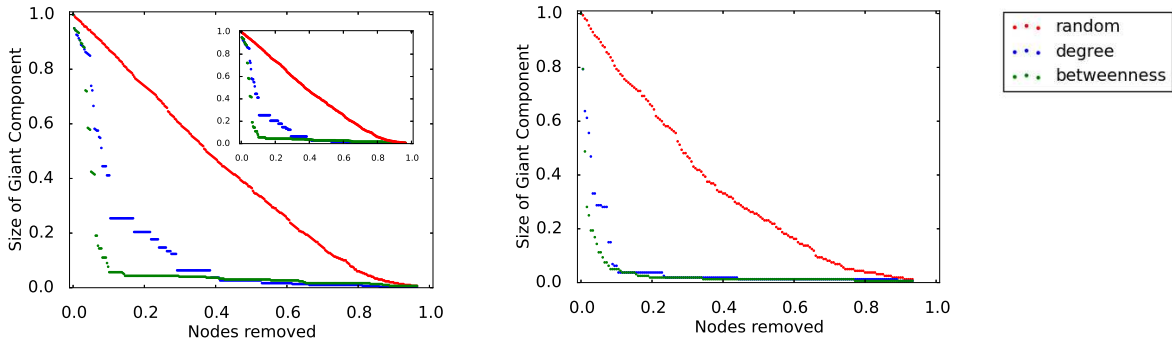


Figure D.1: The relative sizes of the giant components as a function of the percentage of nodes removed. In the left panel the size of the giant component for the unsigned network is given. That of the positive network, which has a very similar decay, is given as an insert. The right panel shows the decay of the giant component of the negative network as nodes are removed. The red data points correspond to random removal of nodes and the blue and green data concern removal by highest degree and betweenness, respectively.

pass through node l as $\sigma_l(i, j)$. The betweenness centrality of vertex l is then defined as

$$g_l = \frac{2}{(N-1)(N-2)} \sum_{i \neq j} \frac{\sigma_l(i, j)}{\sigma(i, j)}. \quad (\text{D.1})$$

If $g_l = 1$, all geodesics pass through node l . If i, j and l represent edges rather than nodes, Eq. (D.1) can be interpreted as the *edge betweenness centrality* instead.

Other measures of importance include nodes' *closeness* and *eigenvector centralities*. The sum of the distances of a given node from all other nodes in a connected graph or component is termed its *farness*. The reciprocal of farness is a measure of how central a node is and is termed its *closeness* [50]. *Eigenvector centrality* characterises node importance in terms of centralities of its neighbours; nodes are deemed influential according to how they are linked to other important nodes [50]. Eigenvector centrality is a variant of the ‘‘pagerank’’ score used to rank websites. The leading characters of *Cogadh Gaedhel re Gallaibh* are listed in Table D.1, ranked according to four different measures: degree; betweenness; closeness and eigenvector centrality.

We present the study of robustness for the networks underlying *Cogadh Gaedhel re Gallaibh* in Figure D.1. The main left panel depicts the relative sizes of the giant component of the unsigned network as nodes are removed randomly (red data points), by highest betweenness (blue) and by degree (green). A similar behaviour is observed for the positive network, shown in the insert. The counterpart information for the negative sub-network is contained in the next main panel. We see that random removal of nodes only has a relatively gradual effect on the giant-component size in all three networks. Removal by betweenness or by degree has far more devastating consequences. Removal by betweenness is particularly damaging for the integrity of the full and positive

Table D.2: The effects of removing the most important characters or of removing characters at random. The entries in the table give the relative size of the giant component after removal of the top 10% of characters systematically and randomly; the top five characters; and after removal of the most important character, namely Brian Boru.

	Remove 10% by degree	Remove 10% by betweenness	Remove 10% randomly	Remove top 5 by degree	Remove top 5 by betweenness	Remove Brian Boru
Unsigned	43%	6%	92%	90%	91%	92%
Positive	47%	7%	83%	85%	85%	86%
Negative	6%	5%	81%	69%	58%	85%

networks whereas, for the negative network, removal by betweenness and degree are about equally effective. Details of the effects of node-removal on the relative sizes of the giant components are given in Table D.2.

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