Facilitating the development of prospective primary school teachers' understanding of the concept of ratio through discussion

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Abstract:

Research on prospective primary school teachers' (PPSTs') understanding of the concept of ratio and its application to everyday life has shown that this can be problematic for many student teachers. As the concept is important in the development of proportional reasoning, which underpins many areas of primary school mathematics, questions arise about the PPSTs' ability to facilitate its development for the children they teach. The "ATEE Ratio Project," started in 2011, has contributed to the research, chiefly using an instrument designed to elicit from individual respondents the meanings that they ascribed to ratio, the uses (both their own and other people's) of ratio that they identified, and the ways in which they represented the concept in particular by symbols and drawings – all measures of understanding. For the present study, the research instrument was utilised differently: as a stimulus to discussion (in "Think, Pair, Share" mode), with the aim of investigating if participants' understanding could be enhanced by this exercise. The work was undertaken in a Mathematics Education module with Professional Masters in Education students in one institution in Ireland, and focused in particular on usage in everyday life. This paper reports on the study, presenting and discussing the findings with reference to the previous Irish and international work. Initial findings suggest that embedding the concept in everyday life contexts helps facilitate PPSTs' understanding and application of ratio, and should therefore help in supporting the development of their own students' understanding of the concept.

Keywords: Ratio, applications, primary teacher education, relational understanding, "Think, Pair, Share", discussion.

1. Introduction

Ratio has long been recognised as an important concept in mathematics education; it is crucial in the development of proportional reasoning, which underpins many areas of school mathematics. Understanding ratio, hence being able to recognise its manyapplications and use it to solve problems, is therefore a key to successful progress in mathematics. However, many school students have an imperfect grasp of the topic; moreover, some prospective teachers carry this limited understanding from their school days into their professional studies. This is the case especially for prospective primary teachers, many of whom did not study mathematics to high levels. Given the importance of appropriate content knowledge for teaching, this is likely to mean that the children whom they teach will in turn gain an imperfect understanding of ratio, and the problem will be perpetuated.

At the Association for Teacher Education in Europe (ATEE) Annual Conference in 2011, a group within one of the Association's Research and Development Communities (RDCs) initiated a project to investigate prospective teachers' knowledge with regard to ratio. An instrument was developed to elicit from individual respondents the meanings that they ascribed to ratio, the uses (both their own and other people's) of ratio that they identified, and the ways in which they represented the concept. Data were collected from four institutions in three countries; analysis of the data led to the identification of a tentative framework for further research. Since then, individuals within the RDC have collected more data within their own countries.

From an early stage, it was recognised that there was potential for widening the scope of the study: for example, aiming not just to collect data but to enhance the prospective teachers' understanding by instigating class discussion around their responses to the questions in the research instrument.

The study described in this paper is in the latter category. In a Mathematics Education module with Professional Masters in Education students – prospective primary school teachers (PPSTs) – in one institution in Ireland, the research instrument was utilised as a stimulus to discussion, addressing the research questions:

- What personal uses and users did PPSTs identify for ratio?
- Does using discussion and application to real-life contexts facilitate PPSTs' understanding of the concept of ratio?

2. Literature review

The literature review addresses two areas. It provides an overview of research on teachers' knowledge, both in general and with reference to the ratio concept, and discusses the role of applications to real-life contexts in the teaching and learning of mathematical concepts. It then examines the project to which the present paper is a contribution.

2.1. Teachers' knowledge and the ratio concept

Teachers' knowledge of mathematics, and in particular their knowledge specifically for teaching the subject, has been studied extensively in recent years, with important work emerging from the 1970s. First, to be an effective teacher of the subject, a teacher requires what Skemp (1976) called relational understanding: knowledge that involves knowing both what to do and why, understanding a concept in relation to other concepts and how it fits into the wider mathematical landscape. So, to have a relational understanding of ratio means to understand ratio and its relationship to fractions, division, decimals, multiplication, proportion, and so forth. Secondly, seminal work by Shulman (1986, 1987) distinguished content knowledge (CK) and pedagogical knowledge (PK) from pedagogical content knowledge (PCK), with the latter referring to that special knowledge of “the ways of representing and formulating the subject that make it comprehensible to others” (Shulman, 1986, p. 9). There are many aspects to this category of knowledge, including useful ways of representing ideas and concepts, and utilisation of powerful analogies, examples, demonstrations, and explanations. In this sense, the ability to apply in a real-life situation forms part of a teacher's PCK.

The relationship between the forms of knowledge is important. Ma (1999) pointed out that even high levels of PK cannot make up for ignorance (lack of CK) of a concept.

More recent work has focused on attempts to measure attainment of CK and PCK, to determine the extent to which they are distinct, and where possible to examine their role in predicting the performance of students. For one cross-national study on preservice teachers, TEDS-M (Teacher Education and Development Study – Learning to teach Mathematics), findings indicate that some CK is necessary for PCK and that the two concepts, though distinct, are difficult to separate (Blömeke, Hsieh, Kaiser, & Schmidt, 2014). For a study based in Germany, COACTIV (Cognitive Activation), in which teacher scores could be related to those of their students, Rowland (2014, p. 97) summarised the findings as follows: “weak CK puts limits on the growth of PCK.However, teacher PCK (as measured by the COACTIV instruments) predicts student progress – in the German secondary setting – better than teacher CK.” Altogether, it can be said that the importance of teachers’ CK is affirmed as necessary although not sufficient for good teaching.

But the question remains: in specific terms, in what way exactly do we want primary school teachers to know about mathematics? One of the more complete explanations comes from Wu (2011). To know mathematics, he describes, is unambiguous in the sense that it is understood to mean how mathematicians know mathematics. He breaks this down into what it means to know a concept and what it means to know a skill.

With respect to knowing the concept of ratio, this means knowledge of why it is needed as well as its role and context in the world. That is, we need ratio to compare values multiplicatively rather than additively, the latter being the case for subtraction.

But teachers also need to be able to recognise when it is preferable to compare multiplicatively rather than additively. Problems involving ratio are not always as straightforward as, say, comparing the number of blue balls to the number of red balls or boys to girls in a class. This leads to the next sort of knowledge: the skill of using ratio. This involves the ability to use the procedure correctly in diverse situations, identifying when it is appropriate to apply such a procedure. Importantly, really to know a procedure means that the teacher should be able to prove that it is correct.

Knowledge of ratio in this sense will ensure the necessary mathematical precision in teaching and also ensure teachers will have the ability to make mathematics both interesting and relevant for their students.

The development of appropriate knowledge and skill can be enhanced in various ways. The role of representations – graphical, tabular, symbolic and verbal – is noteworthy here; there is widespread agreement among mathematics educators on the importance of being able to provide multiple representations of concepts and to translate between them (Bossé, Adu-Gyamfi, & Cheetham, 2011; Janvier, 1987; Pape & Tchoshanov, 2001). The role of applications is also crucial. Knowledge of a diverse range of applications does not necessarily constitute relational understanding. Rather, having a relational understanding of mathematics (in particular, knowledge of the relevant school curriculum) ensures that teachers will have the ability to apply the concepts to a diverse range of situations (Skemp, 1987). This is because relational understanding is organic in quality and if teachers develop an appreciation for relational knowledge then they are more likely to continue to learn relationally and "actively seek out new material and explore new areas" (p. 10).

Consequently, in order really to know mathematics, such an inquisitive attitude is necessary. Ben-Chaim, Keret, and Ilany (2012) highlighted the key role of the topic:

The concepts of ratio and proportion are fundamental to mathematics and important in many other fields of knowledge.... Conceptualization and comprehension of these concepts, not to mention skills and competence in using them, facilitate mathematic awareness.

Even more importantly, these skills foster the ability to use relational reasoning, otherwise known as proportional reasoning, which is crucial to the development of analytical mathematical reasoning. (Ben-Chaim, Keret, & Ilany, 2012, p. 1 {emphasis in original})

However, studies dating back to the 1970s point to the problematic nature of the concept for school students. Comparatively recent summaries of research are provided by Livy and Vale (2011) and Ellis (2013), with the former paper also highlighting the low levels of correct responses to relevant ratio and proportion test items in their empirical study of prospective teachers. The cluster of papers produced for the ATEE Ratio Project (see below) provides further evidence that at least some prospective teachers have significant gaps in their understanding of ratio and their awareness of its applications.

With regard to classroom practice, Mochon Cohen (2012) asserted that textbooks make too rapid a move from emphasis on proportional reasoning to using the "rule of three" – hence, from development of the concept to the practice of skills. Likewise, in their study of Irish PPSTs, Oldham, Stafford and O'Dowd (2016) noted that skills are the main focus and there is very limited reference to applications. This provides a background for the study of Irish PPSTs described in the present paper.

In Ireland, the confusion around the ratio concept is not helped by the lack of attention it receives in the primary school curriculum (National Council for Curriculum and Assessment [NCCA], 1999a, 1999b; Stafford, Oldham, & O'Dowd, 2015) and many of the definitions in mathematics education textbooks. These texts include those for primary and secondary education, as well as those intended for mathematical knowledge for preservice and practising primary school teachers. For example, Humphrey, Reeves, Guildea & Boylan (2011, p. 74) defines a ratio as “comparison between two similar quantities measured in the same units” while Suggate, Davis, and Goulding (2001, p. 79) define it as “comparison between two quantities, which are measured in the same units.” These are very typical of school textbook definitions, where there is no mention of a multiplicative comparison. These definitions, and others like them, are so loosely formulated that, were they not followed up with the typical types of examples involving ratio, they could be used to describe subtraction.Although ratio is not emphasised in the Irish primary school mathematics curriculum, there is a strong emphasis on application of concepts in general (NCCA, 1999a, 1999b; Oldham et al., 2016). This application extends to a variety of contexts relating to both the physical environment and social interactions. Integral to this is the ability of the child to recognise the situations in which mathematics can be applied. The curriculum recognises the need to contextualise mathematics and as such one of its major tenets is the necessity to integrate with other areas of the curriculum, which creates and promotes boundless opportunities for application. Knowledge of application is also widely emphasised in Irish policy documents, particularly in relation to teacher knowledge. For example, a national strategy for improving literacy and numeracy states that such knowledge should be included in Initial Teacher Education courses as well as other forms of professional development for in-service teachers (Department of Education and Skills [DES], 2011). It notes that teachers need to be equipped with the necessary resources to develop their conceptual understanding as well as being able to apply this across a range of everyday applications.

2.2. The ATEE Ratio Project

As pointed out in the Introduction, the ATEE Ratio Project was instigated in 2011 at the ATEE Annual Conference. The work was undertaken by members of the Research and Development Community (RDC) “Science and Mathematics Education” with the intention that it would lead to an ongoing project under the aegis of the RDC (Berenson, Oldham, Price, & Leite, 2013). The original research questions were:

a) What meanings do prospective teachers at primary and secondary levels in [specific institutions] give to the term “ratio”?
b) What multiple representations do these prospective teachers associate with the term “ratio”?
c) Do the prospective teachers’ descriptive meanings and representations indicate different levels of understanding for teaching ratio?

For data collection, an instrument was designed to examine respondents’ CK by addressing their knowledge of both representations and applications, highlighted above as important features. It contained five questions:

1. What does the term “ratio” mean to you?
2a. When do you use ratios?
2b. Who else uses ratios?
3. How do you represent a ratio using mathematical symbols?
4. Draw several representations of how ratios are used.

The intention was that it could be administered in ten to fifteen minutes, for example at the end of a lecture to prospective teachers of mathematics and science or prospective primary teachers. Data were collected in two institutions in the USA, one in Ireland and one (using a translation version of the instrument) in Portugal, and responses from 158 participants were analysed.

Some provided rich explanations and illustrations, but the responses of others were relatively thin, and many referred only to occurrences of ratio that typically appear in middle school curricula rather than at more advanced levels. Overall, the findings offered evidence that some prospective teachers' CK was comparatively poor.

Further analysis of the responses especially to questions 1 and 4, using a grounded theory approach, identified three emergent themes for the meanings ascribed to ratio.

These were: indication of two distinct variables (typically by naming comparison or relationship); reference to uses or applications or special types of ratio (such as rate, scale, or proportion); and focus on part-whole relationships (as in fractions and decimals).

Responses in the first category, together with the use of (correct) multiple representations and reference to appropriate applications, were conjectured to be indicators of Skemp's (1976) relational understanding; this conjecture provides a possible theoretical framework for further studies. The findings were reported at the ATEE Annual Conference in 2012 (Berenson et al., 2013).

Some limitations in the original study were identified, and changes were made to the instrument and its usage for later work. Issues and changes over time were documented, for example by Oldham, Stafford, and O'Dowd (2015).

First, attempts to ascribe different levels of understanding to individual participants, addressing research question (c), were not successful; the responses were too brief and in some cases too inconsistent to allow for reliable categorisation. Subsequent work has therefore focused on the CK displayed by the participants as a group, rather than on classifying the levels of understanding of individuals.

Stemming from the work done for the original paper, several studies were carried out by members of the RDC in their own counties (Amit, 2015; Fernandes & Leite, 2015; Leite, Fernandes, Viseu, & Gea Serrano, 2016; Oldham & Ni Shuilleabhain, 2014; Oldham et al., 2015; Oldham et al., 2016; Price, 2013, 2014a, 2014b, 2015; Stafford et al., 2015; Veiga, Fernandes, & Leite, 2017).

Recent Portuguese studies have addressed the topic without using the research instrument; rather, they have collected solutions to specific exercises on ratio so as to examine skill as well as knowledge (Leite et al., 2016; Veiga et al., 2017). All the studies indicated that at least some prospective teachers had poor CK of aspects of ratio. Collectively, they highlight the need for a greater focus on developing understanding of the concept in teacher education courses.

3. Methodology

This section describes the methodology for data collection for the current (2017) Irish study. The wording of question 2 in the research instrument was modified slightly for this study to include a higher degree of specificity. The aim of this was to reduce misinterpretation of the questions and elicit the most relevant and meaningful responses from participants. For this paper the focus is on question 2, in order to determine the PPSTs' ascribed meanings and applications of ratio in everyday life context. The current instrument is as follows:

1. What does the term “ratio” mean to you?
2. a. When do you use ratios in your everyday life?
b. Who else uses ratios in their everyday life? Please provide examples.
3. How do you represent a ratio using mathematical symbols?
4. What representations – drawings, charts, graphs, words – might you use to explain ratio and show how it is used?

3.1. Sample

The participants were 34 graduate student-teachers preparing for primary teaching (4- 12 year olds). Mathematics is just one of 14 curricular areas these PPSTs are required to teach, although there is a particularly strong focus on mathematics in recent years (for example: DES, 2015, 2016). The participants were in the second year of a two-year Professional Masters in Primary Education (PMEd) degree at the first two authors’ institution. They have varied academic backgrounds (being graduates in many subjects) and varied levels of achievement in mathematics. As part of the PMEd degree, PPSTs take two mathematics modules. One of these is based on teaching methodologies and pedagogy (PK and PCK), while the other is focused mainly on content knowledge understanding (CK). In the first year of the degree, the PPSTs’ learning of content was focused on the whole numbers and their operations. Ratio was not explicitly taught, although it may have been casually referenced during lectures.

3.2. Discussion and “Think, Pair, Share” as a methodology

It was decided to use collaborative discussion for data generation because this methodology allowed the researchers (the first two authors) to immerse themselves fully in the research process and “explore the ordinary talk and everyday explanations” of the participants (Cohen, Manion, & Morrison, 2000, p. 298). The researchers were mere moderators of the discussion, mainly observing and at times asking questions for clarification but being careful not to put leading questions while remaining cognisant of the fact that personal bias may impact on the validity and quality of the data. Observation of this kind allowed the researchers to construct real meaning from the discussion between the participants. Although not a narrative enquiry in the strict sense of the word, it did allow the researchers to capture participants’ knowledge of ratio through their many years of “lived experiences”, both as students and more recently as educators, within the Irish education system (Clandinin, Pushor, & Murray Orr, 2007, p. 22).

This particular methodology was also selected because of its sound educational benefits. Discussion as a teaching methodology is rooted in social constructivist pedagogy, which is both promoted and practised in the researchers' institutions.

Central to this approach is the idea of social interaction and Vygotsky's (1978) Zone of Proximal Development, or ZPD. In his research on the relationship between childlearning and development, Vygotsky suggested there are two distinct levels of cognitive development in children. The first is what he termed the actual developmental level, defined as "the level of development of a child's mental function determined by independent problem solving" (p. 37). The second level of potential development was defined as "that which a child can achieve if given the benefit of support during the task. It is the ability to solve problems under the guidance or in collaboration with more capable peers" (p. 37). He proposed that there is always a difference between these two forms of development, one being individual and the other as collaboration within a group, and that this gap is an indicator of the cognitive functions that have not yet matured, but are in a type of "embryonic state" (p. 86). It is this ZPD that is critical for teaching and learning as it allows learners to reach their potential cognitive growth via collaboration with others. In this sense, ZPD is defined by Vygotsky as: "the distance between the actual developmental level as determined by individual problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers" (p. 38). Although ZPD is typically referred to in relation to children's cognitive development, it is equally relevant to adult learners. For example, Taylor, King and Pinsent (2002) carried out a case study investigating how adult learners improved literacy skills through collaborative peer work, and how these collaborative practices aid learners in their journey through the ZPD.

From this perspective, learning is an inherently social process and it is this element of social interaction that helps children in progressing through the ZPD to reach their potential cognitive development. For this reason, the authors concluded that "Think, Pair, Share" as developed by Lyman (1981) was the best methodology to incorporate discussion in a natural and inclusive way. Discussion in this way also encompasses several of the core principles of the Irish Primary School Curriculum and Mathematics Curriculum (NCCA, 1999a, 1999b); therefore, the researchers felt it was necessary to model such teaching methodologies.

Finally, "Think, Pair, Share" gives students the opportunity to make sense of the mathematics they are engaged in (Reinhart, 2002). It problematises ideas and concepts, leading to an inevitable struggle which is necessary for real learning to happen.

Additionally, collaborative learning through "Think, Pair, Share" is more effective for developing problem solving and critical thinking skills than traditional learning. Again, this is because the process makes knowledge problematic and contested which encourages students to critically discuss, clarify and evaluate peers' opinions (Lin, 2015). Throughout this process, existing ideas are used to make sense of new situations and in this sense new meaning is constructed by making links between new and existing knowledge. The process encourages new ideas, or old ideas presented in new ways, which may be accepted, rejected, critiqued and/or re-evaluated by members of the group.

3.3. Data collection

The research was carried out over the course of two consecutive mathematics lectures in the first two authors’ institution. During the first hour, the research instrument was administered at the beginning of the lecture. Participants were grouped into six groups of either five or six. “Think, Pair, Share” methodology was utilised and this involved the participants firstly taking approximately 10 to 15 minutes to complete the worksheet individually (Think). After this was completed, the participants were asked to discuss their work briefly with one other person within their group (Pair). Then, each participant was asked to share their work with the remaining members in the group (Share). Each group then created a mind map to represent their collective knowledge of ratio. The process culminated in a whole-class discussion. This discussion was generated by each group verbally presenting their posters to the other groups, who were encouraged to give critical responses. Of the two researchers present, one mediated the discussion while the other took notes based on participants’ responses. After the group discussion, participants were given approximately ten minutes at the end of the first hour to write a reflection on how the “Think, Pair, Share” activity enhanced or did not enhance their understanding of the concept and application of ratio in everyday life.

It was decided to follow up the main “Think, Pair, Share” activity with a lecture focusing on relational understanding of ratio. This decision was made because it was anticipated that participants would have ratio fresh in their minds, may have some burning questions on the topic, and consequently may be more motivated to engage in the topic. Through this, further discussion was generated within and between the groups of PPSTs and the lecturer. Significant events were noted during this hour. PPSTs were again given an opportunity to reflect on the process.

3.4. Data analysis

For each part of Question 2, the data were separately studied and tallied by two of the authors and tentative categories were agreed. Categories chosen emerged from the data and were similar to those in previous studies (Fernandes & Leite, 2015; Oldham et al., 2015; Stafford et al., 2015; Oldham et al., 2016). The data were then coded independently, checked and reconciled. Broadening categories as per the previous studies helped with reaching agreement, while recognising limitations in the data. As some of these responses were difficult to interpret, reference was also made to the answers to the other questions (Questions 1, 3 and 4), as well as the PPSTs' reflections and notes from the group discussions to better ascertain what was intended. The same process of analysis was applied to the PPSTs' reflections. Reflections were grouped into those that found the “Think, Pair, Share” activity helped their understanding and applications and those that did not.

4. Results

There were 32 individual responses to the instrument.

4.1. Question 2a

In response to the question “When do you use ratios in your everyday life?,” PPSTs gave a range of answers (Table 1). For some categories, some PPSTs provided more than one example. The majority of PPSTs (87.5%) could provide at least one correct application, mostly cooking/baking, but many of those (60.7%) also gave an incorrect or incomprehensible application (Table 2). Problematic responses such as “filling the car with petrol,” “checking how long is left in class” and “time” were excluded as incorrect or incomprehensible. Four PPSTs provided no correct applications and at least 1 incorrect or incomprehensible application. Only 14 PPSTs (43.8%) could give more than one correct application; however, 5 of these also gave at least one incorrect or incomprehensible application.

Table 1: Categorised responses to question 2a: “When do you use ratio in your everyday life?”

CATEGORY	NUMBER OF EXAMPLES
Statistics/Surveys/Research	3
Dividing	7
Gambling/Betting/Racing	9
Other	10
Cooking/Baking	17
Incorrect/Incomprehensible	26

Table 2: Number of PPSTs giving correct/incorrect applications for question 2a

CORRECT	INCORRECT/ INCOMPREHENSIBLE	NUMBER OF PPSTS
0	0	0
	1 or more	4
1	0	2
	1 or more	12
2	0	6
	1 or more	4
3	0	3
	1 or more	1
	Total	**32**

4.2. Question 2b

In response to the question "Who else uses ratios in their everyday life? Please provide examples," (Table 3), "teachers" was the most common answer, followed by "professional jobs". Many more of PPSTs could provide multiple applications for ratio (81.25%) for this question (Table 4) than the previous question, with the majority (75%) able to provide 3 or more correct applications. Only 4 PPSTs gave just one correct application. Problematic responses such as "children," "workers" and "parents dividing dinners between family" were excluded as incorrect or incomprehensible; however, only 5 PPSTs gave such answers.

Table 3: Categorised responses to question 2b: "Who else uses ratios in their everyday life?"

CATEGORY	NUMBER OF EXAMPLES
Everyone	2
Statisticians/Surveys/Analysts/Researchers	3
Incorrect/incomprehensible	6
Mathematicians/ Scientists	9
Builders/Construction	10
Cooks/Bakers/Chefs	12
Bookies/Betting/Racing	13
Others	13
Professions	18
Teachers	19

Table 4: Number of PPSTs giving correct/incorrect applications for question 2b

CORRECT	INCORRECT/ INCOMPREHENSIBLE	NUMBER OF PPSTS
0	0	0
	1 or more	0
1	0	3
	1 or more	1
2	0	3
	1 or more	1
3	0	11
	1 or more	1
4	0	4
	1 or more	0
5	0	5
	1 or more	2
6	0	1
	1 or more	0
	Total	**32**

4.3. Reflections after the "Think, Pair, Share" discussion

Most PPSTs (75%) responded that the "Think, Pair, Share" discussion enhanced their understanding of the concept of ratio and its application to real-life contexts. Many commented that the discussion opened their minds to a wider range of applications of ratio in their everyday lives and how it is used in other people's lives. One illuminating response is shown in Figure 1.

> "...enhanced my understanding of ratio as it allowed me to take my prior knowledge and the prior knowledge of other students in the group and pool it together to form a broader, more detailed understanding of ratio, and when and how to apply it to everyday life."

Figure 1: PPST reflection

While PPSTs were initially unsure about their own knowledge and confused after the first class, most reported that the exercise helped build on their understanding and knowledge of ratio and most commented on the benefits of hearing one another's ideas. Some positive reflections are shown in Figure 2.

> "it got me thinking more about the topic"
>
> "it stimulated opinions and discussion among us"
>
> "similar ideas give confidence"
>
> "being challenged enhanced ideas"
>
> " it gave an insight into the depth at which ratio is involved in everyday life"

Figure 2: PPSTs' reflections – positive

The other 25% of PPSTs said they felt more confused about the concept of ratio after the "Think, Pair, Share" exercise. Relevant reflections are shown in Figure 3.

> "left me more confused about the topic"
>
> "left us in limbo"
>
> "made the concept of ratio more complicated"

Figure 3: PPSTs' reflections – negative

4.4. Reflections from the follow-up class

PPSTs' reflections after the follow-up Relational Understanding class were also predominantly positive, with only 2 still showing confusion (Figure 4). Many commented that having the class immediately after the "Think, Pair, Share" exercise was particularly helpful.

Figure 4: PPSTs' reflections after the follow-up class

"I now feel more comfortable with what ratio is and answering questions related to ratio."

".. the follow up class helped build on my understanding and knowledge of ratio and I felt much more confident after this class."

"... gave us the answers to the questions we were trying to make sense of in the first session. It allowed us to talk through things in a guided way."

"... follow up class addressed all of the questions from the previous class and gave opportunity to ask further questions."

"More confused. Didn't really clarify if we were right or wrong."

5. Discussion

Initial results indicate that many PPSTs had limited understanding of the application of ratio in a real-life context, both for themselves and others. This supports findings from previous studies of the ATEE Ratio Project, cited above. The categories identified by PPSTs in this study were similar to those in previous Irish and international studies (Stafford et al., 2015). "Teachers" was the most popular answer, higher than for previous groups probably due to the fact that the they had already completed one year of their teaching degree. Horse racing was again a popular category, perhaps reflecting the importance of horse racing in Irish culture. The findings of this study also suggests that the use of "Think, Pair, Share" and discussion as methodology can be beneficial for facilitating PPSTs' understanding of the concept of ratio and that embedding mathematical concepts such as ratio in everyday context can also be useful for their understanding. This should therefore help them in supporting the development of understanding of the concept for children in their classes in the future. The discussion allowed lecturers to gain a deeper understanding of the PPSTs' applications than using the instrument alone, and is also good for debate around meaning and helping to bring misconceptions and confusion in PPSTs' understanding to the forefront. However, findings also highlight the importance of the follow-up lecture after the discussion with teacher input to solidify understanding. The collaboration between the mathematics teacher educator and mathematics educator on this resarch also supports the call by Leite et al. (2016) for the need for mathematics teacher educators and mathematics educators to work together "to foster the development of PPST content knowledge of ratio" (p. 95).

It should be highlighted that the researchers' decision to give no input into the "Think, Pair, Share" methodology was useful for PPSTs only because they had already studied ratio during their primary and post-primary education. The resulting confusion was a necessary by-product of the methodology, which ultimately ensured the maximum effectiveness of the follow-up class. It is hoped that learning, or re-learning as the case may be, a concept in this way will help to minimise the effects of Lortie's (1975) apprenticeship of observation in future mathematics classrooms. On the other hand, it would not be appropriate to use a purely "discovery learning" type methodology like this for a topic that PPSTs have not studied previously.

It is necessary to acknowledge limitation of the study: The group nature and hence seating arrangement for the exercise may have resulted in PPSTs sharing answers during the "Think" phase of the exercise. Some PPSTs put a question mark after their entry suggesting they were not confident of their answer. As the PPSTs in this study have undergraduate degrees in a wide range of subject areas and many have already worked in those fields, areas of the PPSTs' undergraduate degree subjects may have also influenced work related responses.

6. Conclusion

This study sought to answer the questions: "What personal uses and users did PPSTs identify for ratio?" and "Does using discussion and application to real-life contexts facilitate PPSTs understanding of the concept of ratio?" The categories of uses and users identified by PPSTs in this study were similar to those of previous studies in the project. However, many PPSTs had difficulty with providing multiple correct applications for ratio in their own lives suggesting that work needs to be done on helping PPSTs understand how ratio and other mathematical concepts are useful in their everyday lives. Further analysis of all of the data is necessary to give a fuller picture of the results. The findings suggest that "Think, Pair, Share" discussion has potential for use as a methodology in the introduction of other mathematical concepts to PPSTs. As this is a small scale study, it needs to be replicated on a larger scale to ascertain if the findings are applicable in other contexts.

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