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Optimal trajectories, nonlinear models and constraints in wave energy device control

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Abstract: The optimal control problem for a generic, one-degree of freedom Wave Energy Converter (WEC) with dynamical nonlinearities is formulated in the frequency-domain. Previous research, concerning more specifically a heaving point-absorber with nonlinear restoring force, shows that the unconstrained optimal velocity trajectory is influenced neither by the linear inertial terms, nor by the linear or nonlinear static forces. Further to this result, in this paper, we examine the influence of velocity-dependent nonlinear forces on the optimal trajectory, as well as the effect of physical system constraints. In particular, we show that, under state constraints (e.g. position and velocity limitations), the optimal velocity trajectory remains uninfluenced by static forces; but this is no longer true for constraints involving the control force, such as force limitation and passivity constraints. In addition, unlike static terms and linear inertial terms, the velocity-dependent forces, such as viscous drag, significantly influence the optimal velocity trajectory, regardless of constraints, and must be carefully modelled at the control design stage. In any case, even when the optimal velocity trajectory is not affected by some of the forces considered, the optimal control force required to achieve it depends on all the model dynamics (inertial terms, velocity-dependent and static forces). Numerical simulations, in the specific case of a heaving point absorber, are used to validate and illustrate the theoretical results.

Keywords: Marine systems, non-linear systems, power generation, optimal trajectory, constraints

1. INTRODUCTION

In order to improve the economic competitiveness of wave energy converters (WECs), controlling the device motion so as to maximize the average absorbed power is an interesting path to investigate, see for example Ringwood et al. (2014). In particular, optimal control, assuming knowledge of future wave elevation, is the object of this paper.

Optimal control must accommodate accurate models of the WEC dynamics. In particular, it has been highlighted, e.g. by Penalba Retes et al. (2015), that hydrodynamic nonlinearities become more significant under controlled conditions than, for example, when the PTO is a simple passive damper. Indeed, the control generally has the effect of magnifying the range of positions and velocities, hence accentuating effects such as nonlinear hydrostatic restoring force and quadratic viscous damping. Furthermore, nonlinear dynamics may also stem from the characteristics of the PTO machinery or other effects such as moorings. In this paper, a generic formulation for nonlinear forces, depending on the WEC position and/or velocity, is considered. Besides, the analysis is restricted to a WEC with one degree-of-freedom (DoF).

In this paper, a nonlinear frequency-domain formulation provides a unified framework, both for a relatively simple derivation of the theoretical results with respect to optimal WEC control, and for the efficient computation of the optimal control force and trajectories in the chosen numerical examples. The practical issues of control implementation, such as wave-by-wave forecasting, receding-horizon control or trajectory tracking, are beyond the scope of this study.

The paper is organised as follows: Section 2 presents the frequency-domain formulation of the WEC dynamical equations and control problem. In Section 3, the theoretical results in relation to the optimal velocity trajectory and control forces are derived, depending on the characteristics of the hydrodynamic forces considered, and on the existence and nature of the system constraints. Section 4 presents the case study considered, and the corresponding numerical results. Finally, Section 5 discusses the practical implications of the theoretical results and the possibility to extend them to more general classes of WEC models, and formulates a few recommendations.

2. FREQUENCY-DOMAIN PROBLEM FORMULATION

2.1 Dynamical equations in the time domain

Let us consider a WEC with one degree of freedom ζ, subject to control force u. Classically, the various hydrodynamic forces acting on the WEC are linearly separated as

$$\mu\ddot{\zeta} - f_{rad} - f_{res} - f_e - u = 0 \quad (1)$$

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where μ is the WEC inertia, f_{rad} corresponds to radiation forces, f_{res} is a hydrostatic restoring force term, f_e is the wave excitation force and u is the PTO control force. The fully linear version of (1) is the well-known Cummins equation (see Cummins (1962)), where the hydrostatic force is modelled as a linear restoring term, and the radiation force can be computed as the sum of an inertial term and a convolution product between the past values of the velocity and the radiation impulse response function k_{rad}:

$$(\mu+\mu_\infty)\ddot{\zeta}+\int_{-\infty}^{t} k_{rad}(t-\tau)\dot{\zeta}(\tau)d\tau+k_h\zeta-f_e-u=0 \quad (2)$$

In order to take into account more specific hydrodynamic or mechanical effects, a nonlinear modification or extension of (2) can be derived, with nonlinear forces depending on the device position and velocity. The case study presented in Section 4 will provide practical examples of such nonlinear extensions of Cummins equation.

In a general way, (2) and its nonlinear modifications may be written as

$$g_l(\zeta,\dot{\zeta},\ddot{\zeta})-f_{nl}(\zeta,\dot{\zeta})-f_e(t)-u(t)=0. \quad (3)$$

In the modified or extended Cummins equation, g_l and f_{nl} unite all the terms that depend on ζ and its derivatives, in a linear and nonlinear way respectively [1] . f_e represents an external, additive input. The PTO or control force remains represented separately given its role in the optimal control problem, which will be the subject of Section 2.3.

2.2 Dynamical equation in the frequency domain

Assuming periodic inputs - f_e, u - and outputs - ζ and its derivatives - for the system represented in (3), the whole equation may be transcribed into the frequency domain, using harmonic balance over a finite, but arbitrarily big, number of sinusoids. This approach, proposed for example by Spanos et al. (2002), implies that, in spite of the presence of nonlinear forces, the input and output of the system can be reasonably well represented through truncated sums of harmonic sinusoids.

Certainly, some systems, for which the output ζ or its derivatives are discontinuous, may not be easily represented in this way in an actual numerical model [2] . But, in spite of its inherent theoretical limitations, the frequency domain representation is of significant practical interest. In our case, the resulting formalism is much lighter than in the time-domain; in particular, no initial condition or transient state are to be considered. Additionally, dealing with periodic signals greatly simplifies the derivation and understandability of some theoretical results. Finally, in practice, the optimal trajectories and control forces can be numerically derived in the frequency-domain relatively easily.

[1] For the linear terms, the letter g is used instead of f, since they include inertial terms which are not forces properly speaking.

[2] Actually, some (unrealistic) systems may also not even admit a periodic output solution, for example when dissipative terms are so small, that the motion amplitude does not have a steady-state regime.

Let us then assume that the inputs and outputs of the system are zero-mean, periodic signals of period T. Then, f_e, u and ζ can be represented through vectors F_e, U and X of $\mathbf{R}^{2N}$, such that:

$$f_e(t)\approx\sum_{k=1}^{N} F_{e_k}\cos(\omega_k t)+F_{e_{k+N}}\sin(\omega_k t) \quad (4)$$

$$u(t)\approx\sum_{k=1}^{N} U_k\cos(\omega_k t)+U_{k+N}\sin(\omega_k t) \quad (5)$$

$$\zeta(t)\approx\sum_{k=1}^{N} X_k\cos(\omega_k t)+X_{k+N}\sin(\omega_k t) \quad (6)$$

where $\forall k\in 1...N, \omega_k=k\omega_0$ and $\omega_0=\frac{1}{T}$.

The transcription of (3) into the frequency domain is composed of four terms:

- The terms of g_l, which are linearly-dependent on ζ and its derivatives, are transcribed in matrix form as $G_l(X)=MX$. In particular, the linear time-domain radiation terms simplify into the frequency-dependent radiation added mass and damping $A_{rad}(\omega)$ and $B_{rad}(\omega)$. Typically, when both radiation and hydrostatic restoring forces are linearly modelled, $\forall i,j\in\{1...N\}^2$,
$$M_{ij}=\begin{cases}-\omega_i^2(\mu+A_{rad}(\omega_i))+k_h, & i=j\\ 0, & i\neq j\end{cases}$$
$$M_{i+N,j+N}=M_{i,j}$$
$$M_{i,j+N}=\begin{cases}\omega_i B_{rad}(\omega_i), & i=j\\ 0, & i\neq j\end{cases}$$
$$M_{i+N,j}=-M_{i,j+N}$$
- The nonlinear terms of f_{nl} are transcribed as a vector $F_{nl}(X)$ such that $\forall i\in\{1...N\}$
$$F_{nl_i}(X)=\frac{2}{T}\int_0^T f_{nl}(\zeta_X,\dot{\zeta}_X)\cos(\omega_i t)dt$$
$$F_{nl_{i+N}}(X)=\frac{2}{T}\int_0^T f_{nl}(\zeta_X,\dot{\zeta}_X)\sin(\omega_i t)dt$$
where the subscript X denotes the dependence of the (time-domain) periodic solution ζ, and its derivatives, on the choice of coefficients contained in X.
- f_e and f_c are transcribed as F_e and U as in (4)-(5).

Then (3) becomes

$$G(X,U):=MX-F_{nl}(X)-F_e-U=0_{\mathbf{R}^{2N}} \quad (7)$$

Of course, depending on how the various hydrodynamic force terms are modelled, the expression for M may vary slightly. For example, if the hydrostatic restoring force is modelled non-linearly, the terms in k_h on the diagonal of M disappear, and are replaced with additional nonlinear terms in F_{nl}.

2.3 The control problem

The primary objective of optimal control is to transmit as much power as possible from the waves to the PTO system. With the frequency-domain formalism presented in Section 2.2, the function to maximize is the average power extracted over one period of the signal, which is equivalent to the following minimisation problem:

$$\begin{aligned} &\min P(X,U) = \frac{1}{T}\int_0^T \dot{\zeta}_X(t)u_U(t)dt \\ &\text{s.t. } G(X,U) = 0_{\mathbf{R}^{2N}} \end{aligned} \tag{8}$$

The integral in (8) can be computed as

$$P(X,U) = \frac{1}{2}X^{\mathrm{T}}D^{\mathrm{T}}U. \tag{9}$$

where DX is the frequency-domain projection of $\dot{\zeta}$ with

$$D = \begin{pmatrix} 0 & \cdots & 0 & \omega_1 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & 0 & \cdots & \omega_N \\ -\omega_1 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & -\omega_N & 0 & \cdots & 0 \end{pmatrix} \tag{10}$$

Furthermore, one can notice that the constraint $G(X,U) = 0_{\mathbf{R}^{2N}}$ defines a function

$$\begin{cases} \tilde{U} : \mathbf{R}^{2N} \to \mathbf{R}^{2N} \\ X \mapsto \tilde{U}(X) = MX - F_{nl}(X) - F_e \end{cases} \tag{11}$$

Combining (9) and (11), the minimisation problem (8) becomes

$$\min \tilde{P}(X) := X^{\mathrm{T}}D^{\mathrm{T}}\tilde{U}(X) \tag{12}$$

At this stage, it is interesting to note that the terms on the diagonal of M, corresponding to the linear inertial and stiffness terms, disappear from $X^{\mathrm{T}}D^{\mathrm{T}}MX$, so that

$$X^{\mathrm{T}}D^{\mathrm{T}}MX = X^{\mathrm{T}}AX \tag{13}$$

where

$$A = \frac{1}{2}D^{\mathrm{T}}(M - M^{\mathrm{T}})$$

A is a diagonal matrix and, typically, $\forall i \in \{1...N\}, A_{i,i} = A_{i+N,i+N} = -\omega_i^2 B_{rad}(\omega_i)$.

It is now convenient to further decompose $\tilde{P}(X)$ into

$$\tilde{P}(X) = \tilde{P}_l(X) + \tilde{P}_{nl}(X), \tag{14}$$

with

$$\begin{cases} \tilde{P}_l(X) = X^{\mathrm{T}}AX - X^{\mathrm{T}}D^{\mathrm{T}}F_e \\ \tilde{P}_{nl}(X) = -X^{\mathrm{T}}D^{\mathrm{T}}F_{nl}(X) = -\dfrac{2}{T}\displaystyle\int_0^T \dot{\zeta}_X f_{nl}(\zeta_X, \dot{\zeta}_X)dt \end{cases}$$

The first-order optimality condition for (12) is written as

$$\nabla\tilde{P}|_X = \nabla\tilde{P}_l|_X + \nabla\tilde{P}_{nl}|_X = 0_{\mathbf{R}^{2N}} \tag{15}$$

with

$$\begin{cases} \nabla\tilde{P}_l|_X = 2AX - D^{\mathrm{T}}F_e \\ \nabla\tilde{P}_{nl}|_X = -(D^{\mathrm{T}}F_{nl} + J_{F_{nl}}(X)^{\mathrm{T}}DX) \end{cases}$$

In a purely linear case, (15) reduces to solving $\nabla\tilde{P}_l|_X = 0_{\mathbf{R}^{2N}}$, which is a straightforward matrix inversion problem.

3. CONTROL SOLUTIONS: THEORETICAL RESULTS

3.1 Static forces

Here, the case of purely static nonlinear forces is considered, so that

$$f_{nl} = f(\zeta).$$

Furthermore, f is assumed to be continuous, and thus to possess a primitive function $f^{(-1)}$, over its interval of definition. Then:

$$\tilde{P}_{nl}(X) = -\frac{2}{T}\Big[f^{(-1)}\big(\zeta_X(t)\big)\Big]_0^T = 0 \tag{16}$$

due to the periodicity of the solution ζ_X. In other words, the mechanical work of the static forces over the periodic WEC trajectory is cancelled out. This important result had already been formulated in the time domain by Nielsen et al. (2013), with an infinite time-horizon.

Finally, the problem to solve, in order to obtain the optimal trajectory, is the same matrix inversion problem as in the linear case. The optimal velocity trajectory and the excitation forces are then related by

$$2BDX_{opt} = F_e \tag{17}$$

where B is the diagonal matrix with elements

$$B_{i,i} = B_{i+N,i+N} = B_{rad}(\omega_i), \forall i \in \{1...N\}$$

As stressed by Nielsen et al. (2013), Equation (17) is a generalisation, to an irregular sea state, of the well-known condition that the velocity must be in phase with the excitation force. Furthermore, the optimal power does not depend on the nonlinear forces, and is given by

$$P_{opt} = \frac{1}{8}\sum_{k=1}^{N} B_{rad}(\omega_k)(F_{e_k}^2 + F_{e_{k+N}}^2) \tag{18}$$

Although the optimal power and velocity trajectory do not depend on inertia and static terms, from (7) the optimal control force necessary to achieve the optimal velocity trajectory *does* depend on all the WEC dynamics:

$$U_{opt} = MX_{opt} - F_{nl}(X_{opt}) - F_e \tag{19}$$

Let us note that, if additional, linear, velocity-dependent terms are present in the equations of motion (e.g. a linear viscous damping term), then (17) and (18) can be generalised accordingly by complementing $B_{rad}(\omega)$ with more terms.

3.2 Velocity-dependent forces

Now, let us consider a more general situation in which the nonlinear forces is both static and dynamic, i.e.

$$f_{nl} = f(\zeta, \dot{\zeta}).$$

An interesting and common case to consider first is when the nonlinear forces are of the form

$$f_{nl} = f(\zeta, \dot{\zeta}) = f_\zeta(\zeta) + f_{\dot{\zeta}}(\dot{\zeta}) \tag{20}$$

Then, it is easy to show that, similarly to Section 3.1, the term f_ζ disappears from the objective function. In contrast, the velocity-dependant[3] term $f_{\dot{\zeta}}$ does not simplify.

[3] the term velocity is here to be understood in a broad sense and can be, for example, an angular velocity

More precisely, we have

$$\frac{\partial \tilde{P}_{nl}}{\partial X_k} = \int_0^T \omega_k \sin(\omega_k t)(\dot{\zeta} f'_{\dot{\zeta}}(\dot{\zeta}_X) + f_{\dot{\zeta}}(\dot{\zeta}_X))dt$$
$$\frac{\partial \tilde{P}_{nl}}{\partial X_{k+N}} = -\int_0^T \omega_k \cos(\omega_k t)(\dot{\zeta} f'_{\dot{\zeta}}(\dot{\zeta}_X) + f_{\dot{\zeta}}(\dot{\zeta}_X))dt \tag{21}$$

so that the velocity-dependent nonlinear forces indeed enter into account when solving (15).

Finally, all inertial terms and position-dependent forces disappear in (12). Conversely, the velocity-dependent nonlinear forces *do not* simplify, and have therefore an influence on the optimal trajectory, optimal control force and optimal power, as will be illustrated in Section 4.2.

Furthermore, when f jointly depends on ζ and $\dot{\zeta}$, i.e. the static and dynamic forces cannot be separated as in (20), no such simplifications can be expected. Then, all the nonlinear force terms have to be taken into account when solving for the optimal trajectory.

3.3 Position and velocity limitations

Optimal control of a WEC can give rise to exceedingly wide motions. It may then be necessary to implement limitations on position or velocity, either physically (e.g. with end-stop motions), or within the control directly.

In the latter case, the constraints can be expressed on an arbitrarily large, but discrete subset of $[0;T]$, say $t_{i,i\in\{1...m\}}$. Let us introduce $\Phi \in \mathbf{R}^{2N\times m}$ defined as

$$\forall k \in \{1, ...N\}, \begin{cases} \Phi_{k,i} = \cos(\omega_k t_i) \\ \Phi_{k+N,i} = \sin(\omega_k t_i) \end{cases} \tag{22}$$

Let us denote p_x and p_v the numbers of inequality constraints that should be satisfied at each time, for position and velocity respectively (typically, $p_x = p_v = 2$ to express a lower and upper limitation). Let us define $c_x : \mathbf{R}^m \to \mathbf{R}^{p_x m}$ and $c_v : \mathbf{R}^m \to \mathbf{R}^{p_x m}$ that express the inequality constraints on the position and velocity values, at the m selected points in time. The minimisation problem in (12) with position and velocity limitations can be written as:

$$\begin{aligned} &\min \tilde{P}(X) \\ &\text{s.t. } \begin{cases} c_x\big(\Phi^{\mathrm{T}} X\big) \leq 0_{\mathbf{R}^{p_x m}} \\ c_v\big(\Phi^{\mathrm{T}} D X\big) \leq 0_{\mathbf{R}^{p_v m}} \end{cases} \end{aligned} \tag{23}$$

It was shown in Sections 3.1 and 3.2 that the objective function does not contain any inertial term, or position-dependent force terms. Thus, only velocity-dependent forces enter into account in problem (23). The results presented in Sections 3.1 and 3.2 then remain true when the system is constrained in position and velocity.

3.4 Force-related constraints

Let us now introduce force-related constraints. In their most simple form, those are force limitations, so that, similarly to (22) and (23), they are expressed as

$$c_u\big(\Phi^{\mathrm{T}} \tilde{U}(X)\big) \leq 0_{\mathbf{R}^{p_u m}} \tag{24}$$

where $\tilde{U}$ is defined as in (11).

Assuming that c_u is differentiable on $\mathbf{R}^m$, and introducing the vector of nonnegative Lagrange multipliers $\lambda_u \in \mathbf{R}^{p_u m}$ corresponding to the force inequality constraints, the first-order optimality conditions (see for example Bazaraa et al. (2013)) read

$$\begin{aligned} \nabla \tilde{P}|_X + J_{\tilde{U}}(X)^{\mathrm{T}} \Phi J_{c_u}\big(\Phi^{\mathrm{T}} \tilde{U}(X)\big)\lambda_u = 0_{\mathbf{R}^{2N}} \\ \lambda_u \odot c_u\big(\Phi^{\mathrm{T}} \tilde{U}(X)\big) = 0_{\mathbf{R}^{p_u m}} \end{aligned} \tag{25}$$

where $\odot$ denotes element-wise product.

Given the expression for $\tilde{U}$ in (11), in general all linear and nonlinear force terms appear when solving (25). The results presented in Sections 3.1 and 3.2 are then not true anymore when the constraints on the system involve the control force - unless, for example, the force constraints are inactive at all points of the signal period.

4. CASE STUDY AND NUMERICAL RESULTS

4.1 Case study: Spherical heaving point-absorber

For illustration of the results presented in Section 3, a 5m diameter, spherical point-absorber, restricted to heave motions, is chosen. The sphere density is half of the water density, so that the device centre of gravity is on the plane $z = 0$. Several models for the hydrodynamic forces are implemented, and the optimal control problem is solved for each of those models with the same excitation force signal.

In all the models, the radiation and excitation forces are represented linearly. For those two forces, in solving (12) and its constrained variants, only the frequency-domain coefficients $A_{rad}(\omega)$, $B_{rad}(\omega)$ and $F_e(\omega)$, computed with the hydrodynamic software NEMOH [4], are necessary to represent the linear radiation and excitation forces.

The other forces are represented differently depending on the model:

- In model (a), the restoring force is represented linearly, with a hydrostatic stiffness coefficient k_h computed with NEMOH.
- In model (b), the restoring force is nonlinear, taking into account the actual position of the device with respect to the plane $z = 0$.
- Model (c) is similar to model (b), with the addition of a simple quadratic viscous drag term expressed as

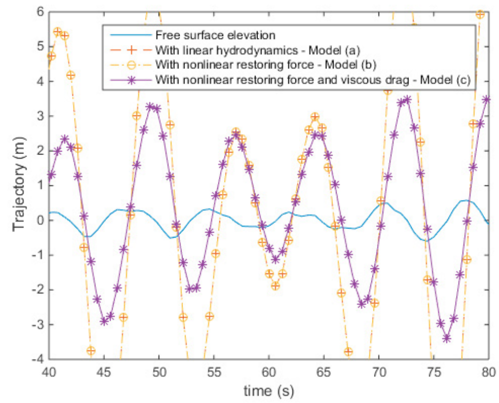
$$f_{\dot{\zeta}} = -b_v \dot{\zeta}|\dot{\zeta}| \tag{26}$$

Three constraint configurations are explored:

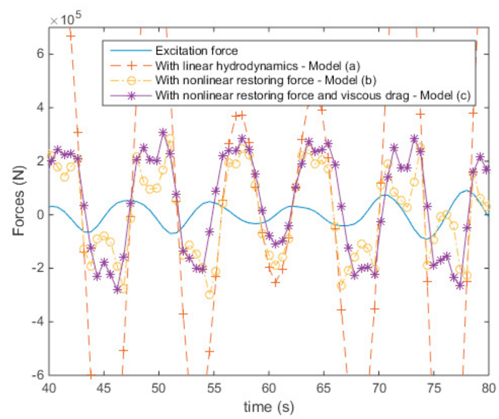
- Variant 1: the unconstrained control problem;
- Variant 2: with position limitations;
- Variant 3: with position and control force limitations.

The analysis period considered is $T = 120$s, discretised into $2N = 200$ time steps. Accordingly, the discrete frequencies are chosen so that $\omega_0 = 1/T$, and $\omega_{k,k\in\{1...N\}} = k\omega_0$. The sea state considered is a JONSWAP wave spectrum (see Hasselmann et al. (1973)) with $T_p = 8$s and $H_s = 1$m.

[4] https://lheea.ec-nantes.fr/doku.php/emo/nemoh/start



(a) Optimal WEC trajectory



(b) Optimal WEC control force

Fig. 1. Unconstrained problem

In practice, in order to solve the unconstrained, optimal control problem, a basic Newton method was used to solve (15). For the cases where constraints are introduced, a logarithmic barrier method (Bazaraa et al. (2013)) was implemented, which led to a sequence of unconstrained control problems, each of which was solved using Newton method. For the sake of conciseness, the complicated and cumbersome details of the residuals and gradient computation in Newton method are not given here.

In spite of their simplicity, the chosen optimisation tools allowed us to obtain an accurate estimate of the optimal trajectory in all the cases considered.

4.2 Numerical results

Table 1. Optimal average power (kW)

	Variant 1	Variant 2	Variant 3
Model (a)	48.1	35.1	26.8
Model (b)	49.5	35.2	34.5
Model (c)	27.6	27.0	26.9

Fig. 1 shows the unconstrained optimal trajectory and control force for the three different hydrodynamic models. It can be seen that the optimal trajectories for models (a) and (b) are identical, which illustrates the results of Section 3.1. In contrast, model (c), which presents velocity-dependent nonlinearities, exhibits a significantly different optimal trajectory, thus confirming the results of Section 3.2.

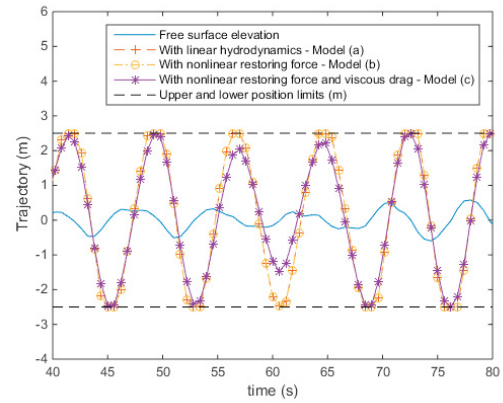


Fig. 2. Optimal WEC trajectory, with position constraints

Furthermore, the optimal control force necessary to achieve the optimal trajectory differs significantly for the three models, thus also confirming the results of Section 3.1. In theory, the optimal average power (Table 1) should be the same for models (a) and (b). The small difference observed in practice may be due to modest numerical issues due to the inherent limitation of the finite harmonic truncation. The optimal power when viscous drag is taken into account is significantly smaller, which was to be expected since control implies higher velocity values, and thus accentuates the effect of viscosity.

Of course, the unconstrained optimal control results are unrealistic, since they give way to large amplitude motions which would imply highly-nonlinear events such as slamming. Hence the interest of considering more realistic, constrained cases (Fig. 2 and 3).

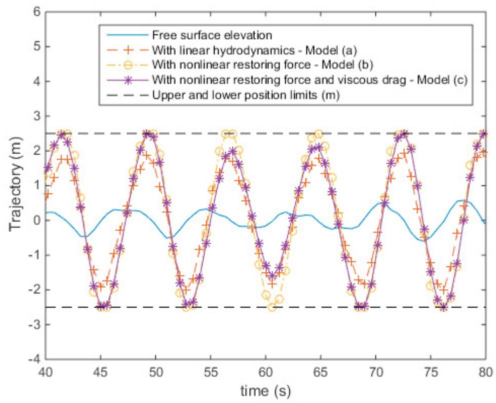
Fig. 2 shows the optimal trajectory under position constraint. As expected from the results of Section 3.3, the optimal trajectory remains identical for models (a) and (b). Furthermore, and interestingly, it can be seen that, even with constraints (that limit the range of possible velocities), the difference between the optimal trajectories with and without quadratic drag force can still be significant at times, and has a non-negligible impact on power production (see Table 1).

Finally, Fig. 3 illustrates the results of Section 3.4, showing that under force constraints, the optimal trajectory is not the same whether the restoring force is modelled linearly or non-linearly. Interestingly, it can be seen from Table 1 that the adverse effect of the chosen force constraint on the optimal average power is much stronger for the linear WEC model, than for the nonlinear WEC models. The optimal control force is also plotted, in order to make it clear that the constraint is taken into account.

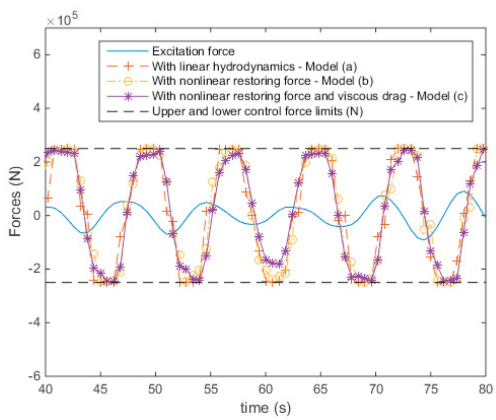
5. DISCUSSION AND RECOMMENDATIONS

5.1 Practical considerations

The results and numerical simulation methodology presented in this paper apply well to periodic signals - or signals with a long duration - but do not provide a solution



(a) Optimal WEC trajectory



(b) Optimal WEC control force

Fig. 3. With position and force constraints

for real-time control. In a practical situation, e.g. in a receding-horizon perspective, the signal cannot be considered as being periodic; therefore another type of resolution method must be adopted. The position-dependent forces don't disappear any more from the objective function and its gradient, and must therefore be taken into account when solving for the optimal control in the time window of interest.

5.2 Extension to more general WEC models

The class of WEC models considered here and described in (3), although already offering a wide range of investigation, exhibits significant limitations.

First, in some cases, for example a heaving point-absorber with a nonconstant, nonlinear cross sectional area, it may be profitable to model the excitation and restoring forces altogether as a nonlinear function, for example as $f_h(\zeta-\eta)$. Unlike terms solely depending on ζ, such a force f_h cannot be simplified in the objective function and its gradient. Thus, the results of Section 3 concerning static forces do not apply to a model of this kind.

Additionally, the cases where the WEC has several degrees of freedom, and/or is composed of two or more bodies, should be studied as well in order to determine precisely in which cases the results of Section 3 can be generalised.

5.3 Recommendations

In view of the results highlighted in this paper, a few recommendations can be expressed with regard to the modelling of optimally-controlled WECs with one DoF.

Unlike inertial and static terms, the velocity-dependent forces, such as radiation or viscous drag, play an essential role in the determination of the optimal trajectory and computation of the optimal power. It is then essential to model velocity-dependent forces accurately, spanning the whole operational space of the controlled device, which can extend beyond the range of validity of linear models.

Under constraints involving the control force, either inertia, velocity-dependent and static terms become important and must be accurately included in the resolution of the control problem.

Finally, even though the optimal trajectory may not depend on inertial and static terms in the cases detailed above, the optimal control force is always influenced by all the terms taking part in the dynamical equations.

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